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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Space Programs Summary 37-62, Vol. III

Supporting Research and Advanced Development

For the Period February 1 to March 31, 1970

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JET PROPULSION LABORATORY
CALIFORNIA INSTITUTE OF TECHNOLOGY
PASADENA, CALIFORNIA

April 30, 1970

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SPACE PROGRAMS SUMMARY 37-62, VOL. III

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Preface

The Space Programs Summary is a multivolume, bimonthly publication that presents a review of technical information resulting from current engineering and scientific work performed, or managed, by the Jet Propulsion Laboratory for the National Aeronautics and Space Administration. The Space Programs Summary is currently composed of four volumes.

- Vol. I. *Flight Projects* (Unclassified)
- Vol. II. *The Deep Space Network* (Unclassified)
- Vol. III. *Supporting Research and Advanced Development* (Unclassified)
- Vol. IV. *Flight Projects and Supporting Research and Advanced Development* (Confidential)

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I. Future Projects

ADVANCED STUDIES

A. A Miniaturized Absolute Gravimeter for Terrestrial, Lunar, and Planetary Research,

R. G. Brereton, B. H. Chovitz,¹ W. M. Greene,²
O. K. Hudson,^{2,3} R. A. Lyttleton,⁴ and R. D. Regan⁵

1. Introduction

A laboratory model of a laser absolute gravimeter is presently being operated at the Marshall Space Flight Center (MSFC). The instrument was designed and developed by O. K. Hudson⁶ of MSFC (Refs. 1 and 2). The Hudson instrument is presently being packaged and miniaturized for use as a portable gravimeter, and it will be utilized by a team of scientists¹⁻⁵ in a variety of lunar, planetary, and terrestrial applications.

2. Instrument Description

The Hudson gravimeter operates as a dynamic Michelson interferometer to make absolute measurements of

gravity accurate to 1 part in 10^7 . Basically, monochromatic light from a highly stable helium-neon laser is passed through a beam splitter (Fig. 1) which directs half of the beam to fixed corner cube reflector A and half to corner cube reflector B (the bird). Both beam halves are reflected back through the beam splitter to a photodetector, producing interference fringes at the detector.

To make the measurement, the bird is released and allowed to fall freely. The bird becomes the moving mirror of a Michelson interferometer. On release of the bird, a preset counter is actuated. As the bird falls, interference fringes move across the photodetector and are detected by the zero-crossing detector. When the preset counter reaches a full count n , it generates a start pulse for counter A and starts counting again. When the preset counter reaches n again, it generates a stop pulse for counter A and a start pulse for counter B. When the preset counter reaches a count of n again, it generates a stop pulse for counter B. The bird is caught by the catch mechanism and is lifted to the drop position. The catcher then returns to the bottom of the drop chamber.

The absolute gravity value is a digital output and may either be displayed visually or routed to the telemetry

¹United States Coast and Geodetic Survey.

²MSFC.

³Principal investigator.

⁴St. John's College, Cambridge, England.

⁵United States Geology Survey.

⁶U.S. patent 3,500,688, Gravimeter, Mar. 17, 1970 (O. K. Hudson).

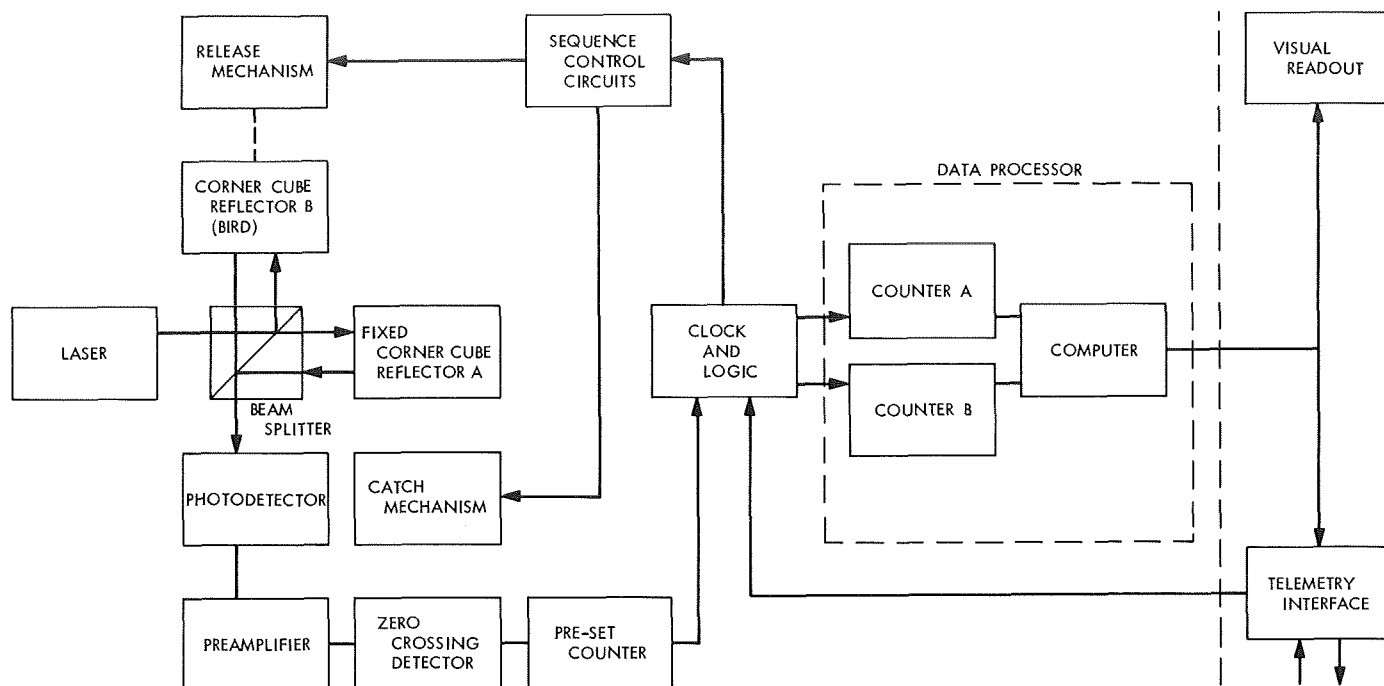


Fig. 1. Laser absolute gravimeter block diagram

interface for transmission to a remote station. Logic circuits are designed for local operation and remote operation through the telemetry interface.

For the purpose of explanation, consider a typical Michelson interferometer (Fig. 2). The mirrors in this case are corner reflectors and the light source is a He-Ne stabilized laser. Such a laser has a vacuum wavelength λ of 6329.9147 Å and a long-term stability of at least 1 part in 10^8 . This laser is a crucial part of the apparatus.

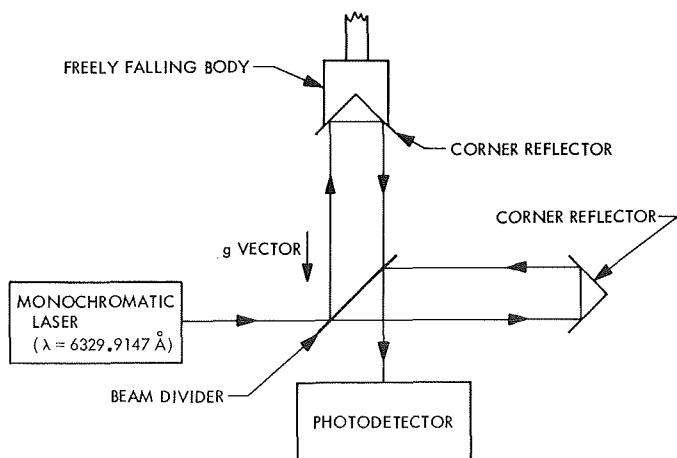


Fig. 2. Typical Michelson interferometer

As shown in Fig. 2, the upper reflector is used as a freely falling body, and it falls along the gravity vector g .

The interference phenomena present at the small aperture of the photodetector result in an electrical current proportional to the instantaneous illumination of the aperture.

a. Mathematical description. If x is the displacement along the light beam, measured positive downward (Fig. 3), then total displacement along the gravity vector g is $x \cos \theta$. The angle θ is the alignment error relative to the gravity vector. It is desirable that this angle be zero.

From the well known principles of the Michelson interferometer, maxima occur periodically with x as

$$2x = 0, \lambda, 2\lambda, 3\lambda, \dots, N\lambda$$

In general, then

$$N = \frac{2}{\lambda} x$$

and the number of maxima N as a function of displacement along the g vector is

$$N = \frac{2}{\lambda} x \cos \theta$$

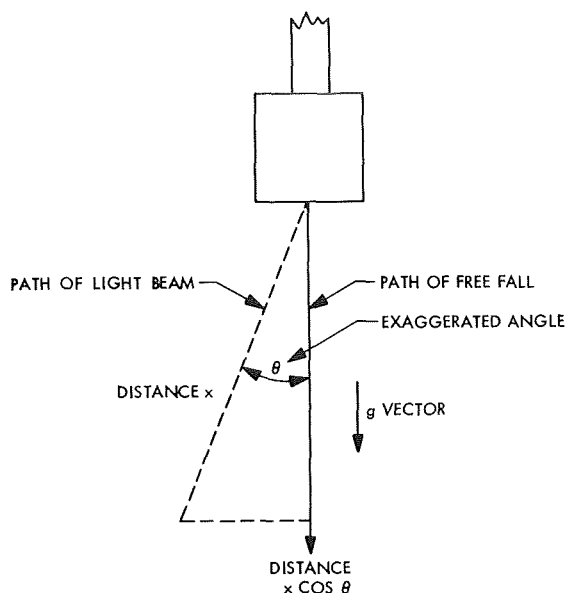


Fig. 3. Light beam displacement

It is assumed that $g \neq g(x)$ for this discussion; thus differentiating twice with respect to time, we define g :

$$-g \equiv \ddot{x} = \frac{\lambda}{2 \cos \theta} \ddot{N}$$

Subject to the initial conditions, $N = 0$, $\dot{N} = \dot{N}_0$ at $t = 0$, the general solution of this differential equation is

$$N = \frac{-g \cos \theta}{\lambda} t^2 + \dot{N}_0 t$$

Time is determined by counting the number of oscillations of a standard oscillator of frequency f , called the clock. If n is the clock pulse number, then time is determined from

$$n = \int_0^t f dt$$

or

$$t = \frac{n}{f}$$

Because n is counted digitally, it is always an integer and, hence, a quantum of error is introduced in a known way.

In terms of the observed variables, the equation of motion becomes

$$N = -\left(\frac{g \cos \theta}{f^2 \lambda}\right) n^2 + \left(\frac{\dot{N}_0}{f}\right) n$$

This equation is regarded as having two unknowns, viz., g and $\dot{N}_0$.

b. Experimental principle. Since the equation of motion contains two unknowns, we break the drop path instrumentally into two segments and solve the resulting equation pair simultaneously to eliminate N and determine g . We also allow each segment to have the same length N . This yields

$$g = \left(\frac{f^2 \lambda}{\cos \theta}\right) N \left[\frac{n_B - n_A}{n_A n_B (n_A + n_B)} \right]$$

Only n_A and n_B are measured to determine the gravitational acceleration. The quantity f is fixed by the frequency standard; the quantity λ is fixed by the stabilized laser. Both of these quantities can thus be related to atomic constants of the materials employed. The quantity θ is adjusted to be as near zero as possible.

The dynamic range of the gravimeter is essentially unlimited, and so it can measure both earth and lunar gravities. Intrinsic noise is not expected to affect output data. Seismic disturbances which might affect output data will be minimized by isolating the laser and optics from the earth or lunar surface. Errors due to the jump effect of the laser and optics when the bird is released are minimized by isolating the drop chamber from the laser and optics. Coarse leveling should be performed manually, but this can be automated, while fine leveling will be done automatically and will be accurate to within 20 arc sec.

Other features of the instrument are:

- (1) No temperature sensitivity.
- (2) No radiation sensitivity.
- (3) Complete portability.
- (4) Complete automation.
- (5) No radiation or interference of any type is generated which would interfere with other lunar experiments or modify the lunar environment.
- (6) This instrument is probably the only one that can permit a tie to be made between the lunar and earth gravitational fields. This is because of the dynamic range characteristic of the gravimeter and because absolute readings are made.

The miniaturized instrument is expected to weigh about 30 lb and so should be quite portable.

3. Applications

Surface gravity measurements have not been made on the moon; however, an orbiting spacecraft will serve as a gravity meter if it is tracked with sufficient accuracy in its orbit to measure perturbations. These types of data from the *Lunar Orbiter* series of NASA spacecraft have provided information on the equipotential figure of the moon and the location of many large-scale gravity anomalies. An example of the latter is the striking positive gravity anomalies associated with many ringed maria (Ref. 3). The true relationship of these gravity anomalies to the geology and geophysics of the lunar crust and their significance to crustal structure and evolution will require detailed surface investigations by manned or automated roving vehicles. A gravity meter will be an essential part of the integrated payload for such investigations (SPS 37-51, Vol. III, pp. 335-341). The experimental objectives and operating problems for lunar surface gravity investigations from both fixed stations and roving vehicles were discussed in SPS 37-57, Vol. III, pp. 1-6.

Lunar surface gravity observations will need to include both fixed-station and exploration-type measurements. Laser absolute gravimeter fixed-station gravity measurements will provide fundamental data towards the task of lunar geodesy, i.e., determining the size and shape of the moon, providing control points for mapping and charting work, and providing a gravity base. In addition, these stations may provide some interesting scientific data on the strength of the lunar crust and the physical properties and nature of the lunar interior by observing the moon's tides through several orbital cycles around the earth. The actual yielding of the solid earth to tidal forces is approximately a foot, but the tidal yielding of the solid moon from the pull of the earth between apogee and perigee may be larger. Optimally, this experiment should be carried out either at the limb or center of the lunar disk where tidal effects are maximum. It should be realized that the equipotential figure of the moon and the location and size of many large-scale gravity anomalies have already been identified from analyses of orbital perturbations of lunar satellites; however, surface observations can provide more details and are supplemental to further geological and geophysical investigations.

A roving vehicle capability or manned surface operations can add a whole new dimension to our task of lunar gravity observations. Specifically, traverse-type gravity observations are expected to have the following objectives:

- (1) Provide information on the anomalous gravity field for such lunar features as impact craters, volcanic

forms, mascons, ridges, faults, rilles, basins, etc. How does the gravity field for these features compare with terrestrial analogs? What is the density of lunar surface and near-subsurface material? Can lunar stratigraphic units be mapped by gravity survey techniques?

- (2) If a crust exists, provide information on the lunar crust. What is the depth variation and strength of the lunar crust? Are the maria and terra surface images of deep crustal phenomena? What is the density relationship between major crustal units?
- (3) Provide information on the possibility of lunar isostasy. Are the mountain masses ringing the Imbrium, Humorum, and other circular maria uncompensated masses supported by a strong lunar crust? What is the rate of adjustment for events ranging in age from the Imbrium through the Orientale? Was the adjustment rate different for different ages and places?
- (4) Provide information on the shape of the moon and in support of lunar geodetic data. Does the moon have a frozen tidal bulge? Is the earth-side of the moon, as indicated by tracking data, slightly depressed?

Measurements of absolute gravity by this experiment will provide *physical* data in relatively isolated areas. Since geodetic objectives involve geometric as well as physical principles, and since these objectives are global as well as local in scope, the usefulness of these measurements will depend a great deal on what auxiliary measurements can be made in conjunction, and on the area-wide extent of the measurements. To understand more specifically how these measurements can be used for geodesy on the moon, it is necessary first to consider the limitations implied by the type and extent of the data.

The broad objectives of geodesy with respect to the moon are:

- (1) To establish relative vertical and horizontal control.
- (2) To determine the size and shape of the surface.
- (3) To determine the exterior gravitational field.
- (4) To establish an absolute (i.e., mass-centered) reference system.

The first two objectives are geometric in nature; the last two are physical. Because various types of measurements yield data which are directed primarily to only the geometric, or the physical aspect, it is often convenient to separate these functions. However, the two aspects are

actually very closely related, and usually, whenever physical measurements are made, geometric data or hypotheses are implicit, and vice versa.

At this point, the value of g as an absolute measurement should be stressed. Although a uniform set⁷ of V by this method is difficult to obtain because of the need of h and a global reference level surface, these limitations do not apply to g . A value of g obtained at one station is directly comparable to that at another station, provided P at both stations is also available. Thus, the great advantage of absolute g is that no physical connection is required between stations to relate values of g to each other. Other important advantages are self-calibration, unlimited dynamic range, and ability to measure variations of g with time. On earth, all gravity measurements are checked against fixed standards, which are ultimately absolute gravity determination. On the moon, such checks will be impossible—at least at this stage of experimentation. Ultimately, when the situation becomes earth-like, with an adequate network of reference stations established, there will be a place for the more convenient spring-type gravimeters. At this stage of lunar exploration, the only meaningful values of g will be those obtained by absolute methods. It should also be noted that the high precision of spring-type gravimeters is due to their very limited dynamic range. This limitation is not a debilitating factor on earth, because the dynamic range required is well known and the calibration constants can be readily obtained. The optimum dynamic range on the moon, however, will not be initially well known, and calibration constants obtained on the earth will not necessarily apply.

The miniaturized portable instrument should be available for field testing during the fall of 1970. This instrument is expected to have a sensitivity of 1 part in 10^7 , i.e., approximately 0.1 milligals of the earth's total gravity field; moreover, continued refinements are expected to push this sensitivity to 1 part in 10^{10} or better. With such an instrument, one cannot only obtain more detailed geological data, but objectives of the study of gravitation as a physical phenomenon can be pursued. For example:

- (1) It has long been questioned whether Newton's Gravitation Law

$$F = G \frac{m_1 m_2}{D^2}$$

has second-order terms.

⁷ V = gravity potential, P = point (x, y, z) , and h = differential height measurement.

- (2) What is the correct value of G (at the present time we have only two significant figures)?
- (3) Some recent theory which postulates that G is a function of time may be investigated.
- (4) By careful long-term absolute measurements on earth and extraterrestrial bodies, along with the careful interpretation of data obtained, one may discover new astrophysical laws.

In addition to obtaining improved gravitational measurements, theoretical work is underway to modify the present apparatus in order to measure gravity gradient and determine the local direction (plumb line) of the gravity vector of the observing station. Such improvements can be expected to become incorporated in an early model of the instrument.

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B. Automated Post-Missions for the Lunar Roving Vehicle, R. G. Brereton

1. Introduction

The LRV (lunar roving vehicle) is presently being built to serve as an astronaut lunar mobility aid for the "J" series of *Apollo* missions. Because of the stretch-out in *Apollo* missions, cancellation of at least one mission, and the general reduction of funds available for lunar research, we need to re-examine lunar exploration strategy. Budget considerations require that maximum use be made of existing *Apollo* hardware, and the stretch-out schedule allows time for more thorough planning and perhaps for minor equipment changes. An obvious consideration here is other uses for the LRV. The vehicle will be left at the *Apollo* site when the astronauts return to earth. Can this vehicle be used in an automated mode, and if so, what needs to be changed on it and what may its scientific utility be? This article is concerned with some automated uses for the LRV—specifically, what can we do with the LRV after the astronaut leaves?

The weight of the LRV is 400 lb; however, it can carry nearly 1000 lb in add-on equipment. Its basic mobility is quite good and it could probably be operated "blind" in an automated mode within the *Apollo* landing area without turning over or hanging up; but some type of guidance and seeing are obviously very desirable for automated operation, especially into rougher areas outside the *Apollo* landing area. The 15-km/h maximum speed of the vehicle should probably be reduced to about 5 km/h for automated operation. The range will be contingent upon how much battery power the astronaut used, on how much astronaut time can be taken for recharge, and upon the availability of additional power sources. Communications are direct between the LRV and earth through the extra-vehicular communications system (EVCS). This unit is presently battery-powered from its own power-pack.

The question, "What can we do with the LRV after the astronauts leave?" can be most completely answered by first considering the utility of a very simple rover, i.e., an LRV that has not been extensively changed, but can perform an automated mission. Then the priorities for changes that would give the LRV a greater scientific mission capability need to be established. These changes will require money, time, and expertise, and their end product should produce the maximum utility rover that can be made from the original LRV design.

2. Simple Rover

There are a number of minor modifications that need to be made to make the LRV into a simple automated vehicle. First, the wheels must be fixed so that the vehicle will generally traverse a selected route, which could be in either a straight line or spiral path out from the origin. The route will deviate from the selected one as a function of the undulation of the terrain. Second, the vehicle's speed should be fixed for about 5 km/h and a stop/start, forward/reverse switch added. This will allow the vehicle to be stopped for science observations and perhaps backed out of hang-up or hold situations. Third, the EVCS should be modified to allow telemetry of data from scientific instruments, and to allow communications to vehicle as required to control motion. The EVCS modification is the major one for making the LRV into a simple rover and it is a primary concern for science. Fourth, a capability should be provided to recharge the LRV batteries or connect new ones. The LRV is expected to have about a 25% power reserve available after it has served its primary mission function. This could be used in the automated mode to accomplish several practical scientific objectives; however, more range and on-board power are desirable.

The scientific instruments for a simple rover mission would include only those that were available for the manned LRV sorties, and could be readily adapted to the automated mission. These would include a TV camera which is already fixed as part of the LRV, a gravimeter which will need to be mounted preferably at some point on the vehicle least subject to oscillations, and a magnetometer staff which needs to be mounted normal to the magnetic axis of the vehicle.

Apollo 11 and *12* results have indicated that the moon does have susceptible magnetic material and that a polarizing effect has produced a remnant magnetic field. This information, in conjunction with the emerging picture of complicated geology, suggests that magnetic surveys on the moon may be an important scientific tool. The configuration of the proposed magnetometer should lend itself to easy mounting; the LRV doesn't have a radioisotope thermoelectric generator (RTG) or complicated electronic equipment, so at "stop science stations" worthwhile observations can be expected from this instrument.

The anomalous lunar gravity features that have been observed need to be observed at higher resolutions from the ground in a survey or traverse mode. The combined free-air and Bouguer for the moon is less than one half of that required in terrestrial gravity surveys, and should be about 0.075 milligals/m. This means that altitude control to perhaps ± 50 m, which is attainable from pre-landing observation for a considerable area around the *Apollo* site, will correspond to corrections in the gravity data of only a few milligals. Considering the size of the gravity anomalies that the simple rover may investigate, this is quite adequate. The major problem for gravity observations from the converted LRV will probably come from oscillations of the vehicle, and information on this isn't now available. Perhaps by allowing several minutes at a science station, these oscillations can be observed with the gravimeter and compensated for, or they may become very small.

The utility of the simple rover will be to some extent dependent upon the features to be found in the environs of the *Apollo* site that cannot be reached by the manned phase, yet need to be observed at close range by cameras, magnetometer, gravimeters, etc. For example, in the Marius Hills site there are some 15 to 20 topographic features that need to be examined; however, time and range restrictions will not allow a manned sortie to each of these. The rover could be aimed to traverse some of these and so provide additional site data. Even more important, the rover could be directed to a critical area

that wasn't accessible to the man to prove an hypothesis that had perhaps evolved late in the mission.

3. Science Missions For The Simple Rover

With the above modifications, the LRV becomes a useful scientific device for lunar missions of reconnaissance, traverse, instrument deployment, and perhaps special applications:

a. Reconnaissance. This mission might be planned to take the rover in an ever-increasing arc out from the point of origin into the environs of the *Apollo* site. This simple mission could provide scientific data on the environs of the *Apollo* site that wouldn't otherwise be available.

b. Traverse. This mission would be similar to the reconnaissance mission above; however, it would be across or to particular features of interest. For example, preliminary data from the astronaut's explorations might have indicated that the *Apollo* site was on the flanks of a gravity or magnetic anomaly. The simple rover could be pointed towards the area or feature of interest and given a go signal.

c. Instrument deployment. The vehicle would be programmed to transport sensors, as part of an array, to remote parts of the *Apollo* site. Distance of spread and not geometric point of deployment would be a factor in experiment selection. Examples here might be an RGM⁸ package or series of seismometer sensors.

d. Special applications. As an example of one special application, we might consider an active seismic experiment. Present data on the moon indicate that head-waves are transmitted through the lunar regolith just as they would be through terrestrial soils; however, at greater distances and with a large seismic source, the signal is anomalous as compared with terrestrial models. This suggests that there is a critical velocity/depth function that needs to be investigated. The active seismic experiment for the manned *Apollo* missions probably will not be able to investigate the phenomenon because the distance between seismic source and sensor will be too small; but by using the automated rover, with a supply of charges to generate seismic waves, in conjunction with the ALSEP⁹ or MALSEP⁹ fixed seismometer, valuable data may be obtained. This is a most critical experiment for understanding the structure and evolution of the lunar surface.

⁸RGM = remote geophysical monitor.

4. Modifications

The modifications discussed above are considered minimal for making the LRV into an automated rover. They represent the least possible dollars and equipment change, in terms of a significant increase in the potential for science return. If the money and time are available, the LRV can be further modified to substantially increase its scientific utility. Some suggested further LRV modifications and their priority are discussed below. It should be realized that increases in the scientific payload or operational complexity for any automated mission will require increased data handling and telemetry capabilities. Thus, there will be system requirements for data storage, a steerable antenna, more power for telemetry, etc. On-board equipment to handle new rover mission requirements can be expected to increase with each major LRV modification.

a. Guidance system (seeing and steering). The addition of a guidance system is the first priority major change for making the manned operated LRV into an efficient automated rover. The guidance system will consist of several integrated parts, viz., an imaging system on the rover for viewing the lunar scene, a high data rate telemetry channel, an SFOF⁹ control link where the lunar images can be viewed and assessed, and finally a control mechanism on the rover. The guidance and control system for an automated rover has been described in two JPL documents.^{10, 11}

b. Science imaging system. Scientific observations from an automated rover on the lunar surface require an observing capability equal to that of a human observer using his unaided eye for panoramic viewing, using a hand-held magnifying glass of 10 power for specimen analysis, and finally using a telescope of 10 power for viewing select portions of the panorama. The requirements and objects for a roving imaging system have been discussed in SPS 37-55, Vol. III, pp. 258-263; SPS 37-60, Vol. III, pp. 1-3; and a JPL document.¹²

c. Extra range. The first modification that should be considered is to allow for a recharge of the existing LRV batteries. This would give the vehicle a range in excess of

⁹ALSEP = *Apollo* lunar surface experiments package; MALSEP = Modified *Apollo* lunar surface experiments package; SFOF = Space Flight Operations Facility.

¹⁰A Study of Lunar Traverse Missions, Sep. 16, 1968 (JPL internal document).

¹¹LRV Navigation and Guidance System Phase "A" Study Report, Oct. 15, 1969 (JPL internal document).

¹²Experiment in Remote Geological Reconnaissance and Landmark Navigation, Sep. 30, 1969 (JPL internal document).

120 km. With the modifications discussed in Footnotes 10 and 11, scientists would have a tool for probing the lunar surface far from each restricted *Apollo* site. The second modification would be to add batteries or more electrical storage capability. The third and perhaps ultimate modification would be to incorporate an electrical recharge capability in the rover itself by the addition of solar panels or an RTG unit. Power systems for an automated rover are described in a JPL document.¹³

d. Surface sampling system. A sampling instrument similar in function and design to the *Surveyor* sampler is very desirable. The instrument must be capable of picking up selected small rock fragments or particulate material under the direction of a TV viewer. It should have sufficient articulation to allow for scratching, trenching, or chipping at the lunar surface on outcrops to allow for acquisition of selected samples from the surface, and finally to present acquired samples to the imaging system or other analytical instruments (SPS 37-55, Vol. III; SPS 37-60, Vol. III; and Footnote 12).

e. Improved science. A roving vehicle that can be guided from point to point on the lunar surface has an imaging system to serve as the eyes of the remote observer, a sampling system to serve as his arms, and finally a range measured in several hundred kilometers, which is a powerful tool for lunar geological reconnaissance. The utility of

this vehicle can be greatly increased by modifying existing LRV scientific experiments and adding others. For example, the gravimeter that was used for the simple rover mission should be mechanized to provide deployment for more stable readings; also, the capability to measure altitude and terrain effects needs to be incorporated into the experiment. Perhaps a new instrument design that avoids the problems inherent in calibration and drift of conventional gravimeters might be considered.

f. Night-time survival. The ability to survive through the lunar night—to live to work another day—is obviously a most desirable feature for the rover.

5. Conclusions

We have only considered the science utility of the simple rover in terms of available scientific experiments from the manned phase of the mission, and this is apparently quite high; however, the simple rover can also serve a variety of instrument deployment and special missions. Some of these may produce data that can only be obtained by means of this mobile mechanism. Finally, given the major modifications suggested in *Subsection 4*, the vehicle's capability can be increased in step functions to perform almost as we had envisioned the DLRV^{14 15} would.

¹³Science Systems for Mobile Lunar Surface Exploration, Summary of Phase "A" Study Results, June 2, 1969 (JPL internal document).

¹⁴Dual-mode lunar roving vehicle—a roving vehicle designed to transport the astronauts and also to do a 1000-km automated science mission.

¹⁵Science Ground Data System and Science Operations Organization From the DLRV Mission, Phase "A" Study Report, Sep. 31, 1969 (JPL internal document).

II. Space Instruments

SPACE SCIENCES DIVISION

A. Gamma Ray Spectrometer for Apollo 16,

L. L. Lewyn

1. Instrument Development

The gamma ray spectrometer (GRS) is housed in a cylindrical thermal shield and structural assembly which is approximately 10 in. in diameter and 19 in. long (Fig. 1). Inside the thermal shield are two major subassemblies: a gamma ray detector (GRD) and the gamma ray electronics (GRE) package, shown in Fig. 2. The detector design is based on a mechanical arrangement devised by the University of California, San Diego (UCSD), and developed into its present configuration by the Radiation Physics Group at JPL. The electronics design is based on a prototype developed by the Analog Technology Corp., Pasadena, under contract to UCSD. The effort was technically directed and monitored by the Nuclear Instruments Group at JPL. The thermal shield and structural assembly were designed by the JPL Engineering Mechanics Division.

2. Functional Description

A functional diagram of the GRS is shown in Fig. 3. The gamma ray flux will be detected by means of a scintillation detector. The kinetic energy of gamma rays traversing the NaI(Tl) scintillation crystal is dissipated by various ionizing processes and subsequently transformed into

light. The light is reflected and transmitted into a 3-in. diameter photomultiplier tube coupled to the scintillator. The output of the photomultiplier is an electrical pulse (57 pC full scale) which is processed within the electronics subassembly.

In the electronics subassembly, the signal is amplified, shaped and digitized. The 9-bit digital words representing the pulse height of each event are commutated into the instrument 4-kilobits serial data output, which is routed to the spacecraft telemetry transmitter.

Not all the events producing scintillations within the NaI crystal are processed by the analog-to-digital converter within the electronics package. When the converter is busy processing a prior pulse, or when the converter is waiting to transfer a pulse-height word to the output data buffer register, no new pulses may be processed and the converter is "dead." The fraction of time during which the converter is not dead must be determined so that the true input counting rate of the instrument may be deduced from the output data. This fraction of time is called "live time." During the live time, fixed period clock pulses are gated into an internal counter. When the converter is busy processing prior pulses, the gate is disabled and the counter stops counting. The live time counter output is commutated into the instrument serial data output.

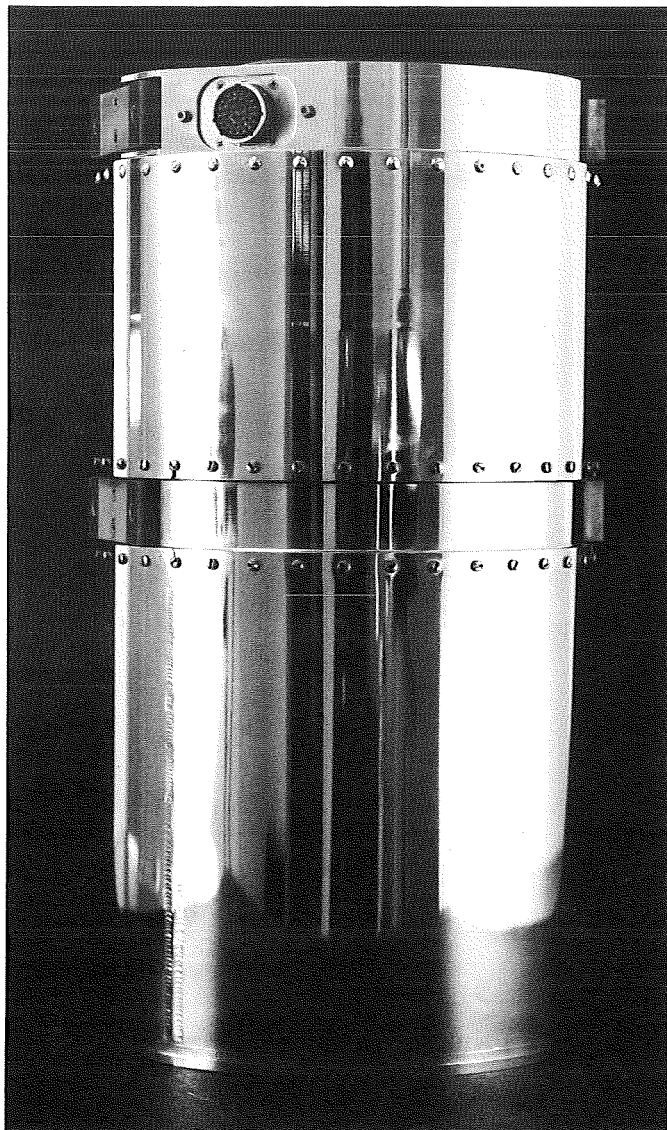


Fig. 1. Apollo 16 gamma ray spectrometer

Some of the events producing scintillations with the NaI crystal result from cosmic ray interactions. A plastic scintillator (Pilot B) cup surrounding the NaI crystal in the detector is used to reject cosmic rays and other charged-particle background events. Light from the plastic scintillator is optically isolated from the NaI crystal and coupled to a 1.5-in.-diam photomultiplier mounted at 90 deg with respect to the photomultiplier in the gamma ray analysis channel. The 1.5-in.-diam photomultiplier signals are amplified and routed to a sensitive threshold discriminator within the electronics subassembly. Signals greater than the 1.8-pC discriminator threshold are designated as "shield events" and will reject simultaneously occurring events in the gamma ray channel. All shield signals are

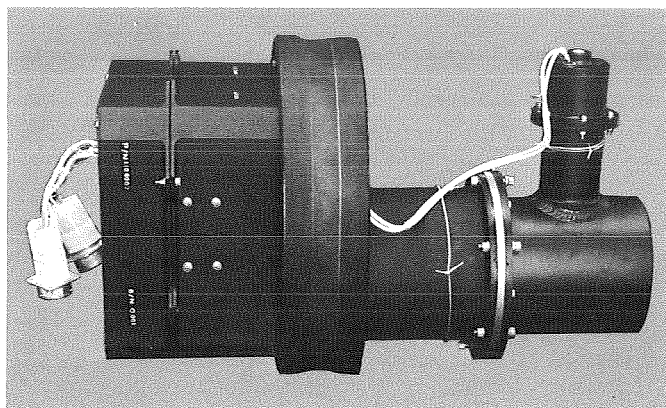


Fig. 2. Apollo 16 gamma ray detector and electronics

counted by the shield event counter. Shield signals which also reject a simultaneous event in the gamma ray channel are counted in the "coincidence event" counter. Both counters are commutated into the instrument serial data output. An astronaut-operated switch contact closure within the command module may be used to override the reject action of the shield signals.

Each photomultiplier in the detector is operated from a separate high-voltage supply. The 1.5-in.-diam photomultiplier operates at a fixed preset voltage and the 3-in.-diam photomultiplier is programmable to any one of eight steps within a preset range. An astronaut-operated switch contact closure within the command module steps the high-voltage supply powering the 3-in.-diam photomultiplier. The state of the high-voltage supply is controlled by a 3-bit digital register. The state of the register is commutated into the instrument serial data output.

Temperatures within the gamma ray detector are maintained at $15 \pm 5^\circ\text{C}$ by an active temperature controller. The controller is a bang-bang servo loop with 1°C hysteresis. The control thermistor is mounted near the 3-in.-diam photomultiplier cathode, and the heater element is contained in a silicone rubber jacket surrounding the detector assembly. Separate thermistors in the detector and electronics provide housekeeping temperature monitoring over the range -40 to $+60^\circ\text{C}$. The thermistor signals are routed to buffer amplifiers which furnish the telemetry system with 0- to 5-V outputs.

The instrument consumes 1.1 W of power when the heater is not operating. The low-voltage power supply uses a switching-mode regulator to convert the spacecraft power into the instrument operating voltages at nearly constant efficiency. This regulator is followed by precision series regulator providing 0.1% stable references for

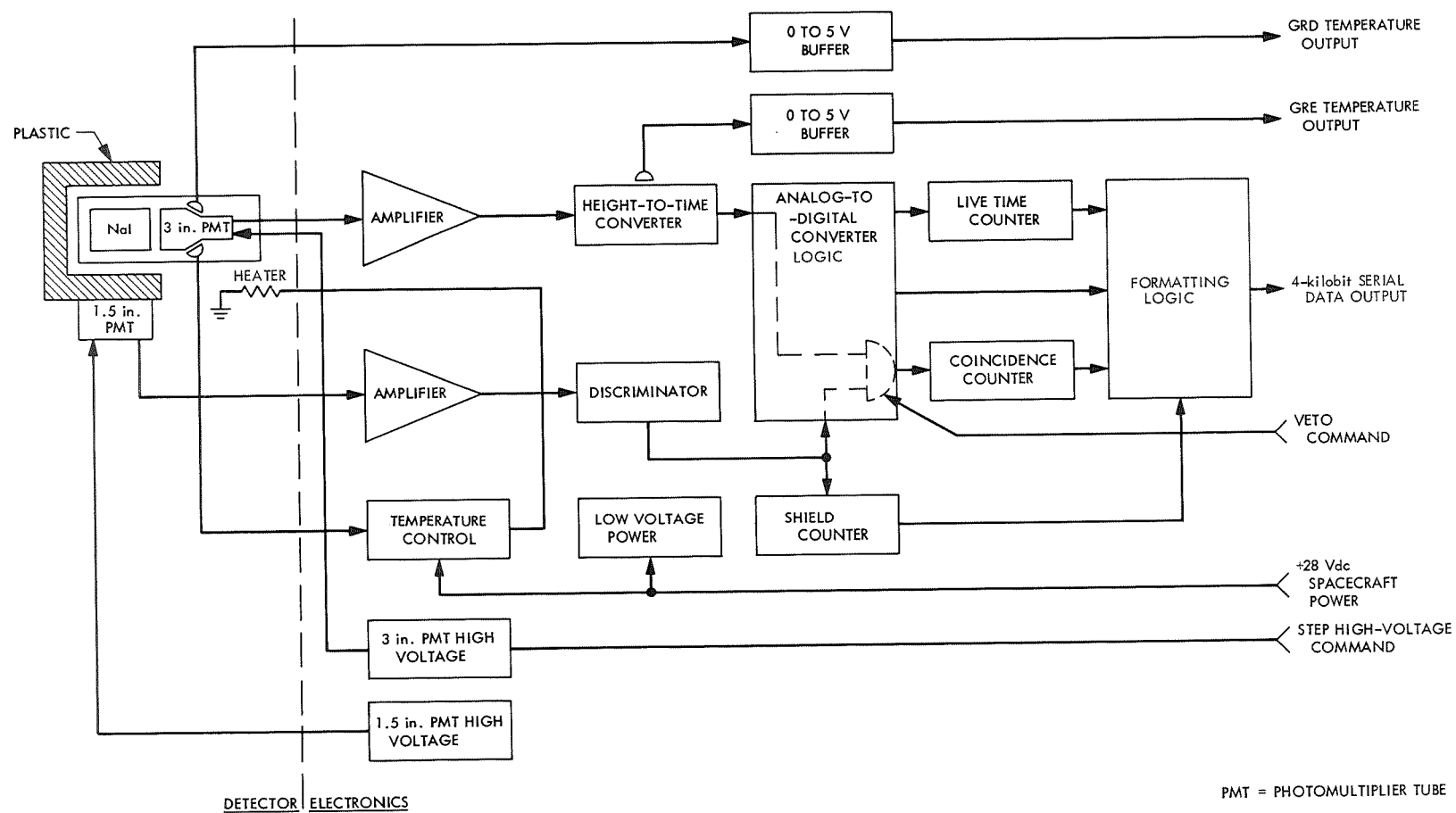


Fig. 3. Apollo 16 gamma ray spectrometer functional diagram

the system. The heater consumes 20 W of power and is switched into the spacecraft 28-V supply on demand.

The total instrument weight is 22 lb. The detector weighs 8 lb; the electronics weighs 7 lb; the thermal shields, brackets and cables weigh 7 lb.

3. Technical Approach

The detector is assembled by the Radiation Physics Group at JPL. The electronics package, structural assembly, and thermal shields are assembled at Analog Technology Corp. After extensive developmental tests are performed on the separate subassemblies, they are integrated and returned to JPL for environmental testing. When environmental testing is complete, the spectrometer is calibrated at JPL and prepared for delivery to the Manned Spacecraft Center.

B. Polarizers and Retardance Plates for a Vector Helium Magnetometer, H. C. Howarth

1. Introduction

Circularly polarized light is necessary to optically pump the atoms in the He cell of the vector helium magnetometer. Circularly polarized light is obtained by passing the 1.083- μ lines of the light from a helium lamp through a linear polarizer and then passing the linearly polarized light through a retardance plate. In practice, it is most efficient to bond the retardance plate to the linear polarizer and term the result a "circular polarizer." In order to obtain optimal thermal capabilities and transmittance to extinction ratios, a more thorough understanding of the theory and manufacturing techniques of circular polarizers had to be undertaken.

2. Retardance Plates

All light traveling in the Z direction (Fig. 4) can be resolved into the X and Y vibration planes. Define the refractive index of the material in the X plane as η_x ; likewise η_y is the refractive index in the Y plane. The difference between η_y and η_x is defined as the birefringence $\Delta\eta$.

The amount by which the component vibrating in the X plane lags the component in the Y plane is defined as the retardance Γ of the material.

Γ is equal to the product of the birefringence and d , the thickness of the material. Since the birefringence is a func-

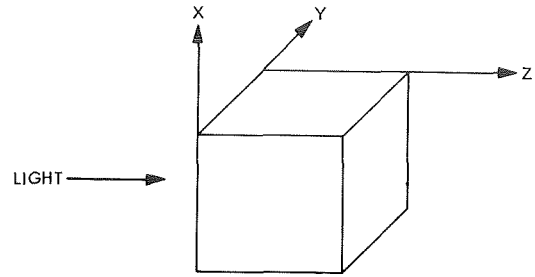


Fig. 4. Light entering retardance plate

tion of the wavelength λ , the retardance is also a function of λ . Thus, the equation for retardance becomes

$$\Gamma(\lambda) = \Delta\eta(\lambda) d$$

For most materials with which we are concerned, i.e., mica and quartz, the refractive indices follow a smooth known curve over the frequency range of interest. Figure 5 shows the relative curves.

It can be seen that even though η_x and η_y vary considerably with λ , $\Delta\eta$ remains a constant. Thus, for a given retardance plate, a wide range of frequencies (including the visible spectrum) can be used to evaluate the value of the retardance. That is, even though the vector helium magnetometer uses 1.083- μ light, visible light can be used to accurately determine the retardance of the plate.

For some materials a singularity in the refraction index can occur. This singularity can be inherent in the material or can be introduced by doping. Furthermore, this singularity is usually only in one plane and is directly related to the absorption characteristics of the material (Fig. 6).

Obviously, since Γ is a function of λ , the plate will only be a $\frac{1}{4}\lambda$ plate at one specific frequency. Thus, when materials that exhibit a singularity in either the X plane or Y plane are used, 1.083- μ light must be used to accurately

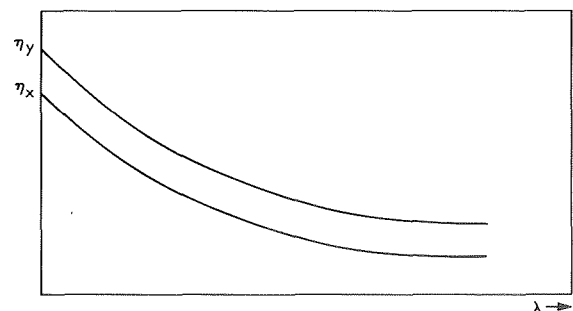


Fig. 5. Typical refractive indices curves

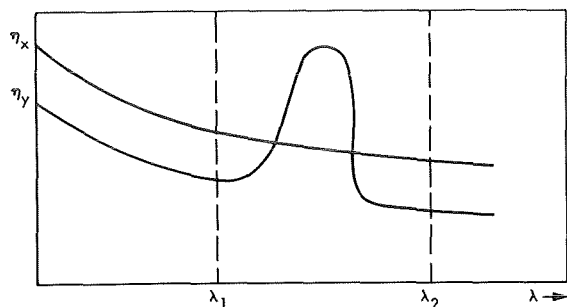


Fig. 6. Typical refractive indices with a singularity in the Y plane

determine the retardance. Since a plate of this type will be a $\frac{1}{4}\lambda$ plate at only one specific frequency, the term "retardance plate" is a preferred term.

To optimize the optical pumping in the helium cell of the vector helium magnetometer, 100% circularly polarized light must be used. The degree to which light is circularly polarized is proportional to the accuracy to which the retardance plate is a $\frac{1}{4}\lambda$ plate at 1.083μ . The most common method of obtaining an exact $\frac{1}{4}\lambda$ plate from a general retardance plate is by rotating the plate about its fast axis (largest refractive index) and slow axis (smallest refractive index). A rotation of up to 45 deg can be made with no adverse effects. The equation for this is

$$\Gamma(\lambda) = \Delta\eta(\lambda) \frac{d}{\cos \theta}$$

where θ is the angle of rotation.

3. Polarizers

H-film material is polyvinyl alcohol (PVA) plus iodine. With the application of heat, PVA is dehydrated, causing the double bonds to alternately bridge two different carbon atoms. The dehydrated PVA is doped with an iodine solution to give H-film material.

For the vector helium magnetometer, the highest transmission of polarized light possible should be obtained. The greater the transmission value of the polarizer the greater the signal for a given magnetic field, and thus a higher signal-to-noise ratio.

Also, as the extinction (absorption) value of the polarizer decreases, the amount of polarized light will increase. A typical curve for H-film material is shown in Fig. 7. By doubling the iodine concentration, one gets the dotted lines shown in this figure. Thus high concentrations of

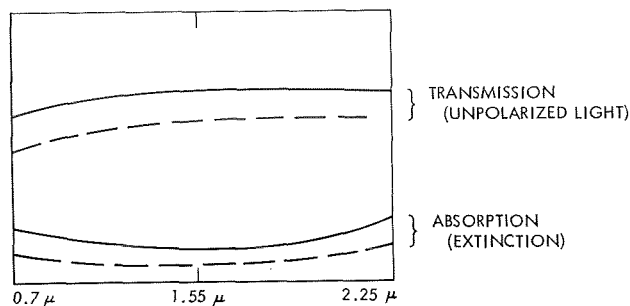
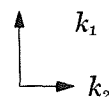


Fig. 7. Typical curves for H-film material

iodine (or more than one polarizer plate) gives better extinction values but at the cost of lower transmission.

Schematically, the waveform passing through the polarizer has the following characteristics:



where k_1 is the magnitude of the polarized light and k_2 is the magnitude of the unpolarized light.

Transmission of unpolarized light is defined as K . Thus,

$$K = \frac{k_1 + k_2}{2}$$

The most common method of increasing extinction is to place two plates parallel to each other. In this case the transmission function (H_0 for parallel plates, H_{90} for crossed plates at 90 deg) becomes

$$H_0 = (1 + V^2) K^2$$

$$H_{90} = (1 - V^2) K^2$$

where V is the degree of polarization.

The degree of polarization

$$V = \frac{k_1 - k_2}{k_1 + k_2}$$

is typically 99.99%. For a pair of identical sheet polarizers with their axes parallel, the transmittance for nonpolarized light is given by

$$H_0 = \frac{k_1^2 + k_2^2}{2}$$

Since $k_2 < 1\%$

$$H_0 = \frac{k_1^2}{2}$$

With axes crossed, the extinction value is

$$H_{90} = k_1 k_2$$

The transmittance of polarized light for a single-sheet polarizer rotated about its own plane is

$$T_\theta = (k_1 - k_2) \cos^2 \theta + k_2$$

The transmittance for nonpolarized light of a pair of identical sheet polarizers is given by

$$T_\theta = H_0 \cos^2 \theta + H_{90} \sin^2 \theta$$

Or substituting $1 - \cos^2 \theta$ for $\sin^2 \theta$

$$T_\theta = (H_0 - H_{90}) \cos^2 \theta + H_{90}$$

Because of the extremely small value of k_2 , the following approximations hold:

- (1) The principal transmittance k_1 of a single sheet polarizer for polarized light is equal to twice the average transmittance value K for nonpolarized light: $k_1 = 2K$.
- (2) The transmittance H_0 for nonpolarized light of a pair of sheet polarizers with their axes not parallel is given by $T_\theta = H_0 \cos \theta$ where θ is the angle between the axes.

Thus from the above relations, one can accurately predict both the transmission and extinction values of any array of polarizers used in the vector helium magnetometer.

4. Mating Retardance Plate With Polarizer

The slow axis of the retardance plate must be 45 deg off the polarizer axis (and to the right) for the right circular light (Fig. 8). In actual fabrication, the alignment of the polarizer with the $\frac{1}{4}\lambda$ plate can only be held to ± 3 deg. This error will give a slightly elliptical array.

The bonding compound is either epoxy or cellulose acetate butyrate. Epoxy exhibits no significant outgassing.

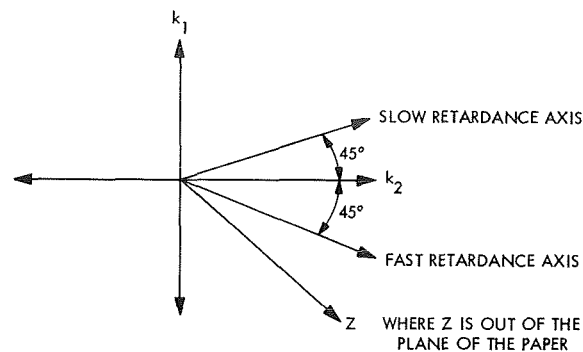


Fig. 8. Relative positioning of linear polarizer and retardance plate

The Naval Research Laboratory has conducted tests on cellulose acetate butyrate, and no significant outgassing was reported. The advantage of cellulose acetate butyrate is its low-stress optical constant. The cellulose acetate bond is equal and opposite to the acetate butyrate bond, thus there are no adverse absorption bands.

Any stress put on glass covers cause adverse absorption bands which cut down on the transmission of polarized light. It is recommended that glass covers not be used if at all possible. Coatings that will prevent damage to the polarizers caused by adverse environmental conditions should be used.

5. Conclusions

Due to the manufacturing process (in which heat is used to dehydrate the PVA) there is no possibility that H-film polarizers will withstand temperatures greater than 100°C without adversely affecting the iodine and hydroxide concentrations. Other types of polarizers must be used to sustain higher ambient temperatures in the vector helium magnetometer.

By rotating the retardance plate about its axes, an exact $\frac{1}{4}\lambda$ plate can be obtained, and therefore, a more effective and accurate circular polarizer for the vector helium magnetometer can be fabricated. Finally, more attention must be paid to mounting techniques for polarizers and cover glasses to eliminate adverse optical stress absorption bands.

Reference

1. *Polarized Light*, Publication FT3374A, Polaroid Corp., Cambridge, Mass., Feb. 1967.

III. Science Data Analysis

SPACE SCIENCES DIVISION

A. Tests of Instruments Proposed for the Scientific Payload of a Lunar Rover, J. N. Strand

1. Introduction

The tests of the instruments discussed in this article were performed by the Advanced Lunar Studies (ALS) team. These instruments were acquired by the ALS team for testing purposes only, and are not intended to be used as flight hardware. However, they were meant to be typical of those instruments that may be used on a future lunar rover. The instruments tested are as follows:

- (1) Surface sampler.
- (2) TV camera.
- (3) Auger system.
- (4) Chipper-and-tongs device.

The tests were conducted at the Science Experiment Test Laboratory (SETL). The purpose of the SETL is not primarily to test equipment, but to test instrument concepts, capabilities, and operating procedures. The use of the SETL to test concepts for the *Surveyor* program is described in Ref. 1.

2. Television Camera

A commercially available TV camera was purchased to provide the test operator with a view of the operations. In the SETL configuration, this TV view can be augmented by looking through the window just above the console or by walking into the operations area, about 25 ft away. These viewing options are shown in Fig. 1.

The TV camera has been used with a beam splitter as shown in Fig. 2. The purpose of the beam splitter is to provide a stereo base line. The beam splitter is composed of two mirrors 6-in. apart. These mirrors reflect the image into a prism system that allows the camera to see a split image, i.e., the left-half of the camera views an object as seen by the left mirror and the right-half of the camera views an object as seen by the right mirror. This split-image effect can be seen in the TV monitors shown in Fig. 1.

A stereo viewer is available for the TV monitor that allows an experimenter to use his left and right eye, individually, to view the left- and right-hand sides of the TV monitor, respectively. This results in the experimenter obtaining a three-dimensional view of the test area. However, tests of this three-dimensional view were disappointing because, on some attempts, it appeared that the surface



Fig. 1. TV viewing options

sampler was directly over an object when it was actually more than a foot away. Therefore, this concept must be modified to provide for accurate depth perception.

The aiming of the TV camera, by means of the pan-tilt mechanism visible in Fig. 2, is controlled at the test console.

3. Surface Sampler

The soil mechanics surface sampler, obtained from the *Surveyor* program, was used to determine the problems

involved with picking up objects and to find means of circumventing these problems. To enable tests of data handling alternatives, the sampler was fitted with three potentiometers to measure the sampler's azimuth, elevation, and arm extension.

For effective deployment, the surface sampler was mounted on the *Surveyor* 18 in. above the surface. However, on a lunar rover test bed, the sampler would be mounted considerably higher than 18 in. and would not be able to reach the surface in some configurations. To eliminate this problem in the SETL, the sampler was mounted

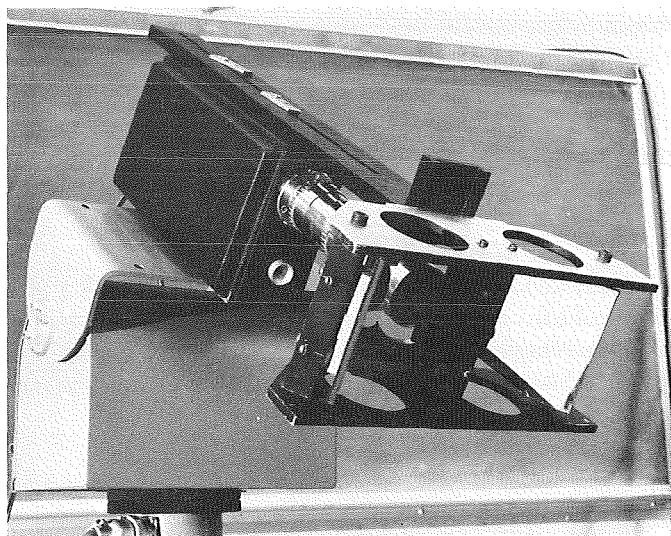


Fig. 2. TV camera—beam splitter configuration

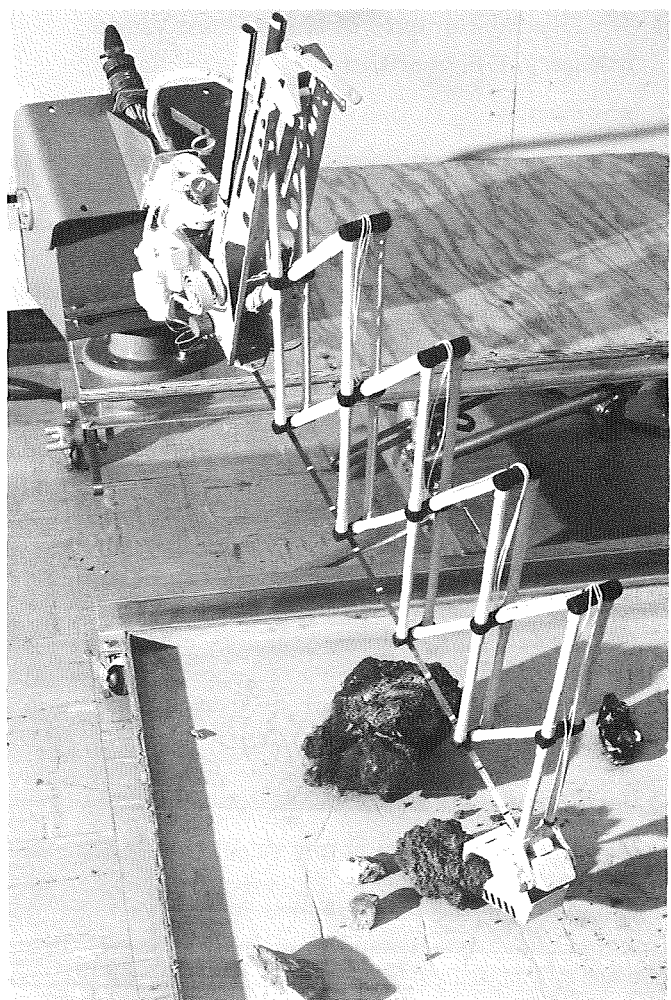


Fig. 3. Surface sampler configuration

on a pan-tilt mechanism. The surface sampler is shown in Fig. 3.

Accurate perception of distance through the TV monitor was a problem in the SETL. This was as expected since, on *Surveyor*, some problems were experienced in trying to position the sampler above a specified object. One solution to the problem of sampler positioning was the use of a beam splitter on the TV camera, which has been discussed in *Subsection 2*.

4. Auger System

It has been proposed that an auger system be included as part of the lunar rover scientific payload. This auger would be employed to drill holes into the lunar surface thus enabling the surface sampler to obtain samples of freshly excavated lunar subsoil. To test this concept, a commercially available auger (Fig. 4) was acquired. However, to avoid excessive weight, it was decided to replace

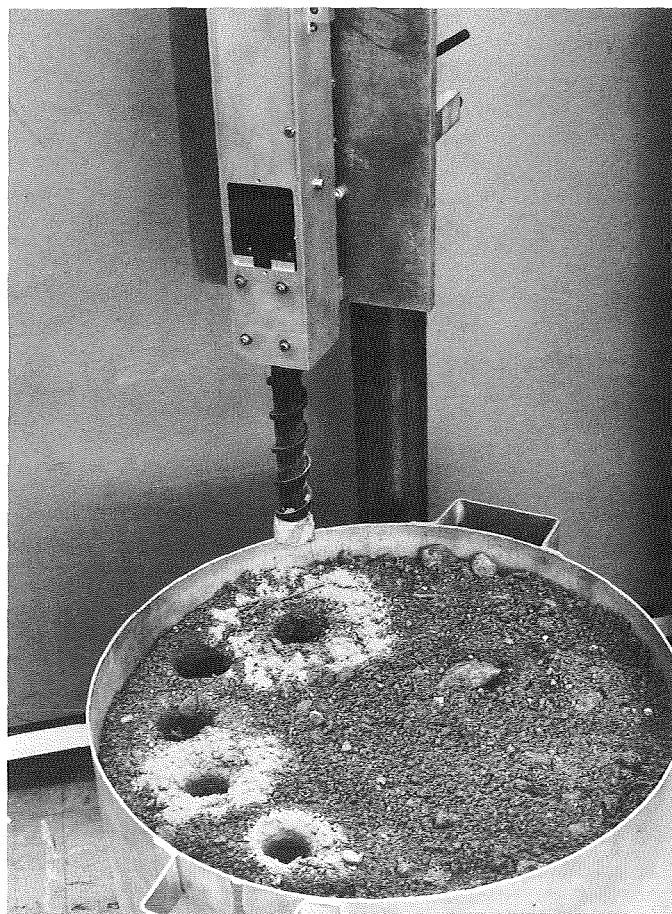


Fig. 4. Auger system configuration

the standard 13-lb steel auger bit with a 2-lb bit composed of an aluminum shaft with a steel tip.

The auger system was equipped with a force-sensitive safety feature, i.e., when the resistance encountered by the auger exceeded 20 lb, the auger motor was turned off. Unfortunately, since the internal friction of the auger approached 20 lb, the auger did very little digging when the safety switch was operative.

5. Chipper-and-Tongs Device

For use in these lunar instrument studies, a combination chipper-and-tongs device (Fig. 5) was obtained and attached to the end of a Ryan boom. (A Ryan boom is an arm which can be wrapped around a drum for compact storage but which is reasonably rigid when extended.) As part of the lunar rover scientific payload, the chipper could be used to chip off a piece of rock and the tongs used to pick up objects of interest.

During testing, it was found that, under earth gravity, the weight of the chipper-and-tongs device cause the Ryan

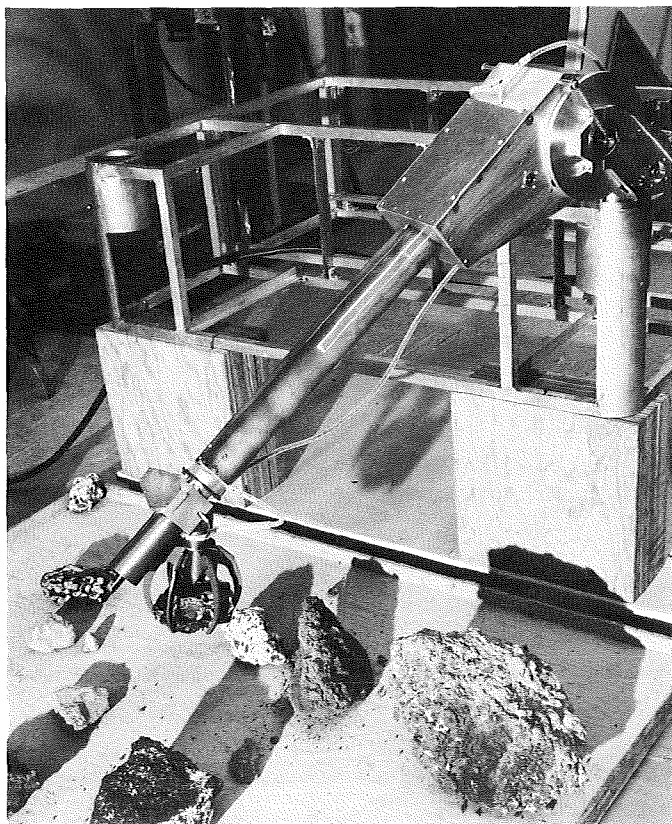


Fig. 5. Chipper and tongs device—Ryan boom configuration

boom to lose rigidity after an extension of more than 36 in. and it was necessary to support the boom externally to prevent it from collapsing. This problem was anticipated, however, and the concept of a compactly storable boom was found useful.

6. Concluding Remarks

Application of the SETL test philosophy has identified the problems associated with the instruments and operating procedures and concepts discussed in this article. The SETL philosophy has also been useful in testing the concepts that could solve these problems.

Reference

1. Peer, W., *The Surveyor Experimental Test Laboratory*, Rev. 1, Technical Memorandum 33-445. Jet Propulsion Laboratory, Pasadena, Calif., Jan. 15, 1970.

B. The Digital Removal of Noise From Video Imagery, T. C. Rindfleisch

1. Introduction

As all scientific instruments influence the data they collect, so do television cameras leave their signatures on the imagery they record. A scientist analyzing the raw spacecraft pictures returned from Mars or the moon is actually seeing the surface through the camera's eye and any distortions introduced by camera processes potentially affect the analyses of the data. It is one of the major functions of digital image processing to remove the various distortions from the raw data (using camera performance characteristics) so that scientific investigations can be based upon imagery as representative of the planet itself as possible. One of the most important elements of this correction process is the suppression of noise so that subsequent enhancement processing and analysis may be performed on maximum signal-to-noise ratio imagery. The purpose of this article is to describe computer techniques that have been developed to remove various types of structured noise from images and to illustrate their application to a *Mariner* Mars 1969 photograph.

2. Description of the Problem

The precise separation of any signal from a composite signal must be based on one or more quantifiable characteristics of the signal of interest that distinguish it uniquely from the other signal components. In most real situations, one has only statistical information about the various components of a signal and thus, even in the theoretical limit, their separation is approximate. This is certainly the case

in the data output of spacecraft video systems where the dominant signal represents a complex natural scene and the noise components range from wideband thermal noise to highly structured periodic noise. The essence of video noise removal is to isolate and remove the various identifiable noise components as rigorously as possible so as to do a minimum of damage to the actual video data. In most cases, the errors introduced to the real signal by the removal process, while small, vary from point to point and are impossible to measure meaningfully since very little is known in detail about the scene being photographed and the efficacy of removal is data dependent. In the following subsections, the removal of periodic, long-line, and spike noises will be described.

3. Periodic Noise Removal

In electronic video systems, a common noise problem arises from the coupling of periodic signals, related to the raster scan and data sampling drives, into the low-signal portions of the video-handling electronics. These noises are generally introduced when the video scene is represented by a one-dimensional temporal signal. Because of the periodicity of this type of noise, a useful method for characterizing it is in terms of a Fourier decomposition. A digitally computed Fourier power spectrum of a test scene photographed by a *Mariner* Mars 1969 camera is shown in Fig. 6. The spectrum was computed from the

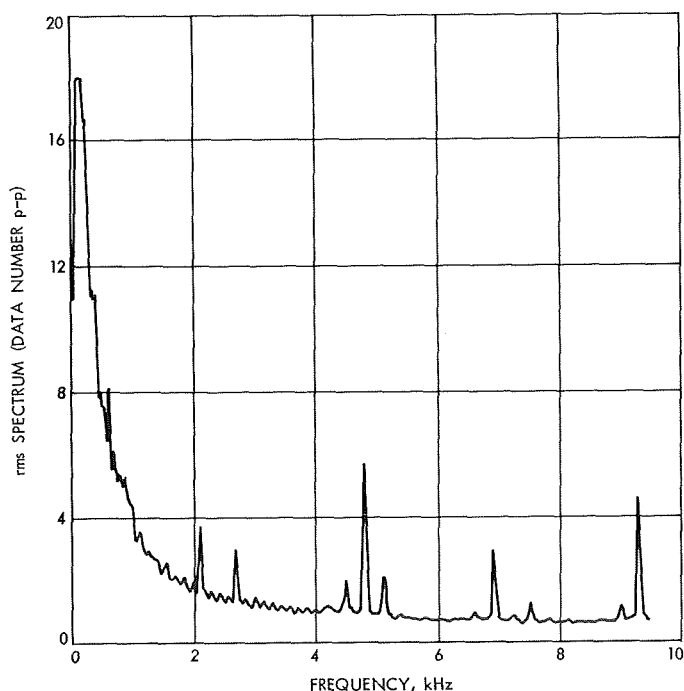


Fig. 6. *Mariner* Mars 1969 rms power spectrum

one-dimensional scan output of the camera. The power spectrum clearly shows a smoothly varying background decreasing in amplitude with increasing frequency and representing the true video spectrum on which is superimposed a number of narrow, abnormally large spikes. These spikes are the various components of the periodic noises present in the *Mariner* system and are related to multiples of the 2400-Hz spacecraft power supply frequency.

For well designed systems, these periodic noises exhibit phase coherence over times that are long as compared to the frame time of the camera. For this reason, when the two-dimensional image is reconstructed, the periodic noise appears as a two-dimensional pattern exhibiting periodicity not only along the scan lines, but perpendicular to them as well. If one computes a two-dimensional Fourier transform of a reconstructed picture, as shown in Fig. 7, this two-dimensional structure becomes evident. Again, the actual video signal spectrum appears as a continuum falling off in amplitude (brightness) toward high spatial frequencies and the noise components appear as well defined two-dimensional spikes.

Figure 7a is a raw photograph of Mars returned by a *Mariner* VI camera. Note the diagonal bands of periodic noise. Figure 7b is a digitally computed two-dimensional Fourier spectrum of a portion of Fig. 7a. The absolute spectrum amplitudes are displayed as gray levels increasing from black to white. The dc point of the spectrum is in the center of Fig. 7b. Positive and negative horizontal spatial frequencies are displayed to the right and left of dc, respectively, and positive and negative vertical frequencies above and below.

Because both the noise and the scene exhibit two-dimensional correlation, a more precise removal can be achieved by filtering the noise from the two-dimensional Fourier space than from the one-dimensional space. This can be seen by considering the effects of one-dimensional filtering as observed in the two-dimensional transform space. Since the removal of a one-dimensional frequency component is made irrespective of its vertical variation, the corresponding spectral components removed in two-dimensional frequency space lie in a vertical bar. Thus, since the noise component is contained in one spike along the bar, a large amount of video signal is unnecessarily removed.

Removing the two-dimensional spikes, one achieves a first-order clean-up of the periodic noise components. By this method, only the portion of the component spectral

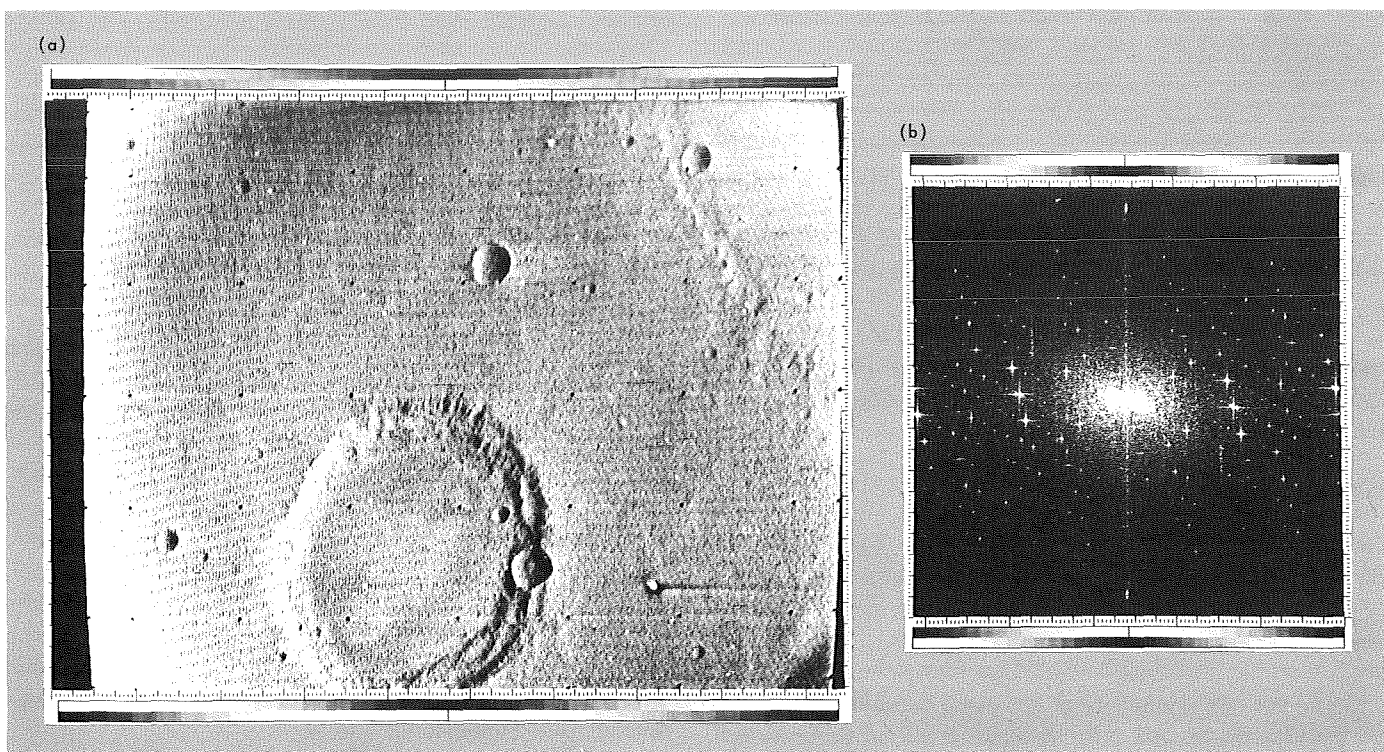


Fig. 7. Two-dimensional Fourier transform of a reconstructed picture

spikes that protrude above the video spectrum continuum can be isolated without unduly removing the video signal. As can be seen in the power spectrum of Fig. 6, the spikes clearly have skirts extending below the continuum. These skirts represent a subtle local modulation (generally amplitude modulation) of the basic noise pattern as might occur with amplitude dependence or saturation effects on the noise signal. These cannot be readily isolated by frequency space filtering. A more effective approach is to determine a local modulation coefficient for the idealized noise pattern derived from careful two-dimensional filtering. This coefficient is (typically) slowly varying relative to the resolution limits of the camera. The criterion used for determining the local modulation coefficient is to minimize the variance of the noisy signal minus a variable fraction of the idealized noise pattern over a local region. If N_{ij} is the noisy image, n_{ij} is the idealized noise pattern, α_{ij} is the local modulation coefficient, and σ_{ij}^2 is the "cleaned-up" signal variance for the local region of dimension M , the definition of local signal variance

$$\sigma_{ij}^2 = \frac{1}{(M+1)^2} \sum_{k=-M/2}^{M/2} \sum_{l=-M/2}^{M/2} [(N_{i+k, j+l} - \alpha_{ij} n_{i+k, j+l}) - (\langle N \rangle_{ij} - \alpha_{ij} \langle n \rangle_{ij})]^2 \quad (1)$$

is used. The bracketed terms are average values defined for any matrix m_{ij} to be

$$\langle m \rangle_{ij} = \frac{1}{(M+1)^2} \sum_{k=-M/2}^{M/2} \sum_{l=-M/2}^{M/2} m_{i+k, j+l} \quad (2)$$

By using the minimization condition

$$\frac{d\sigma_{ij}^2}{d\alpha_{ij}} = 0 \quad (3)$$

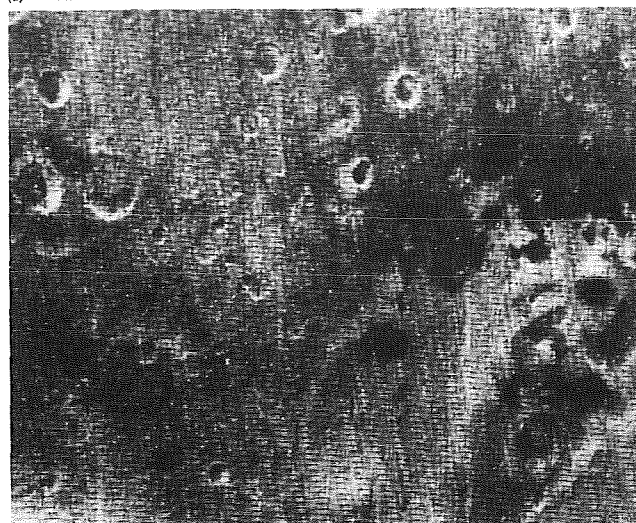
one derives the expression

$$\alpha_{ij} = \frac{\langle [N_{ij} - \langle N \rangle_{ij}] [n_{ij} - \langle n \rangle_{ij}] \rangle}{\langle [n_{ij} - \langle n \rangle_{ij}]^2 \rangle} \quad (4)$$

which defines a modulation coefficient for each point in the picture in terms of the noisy image and the idealized noise pattern. The dimension M of the local area used to determine the coefficient α_{ij} for each point is chosen on the basis of the spatial frequency bandwidth appropriate for α_{ij} .

The results of applying these techniques to remove the periodic noise component from the *Mariner VII* picture shown in Fig. 8a, which has been enlarged two times to

(a) MARINER VII PICTURE OF MERIDIANI SINUS



(b) RESULT OF REMOVING PERIODIC NOISES FROM (a)



(c) PERIODIC NOISE PATTERN REMOVED FROM (a) TO OBTAIN (b)

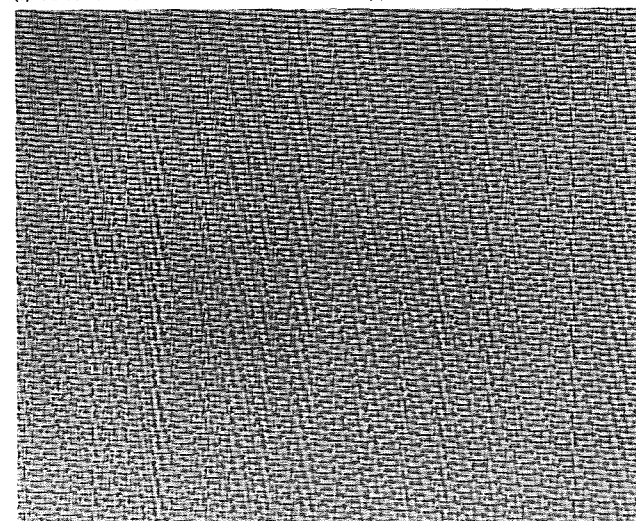


Fig. 8. Periodic noise removal

make the noises apparent, are apparent in Fig. 8b (note the presence of vertical streak noises at this point). The complicated periodic noises that were extracted are seen in Fig. 8c.

4. Long-Line Noises

A variety of mechanisms can produce long-line or streak noises in television images such as gain variations, line bunching, data outages, and analog tape recorder drop-outs. This type of noise, caused by drop-outs in the case of *Mariner* Mars 1969, becomes apparent as vertical streaks in Fig. 8b with the removal of the periodic noise. The characteristic that distinguishes streak noises from the actual scene is their linear correlation along some particular direction and the lack of correlation in the perpendicular direction. This distinction is not complete, however, since linear features are common in natural scenes. The problems of data-dependent noise removal are exemplified by this case since major damage to the true video signal may result in particular localized regions that contain scene components resembling the noise.

The technique developed to correct for these streak noises is to compare the local average intensity value of lines adjacent and parallel to the streaks with the average value of the streak itself and to apply a gain factor to account for any differences. Since the correlation between points in a picture decreases with increasing separation, a linearly decreasing weight was applied to more distant local lines in determining the surrounding average. If N_{ij} is the noisy picture (j is the index along the direction of the streak noises) and G_{ij} is the corrective gain to be applied to the point (i, j) , the expression

$$G_{ij} = \frac{\sum_{k=1}^P (P - k + 1) [\langle N \rangle_{i+k, j} + \langle N \rangle_{i-k, j}]}{P(P+1) \langle N \rangle_{ij}} \quad (5)$$

is used where P is the number of adjacent lines used above and below the streak and the average values are defined by

$$\langle N \rangle_{ij} = \frac{1}{Q+1} \sum_{k=-Q/2}^{Q/2} N_{i, j+k} \quad (6)$$

In Eq. (6), Q is the low-pass filter dimension along the direction of the streaks and is determined by the length-wise correlation of the long-line noise. The dimension P is determined by the perpendicular correlation.

A further refinement was used in determining the numerator of Eq. (5) for each point. The numerator was computed as shown but then a "majority rules" logic was

applied so that terms deviating significantly from the average were eliminated from determining the average on a second pass. This algorithm is a refinement of the long-line filter described in Ref. 1.

The result of correcting for the streak noises in Fig. 9 is shown in Fig. 10. It should be emphasized that this correction is particularly data-dependent and its effect, while closer to the truth in the large, may introduce artifacts in the small.

5. Isolated Spike Noises

The occurrence of a bit error in telemetering digital video data or the presence of sharp spikes of noise from the analog electronics can cause isolated picture elements

to deviate significantly from the surrounding data. The removal of the periodic and streak noises reveals an additional noise component consisting of black and white spikes (Fig. 9a). The characteristic that distinguishes these spike noises from the actual video is the fact that, because of resolution limitations of a Nyquist sampled video system, the picture-element-to-picture-element variation of the true signal is limited.

The logic used to remove spike noises is very simple. Each picture element is examined, and, if it is significantly above each of its neighbors or significantly below its neighbors, it is replaced by the average neighboring intensity. The results of applying this correction to the remaining spike noises in Fig. 9a is shown in Fig. 9b. Note, in comparison with Fig. 8a, that objects of considerably higher resolution are discernible in Fig. 9b.

6. Conclusion

A comparison of Figs. 8a and 9b shows the dramatic improvement in signal-to-noise ratio obtained by using the digital computer to isolate and remove various structured noise components from the raw video. This type of processing, while preliminary to further restorative steps, of itself produces an enhancement that allows analysis of surface detail closer to the resolution limits of the camera system. Techniques for removing noises from video are of widely varying character and, in some cases, strongly video-system and data dependent.

Reference

1. Nathan, R., *Digital Video-Data Handling*, Technical Report 32-877. Jet Propulsion Laboratory, Pasadena, Calif., Jan. 5, 1966.

C. Polaroid Quick-Look System, R. D. Brandt and G. R. Andrews

1. Introduction

The Image Processing Laboratory has been producing high-quality digital photographic prints for several years. The minimum time required to produce a print after the computer-generated magnetic tape is available is approximately 1 hr. This relatively long period of time is a serious disadvantage to image processing whenever the next process is determined by information from the photographic print of the previous process. The Polaroid Quick-Look System interfaces directly with an IBM System/360 model 44 computer and provides a $3\frac{1}{2} \times 4\frac{1}{2}$ -in. photographic print (1024×1024) in approximately $1\frac{1}{2}$ min. The quality of the photographic print is sufficiently good

(a) RESULT OF REMOVING STREAK NOISES FROM FIG. 8b



(b) RESULT OF REMOVING SPIKE NOISES FROM (a)



Fig. 9. Unstructured noise removal

so that almost all of the information needed for further processing can be obtained from the Polaroid print.

2. Design Philosophy

The three basic requirements satisfied by the Polaroid Quick-Look System are as follows:

- (1) Providing a good-quality photographic print to yield information for further computer processing of the image data.
- (2) Making the photographic print available within a few minutes after completion of image processing by the computer.
- (3) Satisfying the above requirements at a minimum cost.

A block diagram of the Polaroid Quick-Look System is shown in Fig. 10.

A Polaroid print was the obvious choice for a quick-look print. The Polaroid print also has a dynamic range wide enough to yield good quality data (Fig. 11). The initial decision was the selection of a print size of $3\frac{1}{2} \times 4\frac{1}{4}$ -in. using the Polaroid 4 $\times$ 5 pack; the next step was to provide an electro-optics system that could record a 1024×1024 square array of picture elements without degrading the image quality by the severe overlapping of picture ele-

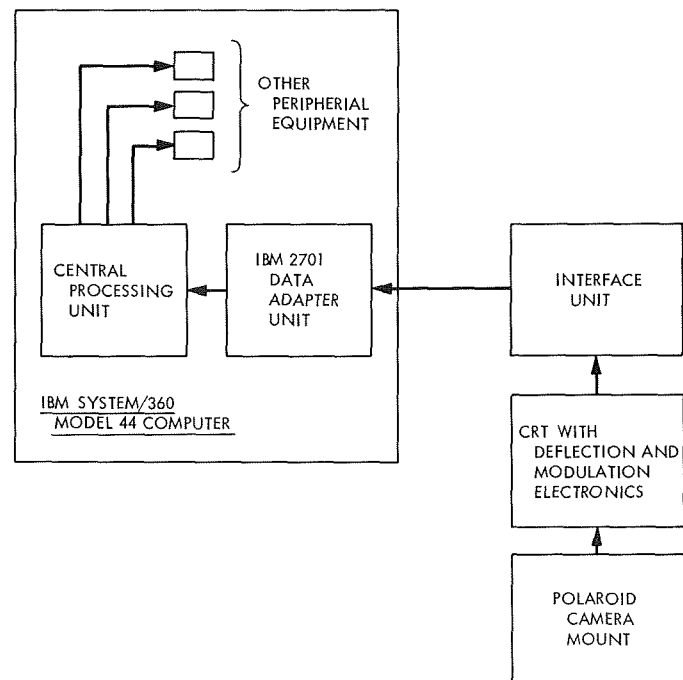


Fig. 10. Polaroid Quick-Look System block diagram

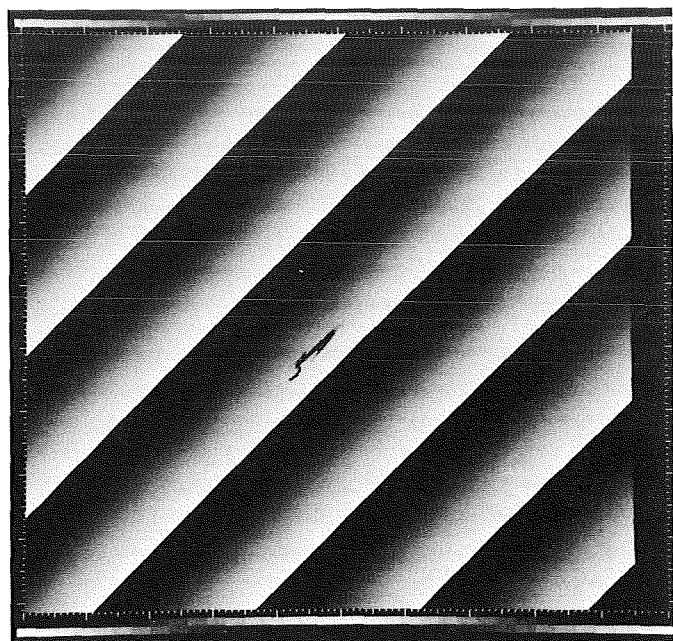


Fig. 11. Polaroid print dynamic range

ments. Some overlap of the picture elements is normally desired to improve the subjective appearance of the print. (Figure 12 is an example of a *Mariner Mars* 1969 Polaroid print.) It was determined that one of the recorders used on the *Ranger* program could be easily modified to interface with the interface unit, thus reducing the cost of the Polaroid Quick-Look System. The cathode ray tube (CRT) and its deflection circuitry have been modified to accept signals from the interface unit. The effective electron beam

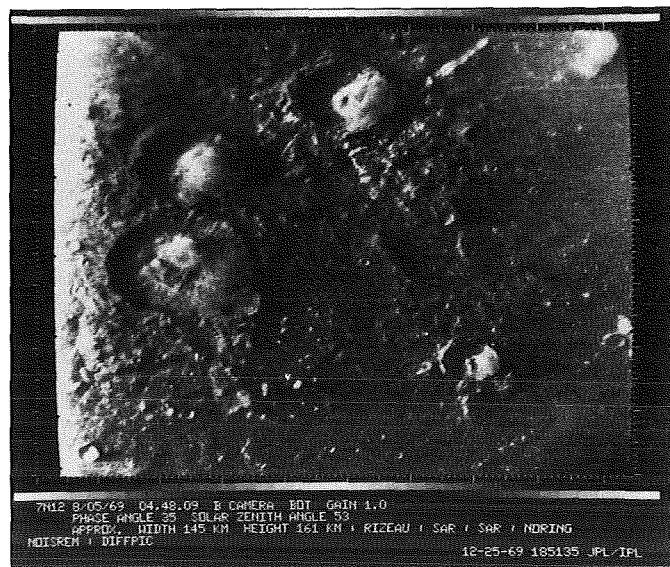


Fig. 12. *Mariner Mars* 1969 Polaroid print

diameter for the *Ranger* CRT is approximately 0.002 in. which would necessitate a 2×2 -in. raster on the CRT. Since the *Ranger* CRT is a 5-in. tube, it has sufficient capability to generate a 2×2 -in. raster. In addition to the above requirements, it was found desirable to utilize as much of the Polaroid print as possible. Hence, a magnification ratio of approximately 1.68 was used to yield a $3\frac{3}{8} \times 3\frac{3}{8}$ -in. raster on the Polaroid print.

3. Hardware Implementation

a. Raster generation. The method of generating the raster was determined by the line-by-line method used in the computer for manipulating image data. Thus, the picture elements of line 1 are sequentially recorded, followed by the sequential recording of the picture elements of line 2, etc. The picture elements of all lines are recorded in the same direction. The vertical deflection signal is generated by a 10-bit digital-to-analog converter. This digital signal was direct coupled into the vertical deflection system of the *Ranger* recorder.

The horizontal deflection signal is also generated by a 10-bit digital-to-analog converter direct coupled to the *Ranger* recorder. The digital signal is derived from a digital counter that counts the number picture elements recorded per line. All picture elements of all lines are recorded at a constant rate of approximately 23 kHz.

b. Intensity modulation of raster. The image information per picture element is 8 bits. An 8-bit digital-to-analog converter is used to convert the digital information to an analog signal used to amplitude-modulate the grid of the CRT. The duration of modulation is constant at approximately 13 μ s per picture element.

c. Interface unit. The interface unit (Fig. 13) accepts image information and control signals from the IBM 2701 for the CRT assembly. The CRT assembly also returns control signals through the interface to the IBM 2701. The IBM 2701 provides for the transfer of 16 parallel bits of image information. Thus, the image information for two picture elements is transferred simultaneously from the IBM 2701 to the interface unit.

4. Future Hardware Configuration

The Polaroid Quick-Look System is being used extensively as an operational part of the Image Processing Laboratory. The next hardware configuration will be to put the camera into a console that may also contain a keyboard for entering information into the computer. It is anticipated that the console will also contain a "live" computer-refreshed display. This console will centralize two important operations of image processing.

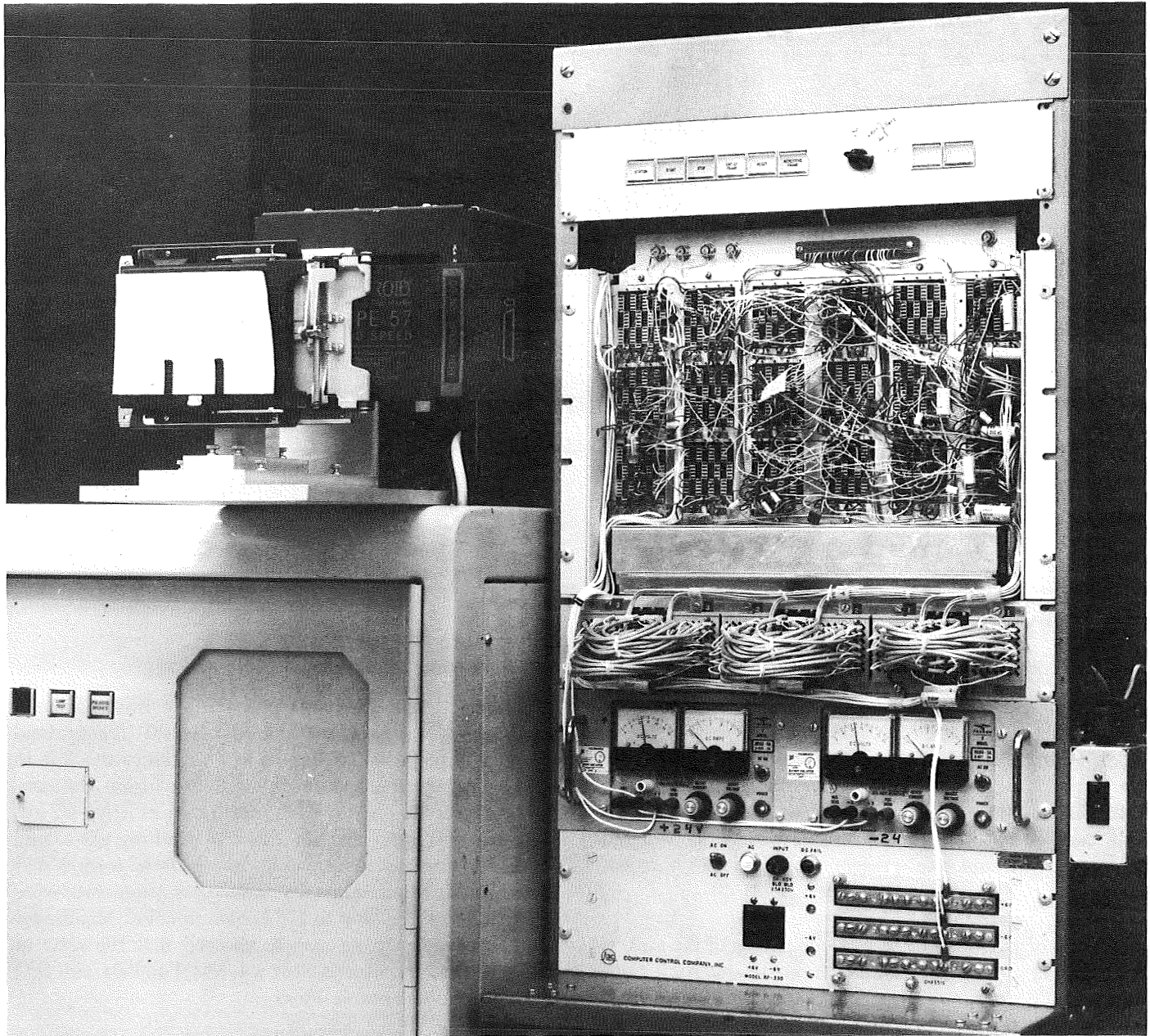


Fig. 13. Interface unit

IV. Lunar and Planetary Sciences

SPACE SCIENCES DIVISION

A. Coloring of Synthetic and Natural Lunar Glass by Titanium and Iron, J. E. Conel

1. Introduction

Part of a study of the luminescence and reflectance properties of lunar samples^{1,2} (Ref. 1) involved preparation of a fresh sample of synthetically produced material with pronounced red reflectance relative to fresh crystalline rock. The significant result is that it was possible by vitrification to produce silicate materials with enhanced red reflectance without the presence of ferric iron oxide, hydrated or otherwise.

A study of two possible causes of coloring of the glass is reported here: (1) The coloration is due to electronic transitions in Ti^{+3} ions, and possibly Fe^{+2} ; and (2) The coloration arises from absorption by colloid-size metallic spheres dispersed in the glass—a phenomenon similar to that pro-

ducing the red coloring of certain gold solutions in water (Ref. 2).

2. Experiments With Synthetic Glasses

Synthetic lunar glass was prepared by fusing a small portion (~50 mg) of crushed rock 10020 (fine-grained holocrystalline vesicular basalt) in a platinum crucible in a high-purity nitrogen atmosphere. Before fusion, the sample was baked out near 300°C for 5 min to drive off adsorbed atmospheric constituents. The resulting glass bead was black, vitreous, and opaque. In powdered form, it was dark brown. Microscopically, thin glass fragments were transparent and yellow to red-brown in color. Mössbauer analysis of iron in the sample showed that the ratio of ferric iron to total iron present was less than 0.04, and that there was no detectable Fe_2O_3 .

Rocks and glasses containing substantial amounts of ferric iron do not necessarily show enhanced red reflectance. For example, spectra given in Fig. 1 of holocrystalline leucite olivine basalt and synthetic glass prepared from this material are quite similar (although characteristically the reflectance of glass is lower at all wavelengths) despite ferric iron contents that differ by a factor of four between the two samples. Both samples are light gray. In

¹Nash, D. B., and Greer, R. T., "Luminescence Properties of Apollo 11 Lunar Samples and Implications for Solar-Excited Lunar Luminescence," *Geochemica et Cosmochemica Acta* (in press).

²Conel, J. E., and Nash, D. B., "Spectral Reflectance and Albedo of Apollo 11 Lunar Samples: Effects of Irradiation and Vitrification and Comparison With Telescopic Observations," *Geochemica et Cosmochemica Acta* (in press).

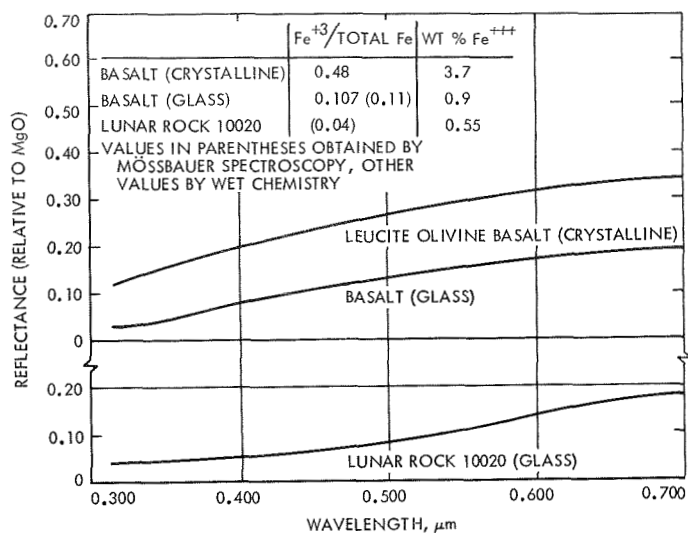


Fig. 1. Visible reflectance of crystalline and fused basalt and synthetic lunar glass

contrast, the fused lunar basalt contains half the ferric iron of leucite basalt glass, yet shows comparatively strong absorption in the violet and blue.

Since orange, red, and brown lunar glasses are characterized by high iron and titanium contents (Refs. 3, 4, 5) with no reported ferric iron, it seems reasonable to seek an explanation of the observed color of both natural and synthetic lunar glass by the presence of these elements. Adams and Jones (Ref. 6) suggested that the red color of some lunar glasses resulted from high iron and titanium content, but left the electronic or other processes involved unspecified.

Ti³⁺ and Fe²⁺ ions in octahedral coordination in silicates produce absorption bands near 0.45 and 1.0 μm, respectively (Ref. 7), the violet or purple color of titan-augite in reflection or transmission being attributed to absorption by Ti³⁺ such that red and blue light are transmitted. In glass, titanium and iron can be expected in both octahedral and tetrahedral coordination (Ref. 7). Characteristically, the Fe²⁺ absorption band near 1.0 μm in a crystalline silicate is shifted to longer wavelengths (between 1.0 and 1.1 μm) in the compositionally equivalent glass. The same might be expected for bands associated with Ti³⁺.

To obtain an idea of the coloring effects expected from bands associated with Ti in silicate glasses, mixtures of TiO₂ and various minerals, both with and without iron, were each fused in extra high-purity helium, using a molybdenum crucible. After fusion, each liquid was quenched in a stream of argon gas. In spite of rapid

quenching, however, some mixtures (especially enstatite) crystallized. The crystalline sanidine (KAlSi₃O₈) and enstatite (MgSiO₃) used were pure synthetic minerals, while the oligoclase (~An₂₀) contained small amounts of both ferrous and ferric iron. Reflectance spectra of crystalline, and glassy materials and glass-crystal mixtures are shown in Fig. 2. In general, all mixtures containing Ti give spectra with a very broad absorption band near 1.0 μm, one wing of which extends into the visible. In all cases, the spectra show sharp peaks near 0.40 μm and other minimums between 0.25 and 0.35 μm. The color of the glasses and glass-crystal mixtures is dark blue to blue-gray. Somewhat surprisingly, both glassy and crystalline samples give similar spectra. The presence of small amounts of iron in oligoclase apparently do not affect the position of the strong 1.0-μm absorption minimum, which is naturally attributed to titanium. It should be emphasized that the

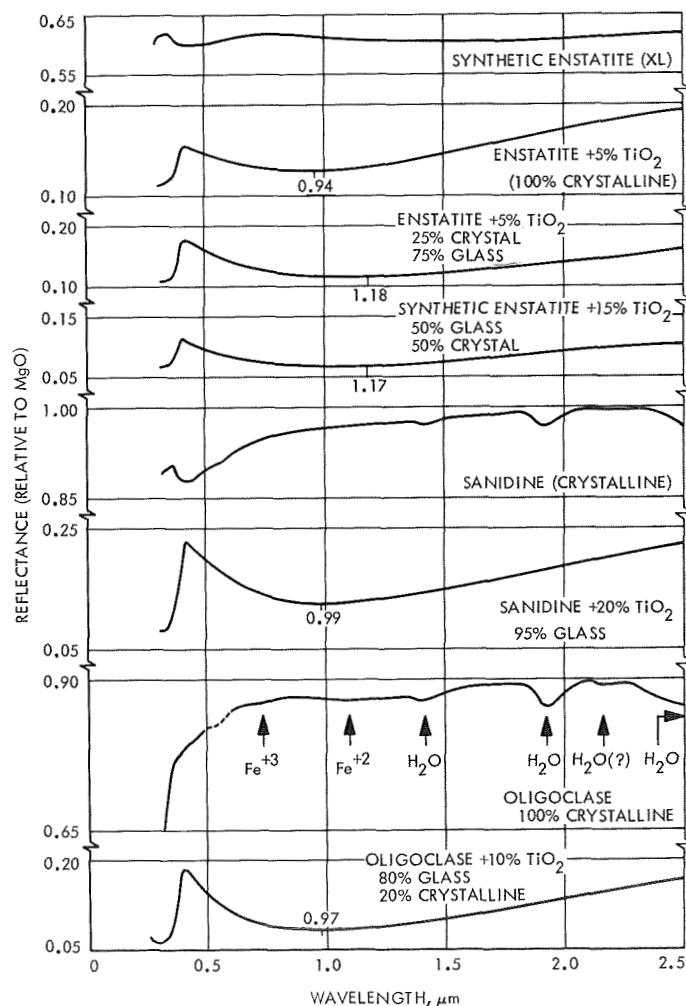


Fig. 2. Ti³⁺ absorption bands in spectra of minerals and mineral glasses

1.0- μm band arising from titanium in these samples is quite distinct from that ordinarily attributed to Fe^{+2} in silicates. In Fig. 3, we show the spectrum of fused oligoclase, illustrating the result typically observed with silicate glasses containing ferrous iron, namely a broad minimum located near 1.0 μm and a maximum near 0.7 μm . (In materials containing abundant iron there is often a second minimum in the reflectance near 1.75 μm .) Also, we observe that water bands in the spectra of unfused feldspars do not generally appear in spectra of these materials after fusion.

The valence state of titanium in these glassy and crystalline materials is inferred to be +3, based on a comparison of glass color with that obtained with titanium-bearing SiO_2 glass prepared in the presence of a reducing atmosphere in which the ion is known to be trivalent (Ref. 8); both are light to deep blue, depending on titanium content. Based on approximate results from crystal field theory, an elementary interpretation of the spectra of titanium-rich glass and crystalline materials is that the major absorption arises from electronic transition in Ti^{+3} , mostly in tetrahedral coordination. The energy separation or crystal field splitting between d -orbitals of trivalent titanium in octahedral coordination with oxygen in aqueous solution determine the absorption band to be near 0.49 μm (Ref. 7). The observed band in silicates is near 0.45 μm . In passing from octahedral to tetrahedral stereochemistry, the crystal field magnitudes are altered by a factor $-4/9$ (Ref. 9), providing the metal-ligand separation is the same. On this assumption, which admittedly may be violated, we determine the absorption band for Ti^{+3} in tetrahedral coordination to lie near 1.0 μm , which is close to that observed. Further evidence that Ti^{+3} may be occupying silicon-like sites in tetrahedral coordination is provided by the spectra for crystals and glass in which these absorption bands are approximately in the same position. (Actually the band in crystalline material is located near 0.94 μm , which may indicate from previous arguments some contribution from titanium in octahedral sites as well.) Thus, either the same tetrahedral-like arrangement involving titanium not on

silicon sites is common to both crystalline and glassy materials, or there is some substitution of titanium on silicon sites in tetrahedral configuration.

It is not conventionally accepted that trivalent titanium substitution occurs on silicon sites in tetrahedra, because even Ti^{+4} is too large to fit into tetrahedral coordination with four oxygens (Ref. 10). However, ferric iron substitution for aluminum in the potassium feldspars is apparently known (Ref. 11).

A spectrum of borosilicate glass with Ti^{+3} on octahedral sites is given in Ref. 12. A major intense absorption band is located near 0.59 μm in glass, whereas the corresponding weaker narrower band in crystalline material is at 0.49 μm . An unresolved question in the present experiments is the reason it has not been possible to produce glasses with Ti^{+3} on (apparently) octahedral sites.

Whatever the actual configuration of titanium in glass structures, the results reported here are taken to indicate coloring effects expected from Ti^{+3} in glass; that is, absorption bands are shifted to longer wavelengths and the bandwidths are increased. From the results reported, it is difficult to see how Ti^{+3} alone or in combination with ferrous iron can account for blue or near-ultraviolet absorption in natural and synthetic lunar glasses. Ross *et al.*, (Ref. 13) report a correlation in lunar pyroxene between high apparently quadravalent titanium content, aluminum abundance, and intense red color. The common hydrogen peroxide test for Ti^{+4} results in the complex $\text{Ti}^{+4}\cdot\text{O}_2\text{H}$ which has an orange color. The absorption bands involved are known to be charge transfer transitions involving electrons from σ -bonding orbitals that are being excited to empty d -orbitals in the metal (Ref. 14). Such ligand-to-metal charge transfer bands are strong and occur in the near ultraviolet (Ref. 15). This mechanism appears promising, but the arguments involving silicates have apparently not been worked out.

3. Scattering Calculations

The explanation of glass coloring as a scattering phenomenon depends upon the existence of metallic or other absorbing particles a few hundred angstroms in diameter being dispersed throughout the medium. To investigate this question for substances of the type expected, and for which the optical constants are known, scattering calculations for spherical particles immersed in a hypothetical isotropic (glassy) medium were made for iron and ilmenite (FeTiO_3) using the series approximation for extinction and

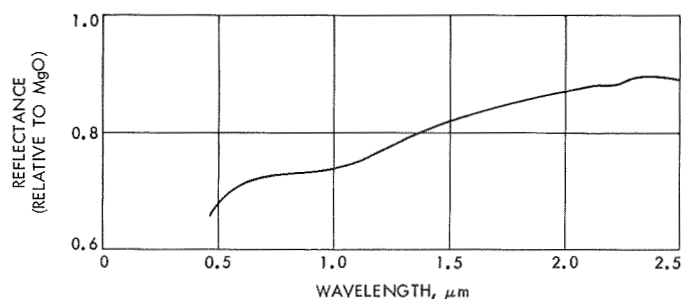


Fig. 3. Reflectance of fused oligoclase

scattering coefficients derived from Mie theory. The relevant expressions are (Ref. 2):

$$Q_{\text{ext}} = -\text{Im} \left\{ 4x \left(\frac{m^2 - 1}{m^2 + 2} \right) + \frac{4}{15} x^3 \left(\frac{m^2 - 1}{m^2 + 2} \right)^2 \left(\frac{m^4 + 27m^2 + 38}{2m^2 + 3} \right) \right\} \\ + x^4 \text{Re} \left\{ \frac{8}{3} \left(\frac{m^2 - 1}{m^2 + 2} \right)^2 \right\} + \dots$$

$$Q_{\text{sca}} = \frac{8}{3} x^4 \left| \frac{m^2 - 1}{m^2 + 2} \right|^2$$

$$Q_{\text{abs}} = Q_{\text{ext}} - Q_{\text{sca}}$$

where Q_{abs} , Q_{sca} , and Q_{ext} are the efficiency factors for absorption, scattering and extinction, and

$$x = 2\pi a / \lambda'$$

a = particle radius, μm

λ' = wavelength in the medium, μm

$m = n - ik$ = complex refractive index of iron or ilmenite in the medium

For iron in the region 0.395–0.668 μm , the real and imaginary parts of the complex index are obtained by least-squares fit of data given by Van de Hulst (Ref. 2) as

$$n(\lambda) = -4.02 + 28.23\lambda - 51.01\lambda^2 + 32.30\lambda^3$$

$$k(\lambda) = -3.80 + 27.63\lambda - 50.06\lambda^2 + 31.96\lambda^3$$

where λ is the wavelength in air. For ilmenite we use the data of Simpson and Bowie (Ref. 16). The approximate polynomial expressions for the ordinary index are

$$n(\lambda) = 10.79 - 36.77\lambda + 52.19\lambda^2 - 23.14\lambda^3$$

$$k(\lambda) = 24.53 - 121.84\lambda + 200.73\lambda^2 - 108.5\lambda^3$$

Calculations were made for spheres in air and for a real refractive index of glass without scatterers of 1.3, independent of wavelength. Silicate glasses typically have refractive indices near 1.5 to 1.6, depending on SiO_2 abundance. The low value used here is necessitated by the narrow range of wavelengths over which the refractive indices of the assumed scattering materials are known. In a medium of refractive index n , the wavelength λ' is given by $\lambda' = \lambda_{\text{air}}/n$, where λ_{air} is the wavelength in air. Thus for $\lambda_{\text{air}} = 0.4 \mu\text{m}$, we must know the refractive indices of the scatterers at $\lambda' = 0.3 \mu\text{m}$, which is possibly outside the region of valid extrapolation of our polynomial approximations. However, the results can be expected to approximate those from dispersions in silicate glasses with higher and more realistic indices.

Results for scattering calculations for iron are presented in Fig. 4, for particles 50, 100, and 250 Å in radius. Typically, the extinction coefficient is always greater in the blue than in the red, and it is greater for spheres in glass than in air. Q_{ext} is dominated by Q_{abs} in these cases. Interestingly, there is a peak in extinction in the indigo blue region for spheres 500 Å in diameter, but all of these hypothetical cases suggest some enrichment in red light in the transmitted or reflected intensity from such dispersions. For ilmenite (Fig. 5), generally similar results are obtained; but the extinction, scattering, and absorption coefficients are always less than for iron, and there are no pronounced maxima or minima in the curves.

Stronger structure in these curves was expected since both the reflectance and real and imaginary parts of the complex index have minimum values in the neighborhood of 0.5 to 0.6 μm (Ref. 16).

An experimental test of these ideas would presumably involve direct identification of particles of the dispersions, using electron microscope or microprobe techniques. Preliminary attempts to detect inhomogeneities in lunar glass chemistry by microprobe techniques and with spot resolution of $\sim 6 \mu\text{m}$ have been unsuccessful; the synthetic lunar glass appears homogeneous at this resolution in both iron and titanium content.

In addition to small particle scattering and absorptions arising from electronic transitions, we could, of course, assume some combination of such processes and conceivably produce a wide variation of color phenomena. In order to make such calculations, the complex refractive indices of both scatterers and glass must be known for a range of wavelengths both inside and outside the visible region. However, such data have not been secured, and these calculations have not been made.

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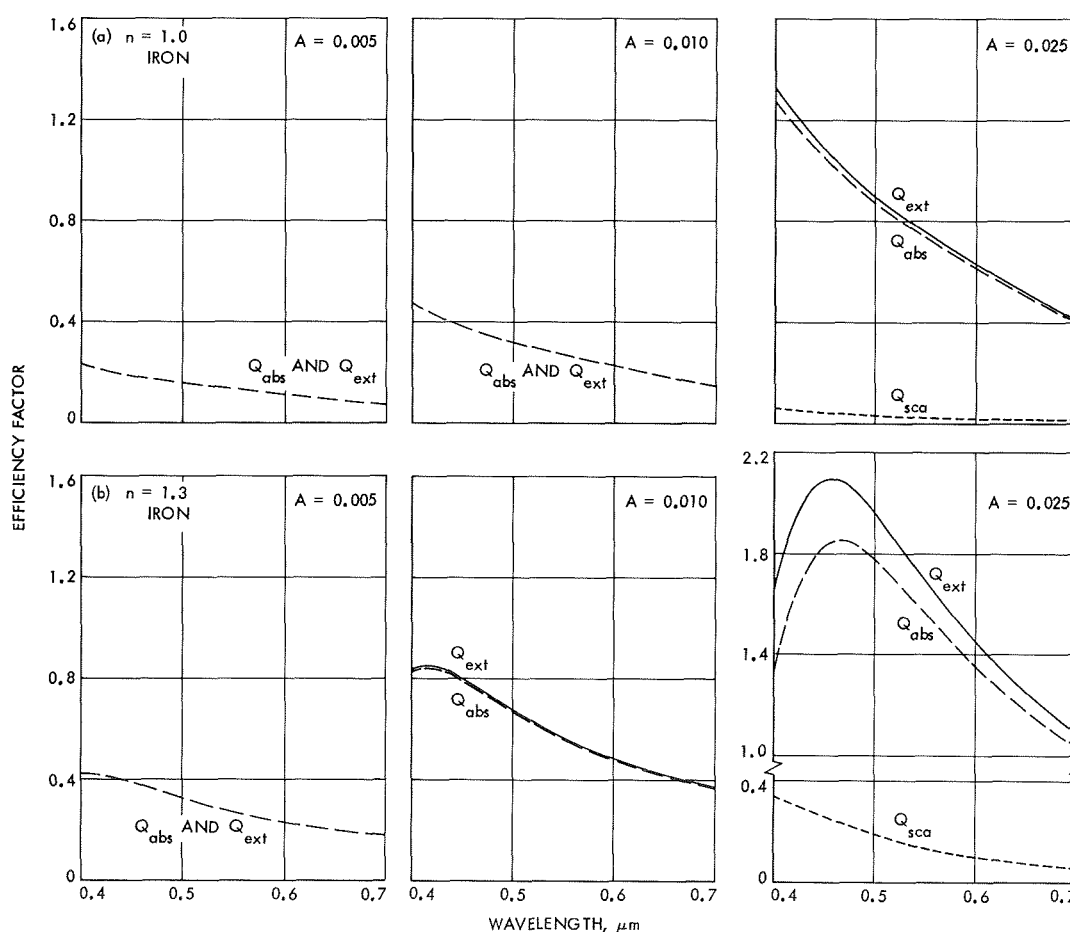


Fig. 4. Extinction, absorption, and scattering coefficients for iron spheres in air and in isotropic medium with real refractive index of 1.3. A is particle radius.

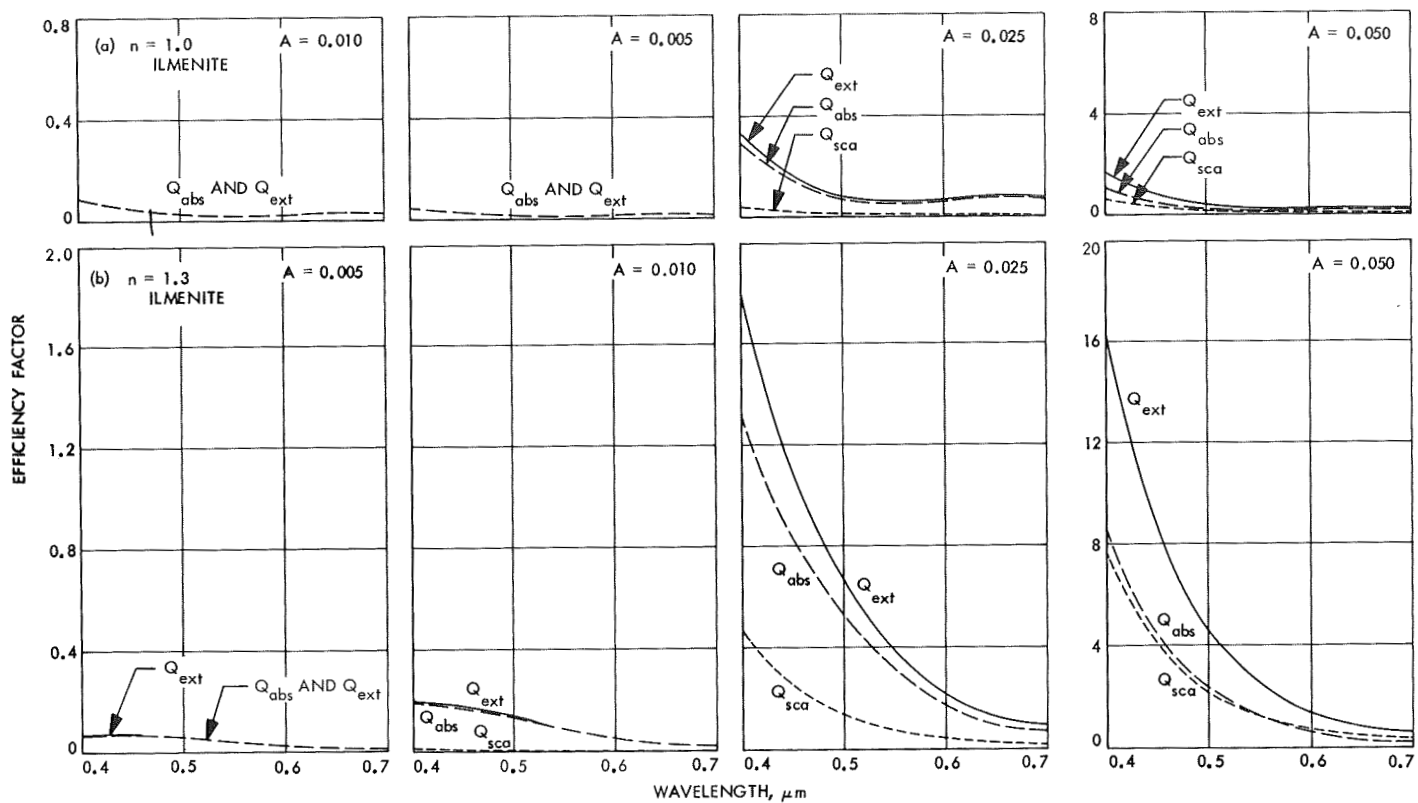


Fig. 5. Extinction, absorption, and scattering coefficients for ilmenite spheres in air and in isotropic medium with refractive index of 1.3. A is particle radius

V. Physics

SPACE SCIENCES DIVISION

A. An Ion Cyclotron Resonance Study of the Ion-Molecule Reactions in Hydrogen-Methane Mixtures, M. T. Bowers and D. D. Elleman

Ion cyclotron single- and double-resonance techniques have been established in the last several years as a method to identify ion-molecule reactions, determine ion-molecule reaction mechanisms, and measure energy dependence of ion-molecule reaction rates (Refs. 1-4). One of the primary goals in our laboratory is the use of ion cyclotron resonance (ICR) to determine quantitative kinetics data (Refs. 4-7)¹ and detailed mechanistic information (Refs. 6 and 7)¹ on simple systems of atmospheric importance. Investigations to date have dealt with H_2 , $H_2 + N_2$ mixtures (Refs. 6 and 7), $H_2 + Ar$ mixtures (Refs. 6 and 7), $H_2 + Ar$ mixtures (Ref. 7), and $CH_4 + NH_3$ mixtures.² In this article, we extend these studies to include the various ion-molecule reactions in H_2-CH_4 mixtures, a system that is considerably important in the study of the Jovian atmosphere (Ref. 8) and possibly crucial to the understanding of the evolution of the earth's primitive reducing atmosphere.

The hydrogen-methane system has been studied by several authors (Refs. 9-11), the most recent work being by Aquilanti and Volpi (Ref. 11) using a high-pressure mass spectrometer with a tritium decay ionization source. The experiments of Aquilanti and Volpi were restricted to measuring the rate of reaction of H_3^+ with CH_4 and to observing the resulting $CH_5^+/(CH_3^+ + CH_5^+)$ ratio as a function of H_2 pressure.

Our ratio data agree very well with Aquilanti and Volpi at high H_2 partial pressures. At lower pressures, however, our data conclusively show curvature and a non-zero intercept while Aquilanti and Volpi assume the linear decrease in CH_5^+/CH_3^+ continues to a value of zero at zero H_2 pressure. Aquilanti and Volpi interpret their ratio data by assuming collisional stabilization of an excited $[CH_5^+]$ by H_2 . Our additional low-pressure data indicate a much more complicated reaction mechanism is needed. Specifically, there is evidence for including reactions of two kinds of H_3^+ ion with CH_4 ; an excited $[H_3^+]$ ion and a ground-state H_3^+ ion.

The ICR spectrometer used for these studies was constructed at JPL and has been described in previous reports (Refs. 6 and 7; Footnotes 1 and 2). Modifications to the

¹Bowers, M. T., Elleman, D. D., and King, J., Jr., *J. Chem. Phys.* (in press).

²Huntress, W. T., and Elleman, D. D., *J. Am. Chem. Soc.* (in press).

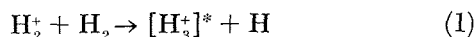
spectrometer allow for standard double resonance and pulsed ion-ejection experiments to be performed.

Figure 1 shows the dependence of the standard single-resonance ICR peaks of single carbon ions in $\text{CD}_4\text{-H}_2$ mixtures as a function of H_2 pressure. CD_4 was at a constant pressure of 6×10^{-6} torr, the electron energy was 40 eV, the electron current was 1×10^{-8} A, and residence time in the cell $\tau \cong 7 \times 10^{-4}$ s.

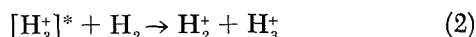
A similar plot for the ratio of

$$\frac{[\text{H}_2^+]}{[\text{H}_2^+] + [\text{H}_3^+]}$$

is given in Fig. 2:

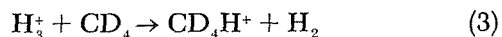


where $[\text{H}_3^+]^*$ has of the order of 2-eV internal energy (Ref. 12) and the residence time in the cell $\tau \cong 5 \times 10^{-4}$ s. The internal energy of $[\text{H}_3^+]^*$ is rapidly moderated by subsequent collisions with H_2 :

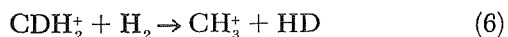
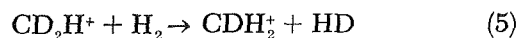
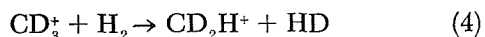


Levanthal and Friedman (Ref. 13) have shown that as few as one collision may completely moderate the $[\text{H}_3^+]^*$ ion. At the higher H_2 partial pressures studied here, a large majority of the $[\text{H}_3^+]^*$ is thus moderated, while at lower H_2 partial pressure, the reaction of $[\text{H}_3^+]^*$ must be considered.

In Fig. 1, the major single carbon product at 21 amu is protonated CD_4 , formed by the reaction



In addition to the major CD_4H^+ product, minor products are detected at 15, 16, 17, and 19 amu. The mass 17, 16, and 15 products correspond to CD_2H^+ , CDH_2^+ , and CH_3^+ , formed principally via the chain reaction sequence



as confirmed by our ICR double-resonance experiments and the pressure dependence data.

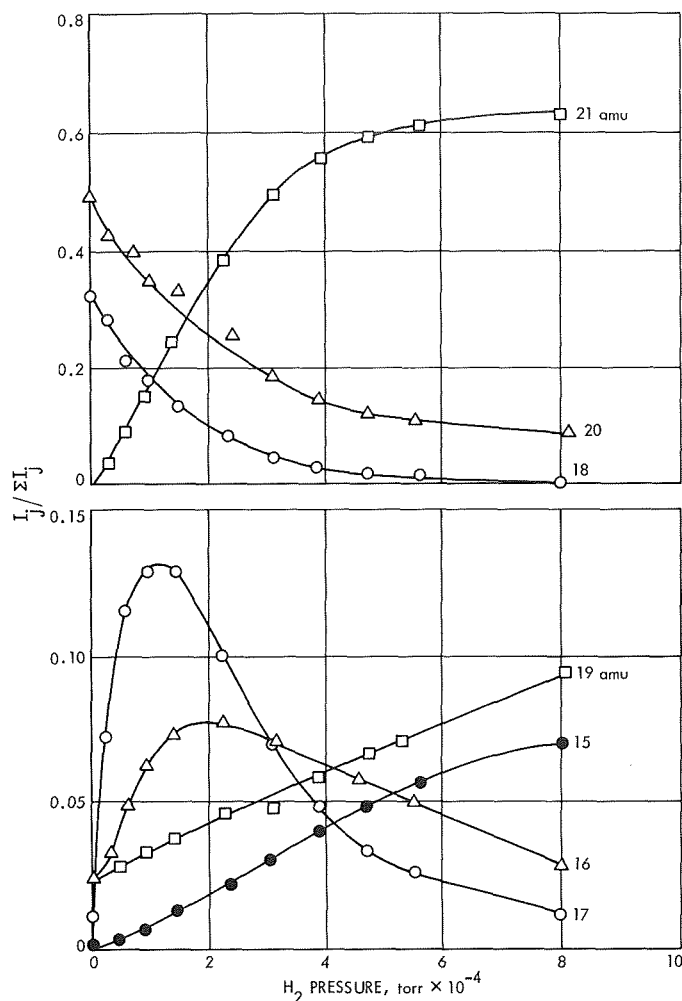


Fig. 1. Single carbon relative intensities $I_i/\Sigma I_i$ vs H_2 partial pressures for $\text{H}_2\text{-CD}_4$ mixtures

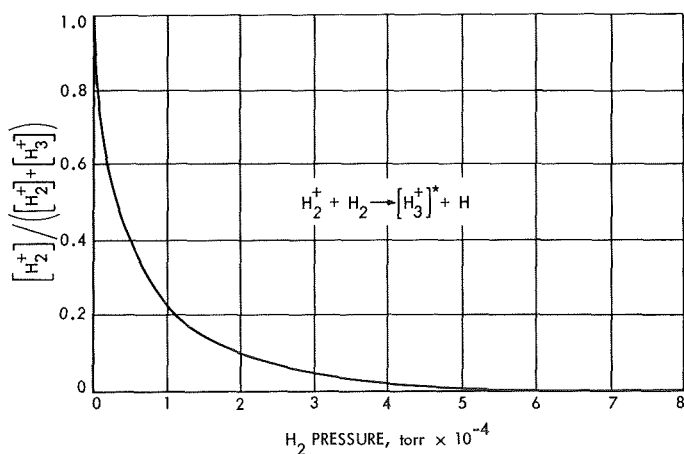


Fig. 2. Relative abundance of H_2^+ vs H_3^+ as a function of H_2 pressure

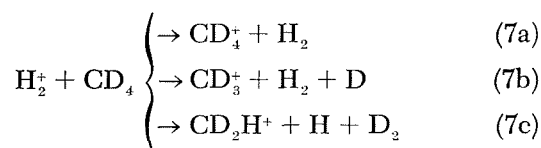
The ion-ejection technique recently introduced by Beauchamp and Armstrong (Ref. 14) is very useful for studying systems where reactant and product ions differ considerably in mass and concurrent reactions make direct interpretation of single-resonance spectra difficult. The reaction of H_2^+ and H_3^+ with CD_4 in H_2 - CD_4 mixtures is a system that lends itself well to the ion-ejection technique.

Figure 3 shows the results of ion-ejection experiments on the H_2 - CD_4 system. Partial pressures of CD_4 are

7.5×10^{-6} torr and of H_2 are 5×10^{-5} torr, the electron energy was 40 eV, the electron current was 1×10^{-8} A, and residence time in the cell $\tau \cong 7 \times 10^{-4}$ s.

The H_3^+ ejection spectrum in Fig. 3c clearly shows that essentially all of the CD_4H^+ originates from H_3^+ ions. Minor contributions of H_3^+ to the CD_3^+ and CD_2H^+ masses are also evident. The small response at mass 20 may indicate an H_3^+ - CD_4 charge exchange process, but the energetics of the reaction are unfavorable.

The H_2^+ ejection results in Fig. 3b are not as simple to quantitatively discuss as the H_3^+ data since ejection of H_2^+ necessarily ejects most of the H_3^+ ions also (due to Reaction 1). It is clear, however, from comparison of the H_2^+ and H_3^+ ejection results, that H_2^+ undergoes the reactions



The mechanism for Reactions (7b) and (7c) cannot be unambiguously specified. There may be a $[CD_4H^+]^*$ complex intermediate or the reactions may be direct. Reaction (7c) clearly eliminates charge-exchange-induced decomposition of CD_4^+ as the sole source of CD_3^+ .

The dependence of the CH_5^+/CH_3^+ ratio as a function of H_2 pressure has been previously reported (in part, Refs. 11 and 13) with two differing mechanisms to explain the data. Aquilanti and Volpi (Ref. 11) explain the data in the high H_2 pressure limit by assuming that H_3^+ and CH_4 form an excited $[CH_5^+]^*$ complex which can either decompose to CH_3^+ or can be stabilized to CH_5^+ by collision with H_2 . Leventhal and Friedman (Ref. 13) have shown that at lower pressures the reaction of excited $[H_3^+]^*$ is important. We find the complete mechanism to contain both features.

In Fig. 4, the ratio $CD_4H^+/\Sigma [C(H,D)_3^+]$ is plotted as a function of H_2 pressure with a constant CD_4 pressure of 7×10^{-6} torr. The electron energy was 40 eV, the electron current was 0.5×10^{-8} A, and residence time in the cell $\tau \cong 7 \times 10^{-4}$ s.

The data of Fig. 4 were taken using H_3^+ ion-ejection techniques similar to Fig. 3c. Figure 5 gives the complete mechanism of reaction necessary to explain the details of the pressure dependence of Fig. 4. At the lowest H_2 pressure, the reactions of $[H_3^+]^*$ with CD_4 will dominate. This

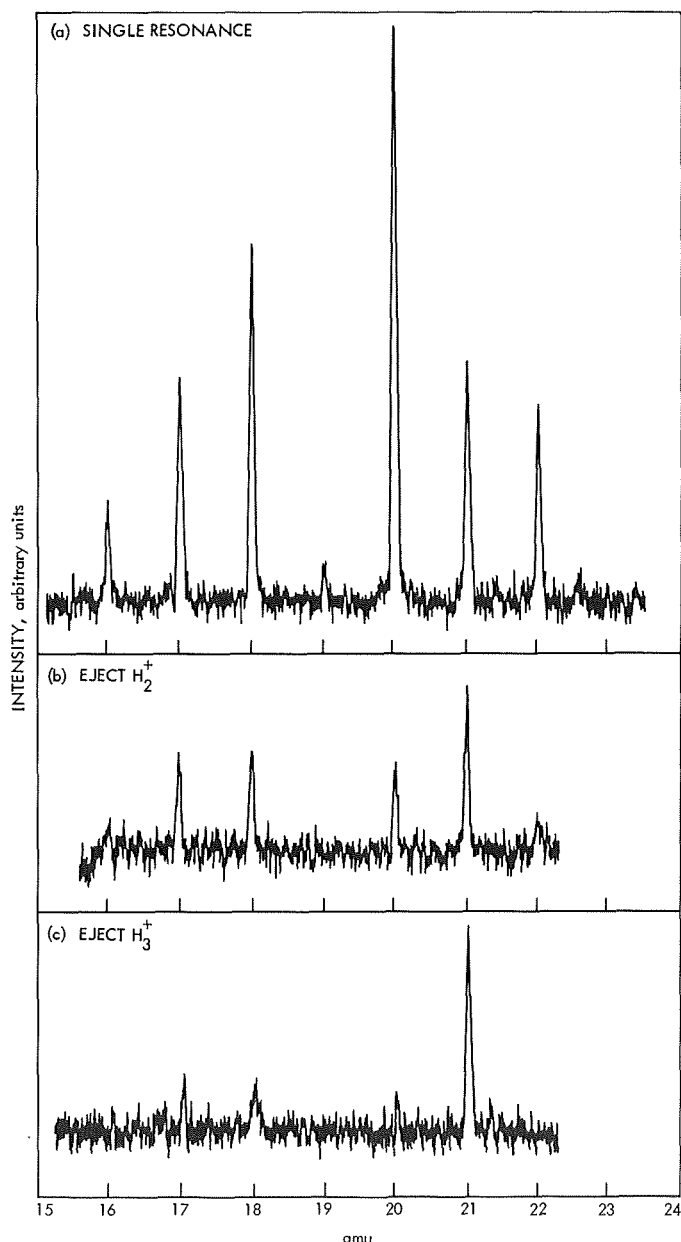


Fig. 3. Pulsed-ion-ejection spectra of CD_4 - H_2 mixture

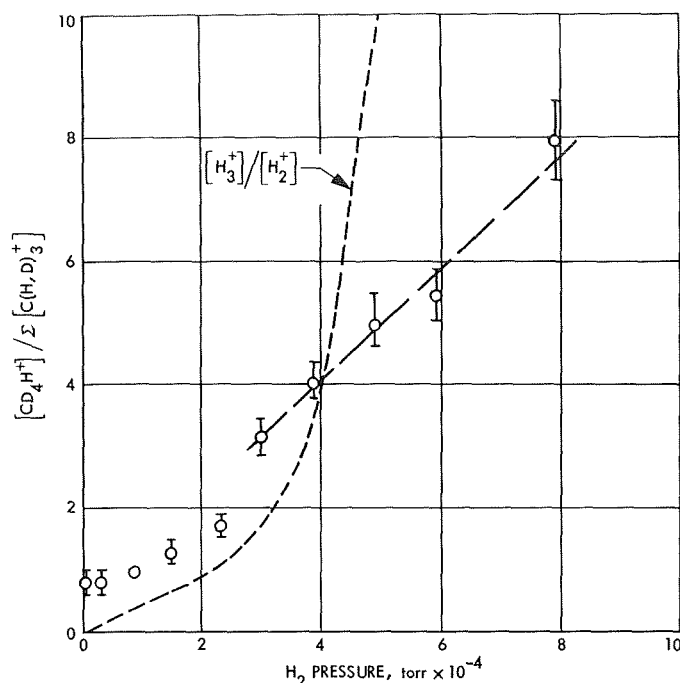


Fig. 4. $[CD_4H^+]/\Sigma [C(H,D)_3^+]$ vs H_2 partial pressure at a constant CD_4 pressure of 6×10^{-6} torr

yields a value of the ratio of the rates of formation of CD_4H^+ and $C(H,D)_3^+$:

$$R_0 = \frac{R [CD_4H^+]_T}{R [\Sigma C(H,D)_3^+]_T} \bigg|_{H_2 \rightarrow 0}$$

$$= \frac{[H_3^+][CD_4]k_2 - [CD_4H^+](k_{4a} + k_{4b})}{[H_3^+][CD_4](k_{3a} + k_{3b}) + [CD_4H^+](k_{4a} + k_{4b})} \quad (8)$$

where

$$R [CD_4H^+]_T = R [CD_4H^+] + R ([CD_4H^+]^{**})$$

If the last terms in Eq. (8) can be ignored relative to the first,

$$R_0 \cong \frac{k_2}{k_{3a} + k_{3b}} \quad (9)$$

It is important to point out that it is essential to include the direct reactions k_{3a} and k_{3b} to predict an intercept other than zero at $H_2 = 0$ (Ref. 10). Aquilanti and Volpi (Ref. 11) extrapolated their CH_5^+/CH_3^+ ratio data to zero at zero H_2 pressures. However, they did not report data for CH_5^+/CH_3^+ ratios of less than unity. If we ignore the two points in our Fig. 4 at lowest pressure, a zero intercept would also be predicted. Leventhal and Friedman (Ref. 14) report a non-

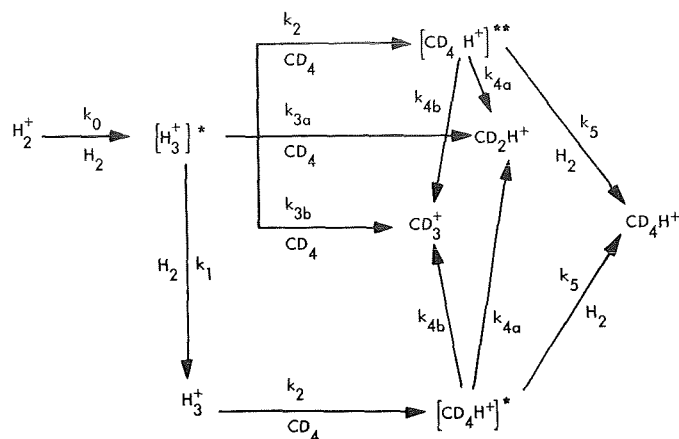


Fig. 5. Reaction mechanism of $H_3^+-CD_4$ system necessary to explain data of Fig. 4

zero intercept of CH_5^+/CH_3^+ for the reaction of $[H_3^+]$ with CH_4 , in spite of the fact their $[H_3^+]$ ions had 3.5-eV translational energy. The reason for the curvature at lowest pressure must be due to the direct reaction of $[H_3^+]$ to form CD_3^+ and CD_2H^+ .

The ion-molecule reactions in methane-hydrogen are seen to be quite complex. A variety of condensation, atom exchange, and charge transfer reactions take place even in a simple system such as this. It is seen that the relative pressures of the two gases, as well as the total pressure, play important parts in determining which reaction pathways will dominate.

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B. Vibrational Excitation of H₂ by Low- and Medium-Energy Electrons, S. Trajmar, D. G. Truhlar,³ J. K. Rice,³ and A. Kuppermann³

1. Introduction

It was realized in the early 1930s that low-energy electrons could excite molecular rotations and vibrations. The indications came from diffusion (Ramien) and swarm (Baily) experiments. It was found that the energy loss of the electrons diffusing through a gas was more than one would expect from momentum transfer considerations. A detailed study of this phenomenon became possible, however, only recently with the advances of electron beam techniques. Besides being of academic interest, these studies have direct implications concerning the fate of low-energy electrons in planetary atmospheres. Excitations of molecular rotation and vibration are the most important energy-loss mechanisms for slow electrons in the nonionized molecular gases.

In discussing vibrational excitation by electron impact, it is convenient to divide the field into three regions, according to the impact energy of the electron:

Impact energy, eV	Region
0-10	Low
10-100	Medium
100-100,000	High

The low-energy region has been studied quite extensively both theoretically and experimentally. The cross sections in this region are comparatively large (for H₂ the integral cross section for vibrational excitation is 2.0 a₀² at 3.5 eV and 0.03 a₀² at 50 eV) and frequently have sharp structures as a function of impact energy. This structure is associated with the energy states of the negative molecular ion that is formed by the capture of the electron. The magnitude and the behavior of the cross section can be well described by resonance theories.

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At high-impact energies (~30 keV), there have been some studies, and it was found that the excitation processes follow the optical electric dipole selection rules.

In this article, we will be concerned with the intermediate energy region, which has been almost completely unexplored; our experimental measurements and theoretical calculations on e-H₂ vibrational excitations will be discussed.⁴⁻⁶

Since elastic scattering is a special case of vibrational excitation in the calculations, and since the elastic cross sections are determined together with the vibrational ones in experiments, measurements and calculations concerning elastic scattering will also be presented.

2. Theory

The integral cross section is defined by the following equations:

$$Q_{pp'}(E) = \int \int \sigma_{pp'}(E, \theta, \varphi) \sin \theta d\theta d\varphi \quad (1)$$

where

$$\sigma_{pp'}(E, \theta, \varphi) = \frac{k_{p'}}{k_p} |f_{pp'}(\theta, \varphi)|^2 \quad (2)$$

and

$$f_{pp'}(\theta, \varphi) = \langle \Psi_{p'} | V | \Psi_p \rangle \quad (3)$$

Equation (2) can be derived from particle flux considerations, and Eq. (3) is obtained by formally solving the scattering problem by the methods of Green's functions. These equations are general without any approximation. p and p' are the quantum numbers associated with the initial and final states of the molecule. Q and σ are the integral and differential cross sections, respectively; E = impact energy, θ = scattering angle; k_p and $k_{p'}$ are the

⁴Truhlar, D. G., and Rice, J. K., "Electron Scattering by H₂ With and Without Vibrational Excitation. I. Quantum Mechanical Theory," *J. Chem. Phys.*, 1970 (in press).

⁵Trajmar, S., Truhlar, D. G., and Rice, J. K., "Electron Scattering by H₂ With and Without Vibrational Excitation. II. Experimental and Theoretical Study of Elastic Scattering," *J. Chem. Phys.*, 1970 (in press).

⁶Trajmar, S., Truhlar, D. G., Rice, J. K., and Kuppermann, A., "Electron Scattering by H₂ With and Without Vibrational Excitation. III. Experimental and Theoretical Study of Inelastic Scattering," *J. Chem. Phys.*, 1970 (in press).

initial and final wave vectors, respectively; f = scattering amplitude; Ψ is the solution of the Schrödinger equation of the composite system (molecule and electron) with the appropriate boundary conditions; and V is the interaction potential between the scattering electron and the molecule.

We can not calculate f_{pp} rigorously even for the simplest electron atomic (or molecular) systems. Therefore, we introduce the following three basic approximations:

- (1) Use a *semi-empirical* expression for the "interaction potential" (more precisely for the integral of the potential over the molecular electronic wave functions).
- (2) Apply the customary *Born-Oppenheimer adiabatic* approximation

$$\Psi_p = \psi_n(\mathbf{r}_1, \mathbf{r}_2, \mathbf{R}) \phi_{vJM}(\mathbf{R}) \varphi_p(\mathbf{r}_0) \quad (4)$$

- (3) Treat the direct scattering in the first *Born approximation* and the exchange scattering in the *Born-Ochkur-Rudge approximation*.

We will consider the molecule as a polarizable, aspherical, rotating, anharmonic oscillator with a realistic electronic ground state charge distribution.

a. Interaction potential. We write for the interaction potential:

$$V(\mathbf{r}, \mathbf{R}) = V^1(\mathbf{r}, \mathbf{R}) + V^2(\mathbf{r}, \mathbf{R}) \quad (5)$$

The first term on the right-hand side of Eq. (5) represents the static potential (interaction energy between the electron and the unperturbed molecule), and the second term is the polarization potential (correction to V^1 due to relaxation of the molecule). V^1 has been calculated accurately for H_2 at $R = R_e$ (the equilibrium internuclear separation) by Ardill and Davison. It is a quite tedious calculation and we need V^1 as a function of R ; therefore, we approximate V^1 by calculating the potential due to the interaction of an electron with two spherically symmetric atomic charge distributions centered on the two nuclei, where the atomic charge distributions have R dependent screening parameter, and we take the first two terms of the multipole expansion of this potential (V_0^c and V_2^c). Such a potential has no quadrupole term in it. The importance of the quadrupole interaction term, however, has been demonstrated in electron-molecule collisions that cause rotational excitation; therefore, we included a quadrupole term.

We have, therefore,

$$V^1(\mathbf{r}, \mathbf{R}) = V_0^c(r, R) + V_2^c(r, R) P_2(\hat{\mathbf{r}} \cdot \hat{\mathbf{R}}) - \frac{Q(R)}{2r^3} f(r) P_2(\hat{\mathbf{r}} \cdot \hat{\mathbf{R}}) \quad (6)$$

where $Q(R)$ is the molecular electric quadrupole moment and $f(r)$ is a cut-off function which is supposed to correct the expression which is only valid at $r = \infty$ (negates the singularity at $r \rightarrow 0$). It is much more difficult to evaluate $V^2(\mathbf{r}, \mathbf{R})$; however, it has been shown to be important in the scattering process. One could adjust $V^2(\mathbf{r}, \mathbf{R})$ to give best agreement with experiment. This would, however, compensate for the inherent inaccuracies of the scattering approximation; therefore, it is more reasonable to construct V^2 on the basis of our best understanding of electronic interactions.

In the *adiabatic approximation*, the electric dipole polarizability is given by

$$V^2(\mathbf{r}, \mathbf{R}) \xrightarrow[r \rightarrow \infty]{E \rightarrow 0} -\frac{\alpha(R)}{2r^4} - \frac{\alpha'(R)}{2r^4} P_2(\hat{\mathbf{r}} \cdot \hat{\mathbf{R}}) \quad (7)$$

where $\alpha(R)$ and $\alpha'(R)$ are the symmetric and asymmetric dipole polarizabilities, respectively. To be precise, one should really consider dynamic polarizabilities; however, it is very difficult to do. There are various ways to try to correct for nonadiabaticity. We take the approach to correct our adiabatic expressions to some extent by appropriate cut-off functions and take

$$V^2(\mathbf{r}, \mathbf{R}) = -\frac{\alpha(R)}{2r^4} g(r) - \frac{\alpha'(R)}{2r^4} g'(r) P_2(\hat{\mathbf{r}} \cdot \hat{\mathbf{R}}) \quad (8)$$

The cut-off functions are chosen to give best agreement for $V(\mathbf{r}, \mathbf{R}_e)$ with best elaborate calculations. $Q(R)$, $\alpha(R)$, and $\alpha'(R)$ were obtained from accurate calculations of Kolos-Wohniewicz (tabular forms).

With this procedure, we have avoided the integration in Eq. (3) over the electronic coordinates of the molecule, and the integrals we have to carry out to get the matrix elements for the scattering amplitude are only over the vibrational rotational coordinates of the molecule and over the coordinate of the scattering electron.

b. Born approximation and treatment of the nuclear motion. We take

$$\phi_{vJM}(\mathbf{R}) \varphi(\mathbf{r}) = \phi_v(R) Y_J^M(\hat{\mathbf{R}}) \exp(i\mathbf{k}_p \cdot \mathbf{r}) \quad (9)$$

and for the scattering amplitude in the Born approximation,

$$f_{pp'}(\theta) = -\frac{1}{2\pi} \int \exp[i\mathbf{q} \cdot \mathbf{r}] \int \phi_{v'}(R) Y_{J'}^{M'}(\hat{\mathbf{R}}) V(\mathbf{r}, \mathbf{R}) \phi_v(R) Y_J^M(\hat{\mathbf{R}}) d\mathbf{R} d\mathbf{r} \quad (10)$$

where

$$\mathbf{q} = \mathbf{k}_p - \mathbf{k}_{p'}$$

In Eqs. (9) and (10), the following approximations are included:

- (1) *Neglected vibrational-rotational interaction.*
- (2) *Born scattering approximation.* (The Born approximation and its applicability can be formulated in many different ways. Basically it assumes that the scattering interaction is just a weak perturbation on the incoming electron and that the electron after scattering is represented by a plane wave and neglects all scattered waves in solving the Schödinger equation of the problem.)

Carrying out the $\hat{\mathbf{R}}$ integration in Eq. (10), we average over all molecular orientations and get rid of the φ dependence in f . The results are then directly comparable to experiments where the molecules are randomly oriented.

Carrying out the integration in $\hat{\mathbf{R}}$ yields

$$f_{pp'}(\theta) = -\frac{1}{2\pi} \int \exp[i\mathbf{q} \cdot \mathbf{r}] \left\{ V_{0,v'v}(r) \delta_{JJ'} \delta_{MM'} + (-1)^{M-M'} \left(\frac{4\pi}{5}\right)^{1/2} V_{2,v'v}(r) C^{(2)}(J'M', JM) Y_{2,M-M'}(\hat{\mathbf{r}}) \right\} d\mathbf{r} \quad (11)$$

where

$$C^{(2)}(J'M', JM) = \left(\frac{4\pi}{5}\right)^{1/2} \int Y_{J'M'}^*(\Omega) \gamma_{2,M'-M}(\Omega) Y_{JM}(\Omega) d\Omega \quad (12)$$

and

$$V_{L,v'v}(r) = \int \phi_{v'}^*(R) V_L(r, R) \phi_v(R) R^2 dR \quad (13)$$

and L refers to the first and second terms of the multipole expansions ($L = 0$ or 2 , respectively). (Equation 13 implies that $\Delta J = 0, \pm 2$, and applies to either ortho or para H_2 .)

ϕ_v was determined numerically for the accurate potential well of Kolos and Wolniewitz. Then Eq. (13) was solved numerically, and $f_{pp'}$ and $\sigma_{pp'}$ were obtained from Eqs. (11) and (2). We sum σ over J' and M' (upper state) and average over J and M because we do not resolve rotational structure and the resulting cross sections are then directly comparable with experiments.

c. Inclusion of exchange. So far, we have assumed that the electrons were distinguishable and that the electron that came in was scattered and left the target. In reality, the electrons are indistinguishable and we have to assume the possibility that an electron other than the incoming one leaves the target after scattering. The effect of exchange can be treated as another semi-empirical term in the potential or by the Born-Ochkur-Rudge method. In our calculations, the latter approach was applied.

The scattering amplitude for exchange scattering is separately calculated and then combined with the direct scattering amplitude to give the overall cross section

$$\sigma_{p'p} = \frac{k_{p'}}{k_p} |f_{p'p} - g_{p'p}|^2 \quad (14)$$

$g_{p'p}$ is given by

$$g_{p'p}(\theta) = -\frac{E_2}{2\pi} \iint \phi_{v'J'M'}^*(\mathbf{R}) \phi_{vJM}(\mathbf{R}) V(\mathbf{r}, \mathbf{R}) \exp(i\mathbf{q} \cdot \mathbf{r}) d\mathbf{R} d\mathbf{r} \quad (15)$$

where

$$E_2 = \frac{q^2}{2[k_{p'} - i(2I)^{1/2}]^2}$$

I = ionization point

This expression is identical to the one giving the direct scattering amplitude except for the factor in front of the integral. In deriving it, however, the exchange of electrons was considered.

3. Experimental

Measurement of scattered signal intensities corresponding to vibrational excitation requires an instrument with high energy resolution and sensitivity. The vibrational features are separated by a few tenths of eV from the elastic peak, which in some cases is 10^4 times stronger than the vibrational feature. Our resolution is between 0.10 to 0.03 eV and our detector system can measure a current corresponding to one electron every 20 s. The electron scattering apparatus has been described previously (Ref. 1).

Figure 6 shows the vibrational excitation spectrum of H_2 at $E = 10$ eV, $\theta = 58.5$ deg, 58 scans, 1-s dwell time, 1.75 mV/channel. From the spectrum, one can obtain the relative scattering intensities corresponding to different features (transitions) for each fixed E_0 and θ .

To study angular dependence of cross sections, one has to obtain the scattering intensities from many spectra taken at many different angles. To be able to make a meaningful comparison of $I(\theta)$, it is required that

- (1) The experimental conditions be the same for each spectrum.
- (2) The efficiency of the instrument be independent of θ .
- (3) The proper correction for the change of "effective scattering path length" with θ be applied.

It takes a long time (weeks) to collect all angular spectra at a given E_0 , and the instrument usually drifts somewhat or breaks down during this time, which makes the study

of the angular behavior meaningless. To overcome this difficulty, we generate a spectrum which contains both the elastic and the vibrational features by superimposing many scans. From each spectrum, the inelastic to elastic scattering intensity ratio

$$R_{v'} = \frac{I_{0v'}}{I_{00}} = \frac{DCS_{0v'}}{DCS_{00}}$$

is determined. Then a calibration of the elastic scattering intensity alone is carried out under identical instrument conditions (which takes only about 1 h and can be well controlled).

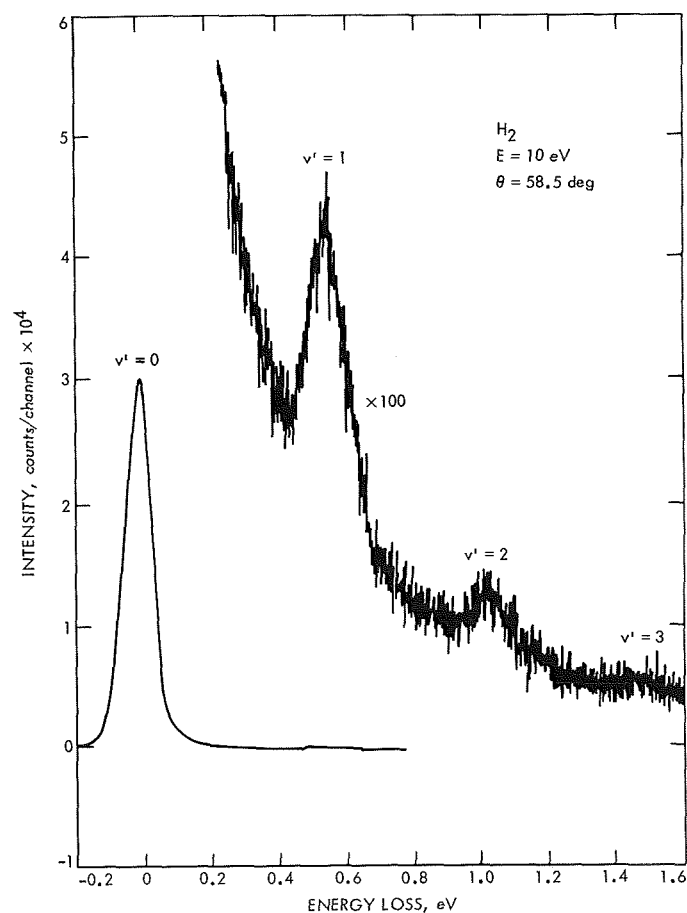


Fig. 6. Vibrational excitation spectrum of H_2 by 10-eV electrons

As the scattering angle changes, the volume from which scattering can be observed also varies; therefore, intensities measured at different scattering angles cannot be directly compared. The "volume-correction" procedure has been discussed elsewhere (Ref. 2).

From the volume-corrected elastic intensities and from the ratios, we can now get the relative differential elastic and vibrational cross sections in the same arbitrary units. (With the present apparatus, it is not feasible to measure absolute cross sections directly by measuring the absolute values of the incoming and scattered beam intensities, pressure, etc.) To put our measurements on the *absolute scale*, we need the absolute value of a differential cross section or an integral cross section, neither of which is available even for H_2 . Fortunately, total H_2 scattering cross sections have been determined by Golden, Bandel, and Salerno (Ref. 3). To achieve the *normalization* of the data by using the total cross section there are two requirements:

- (1) Differential cross section (DCS) is measured only in the 10–80 deg range; therefore, an extrapolation has to be carried out to 0 deg and to 180 deg.
- (2) Inelastic contributions, other than pure vibrational ones, have to be estimated at energies where their channels are open.

We were able to calibrate our cross sections at 7-, 10-, 13.6-, and 20-eV impact energies this way. At 45-, 60-, and 81.6-eV, we normalized the elastic cross section to our calculated integral elastic cross section, which seems to be quite justified because the experimental and calculated cross sections agree well both in magnitude and shape at and below 20 eV and in shape above 20 eV. Furthermore, one would expect that the calculation will be more accurate above 20 eV.

Figure 7 shows a comparison of experimental and theoretical (B/P and BOR/P, present calculation; WT/E, Wilkins and Taylor; R-HL, Rozsnyai; H, Hara) calculations of elastic differential cross sections at 10-eV impact energy.⁷ The previous relative experimental data were normalized to ours at 60 deg. For more details on the methods used in the different calculations, see papers listed in Footnotes 4–6.

⁷See original papers for meaning of * associated with numbers 1, 2, and 3 on right-hand side of Figs. 7, 8, 10, 11, and 12.

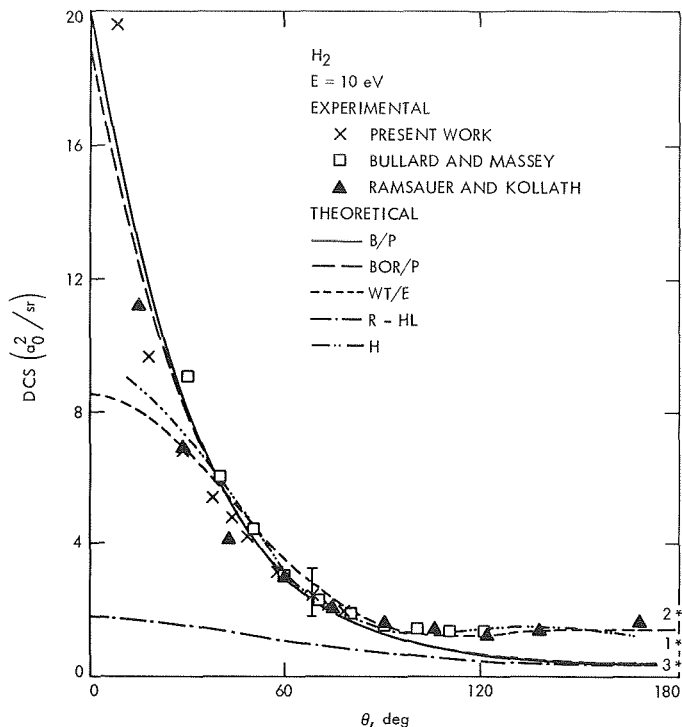


Fig. 7. Experimental and theoretical elastic differential cross sections at 10-eV impact energy

The same quantities at 60-eV impact energy are shown in Fig. 8. Our calculation, which includes the polarization term, gives excellent agreement with experimental data, while the other calculations that neglected polarization (R, Rozsnyai; BORXP₀, present calculation without polarization and with only symmetric terms in the potential included) deviate from the experimental data at low angles. The inclusion of exchange (BOR/P curves) is not important at impact energies above a few eV. The integral elastic cross section Q_{00} as a function of impact energy is shown in Fig. 9. The calculations that neglected exchange (B/P, BXP, and WTXE curves) give unreasonably large cross sections at energies below about 5 eV.

Figures 10 and 11 show the differential cross sections for the various vibrational excitations $\sigma_{0v'}$ at 10-eV impact energy. The curve marked RESONANCE is the prediction of resonance theory (with the scaling parameter adjusted for optimum agreement with experiment). The open circles are the measured values of Ehrhardt et al. (Ref. 4) normalized to our measurement at 60 deg. The other symbols are the results of our measurements. There is an excellent agreement between the experimental values and our calculated values (B/P and BOR/P curves) both in shape and in magnitude for the first vibrational

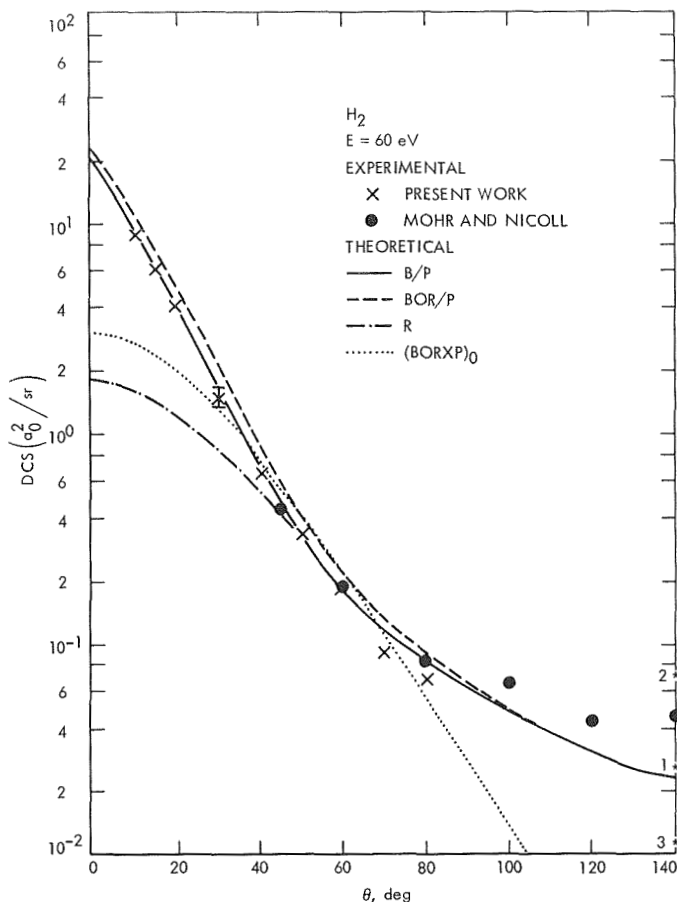


Fig. 8. Experimental and theoretical elastic differential cross sections at 60-eV impact energy

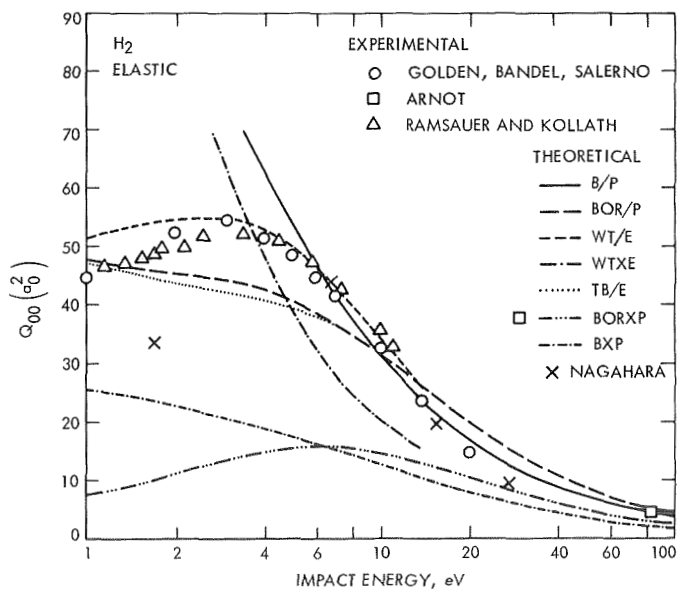


Fig. 9. Integral elastic cross section as a function of impact energy

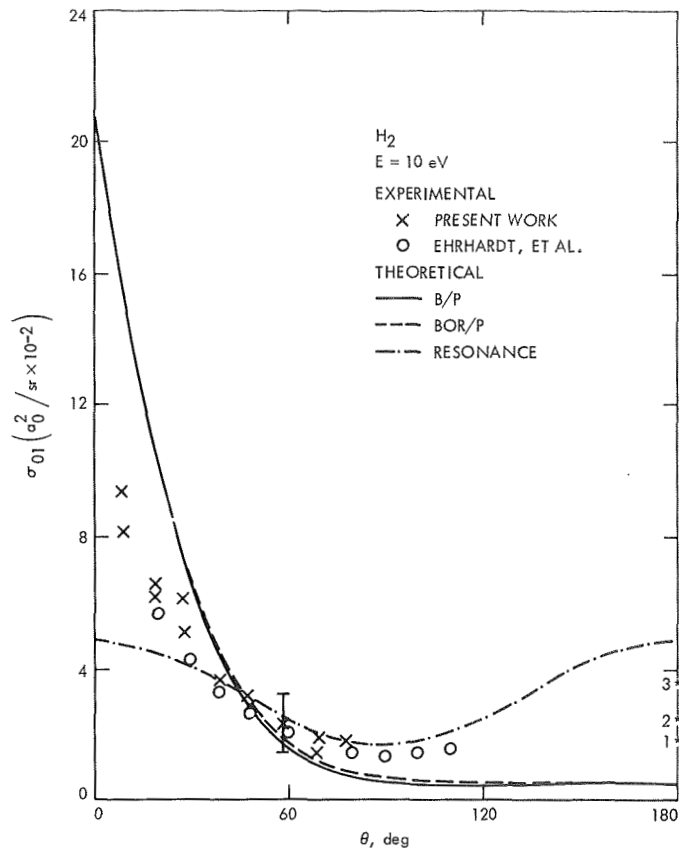


Fig. 10. Differential cross sections for excitation of the first vibrational levels of H_2 by 10-eV electrons

excitation up to about 100-deg scattering angle. The failure of first-order theories to predict the correct rising cross sections at very high angles is due to the neglect of the distortion of the scattered electron wave function. The calculated cross sections for the excitation of the first- and second-overtone vibrations are too low. The reason for this large discrepancy is that our calculation yields only the potential scattering contributions to the cross section, and for these excitations the resonance excitation still dominates at 10 eV (see Footnote 6).

At 60-eV impact energy, resonance effects are absent and the calculated potential scattering cross sections for the overtone agree with the experimental ones (Fig. 12).

Figure 13 shows the integral cross sections for the excitation of the first vibrational level of H_2 (Q_{01}) as a function of impact energy. The symbols on the dotted curves indicate the terms of the electron molecule interaction potential that were included in the calculation to get that specific curve. The abbreviations have the following

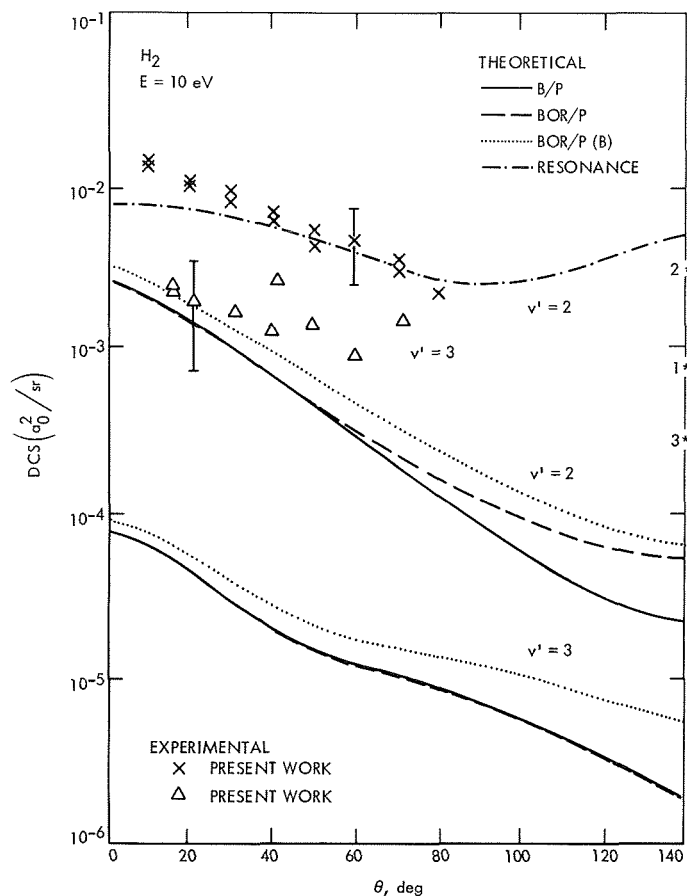


Fig. 11. Differential cross sections for excitation of the second and third vibrational levels of H_2 by 10-eV electrons

meanings; S = short range, P = polarization, Q = quadrupole. The indexes 0 and 2 refer to the spherically symmetric and asymmetric part of the potential, respectively.

4. Conclusions

The calculations with our semi-empirical potential give very good agreement with experiment for the elastic cross sections in the 1-912 eV energy and 0-90 deg angular ranges.

The agreement in the vibrational cross sections is somewhat poorer but still better than any previous calculation can claim. There are large deviations at low energies in the first and second overtone vibrational cross sections as compared to experiment. This discrepancy can be explained by the resonance scattering contributions to the cross section.

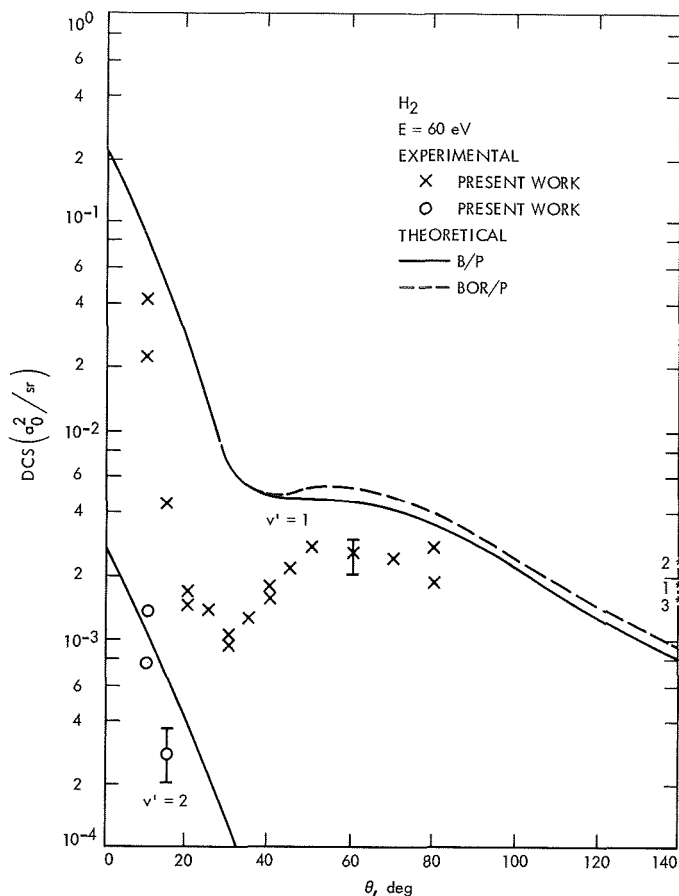


Fig. 12. Differential cross sections for excitation of the first and second vibrational levels of H_2 by 60-eV electrons

The conclusions concerning the interaction potential are:

- (1) Most of the contributions to the cross sections come from the symmetric part of the potential for both elastic and inelastic scattering.
- (2) Polarization is very important in determining the low-angle and low-energy behavior of the cross sections.
- (3) Exchange is important for low-energy elastic scattering only.
- (4) The distortion of the scattered wave has to be included in the calculations to yield correct cross sections at very high angles.
- (5) The short range interactions are much less effective in causing vibrational excitation than elastic scattering.

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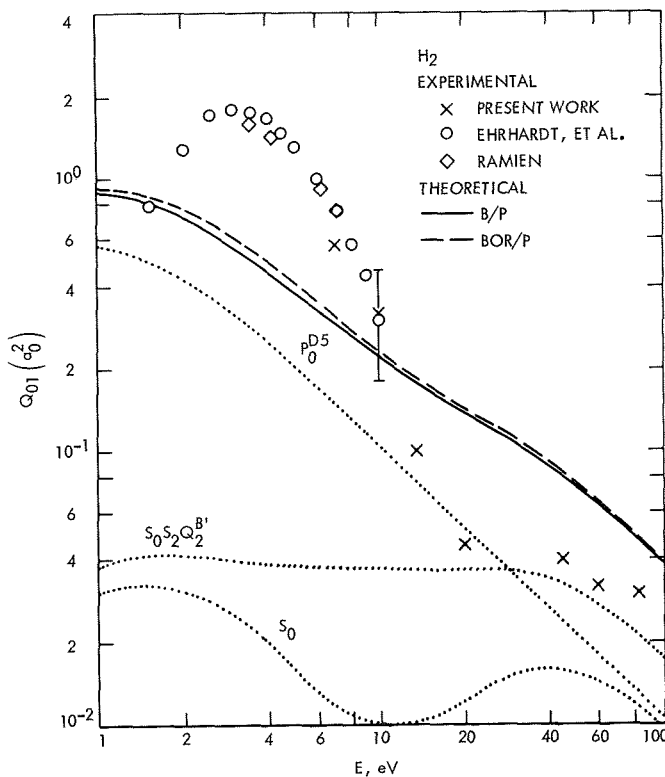


Fig. 13. Integral cross sections for the excitation of the first vibrational level of H_2 as a function of electron impact energy

C. Supersonic Boundary Layer Transition Studies, J. M. Kendall, Jr.

It is of importance in the engineering design of fluid flow systems to know whether boundary layers are laminar or turbulent. An enormous effort has been expended during the last two decades on the study of the transition to turbulence at supersonic and hypersonic speeds, and has resulted in various rules contrived for the purpose of predicting the occurrence of transition under specific circumstances. These are purely empirical correlations of observational data, and do not embody any specific information

concerning the process by which transition occurs. They yield no more than an exceedingly rough prediction. There exists virtually no understanding of the transition process in the high-speed range which might temper or supplant the use of such rules.

For the case of flows in supersonic wind tunnels, it has been proposed that sound-like disturbances in the air stream cause perturbations of the boundary layer on a test model which are amplified by the Tollmien-Schlichting instability to such level that turbulence ensues in the manner occurring at low speeds. It recently became possible to put this idea to a test. Through a lengthy investigation of the boundary layer stability theory,⁸ Mack has shown that, for a Mach number $M = 4.5$, the insulated-wall flat-plate boundary layer can amplify the fluctuation energy by not more than a factor of 10 in the region of the layer preceding the observed site of transition. This, together with the known amplitude of the sound field, which is taken as an input, predicts a final amplitude substantially lower than that which exists at transition. Hence, in this particular case, the instability mechanism is inadequate to explain the occurrence of transition. It is to be emphasized that this conclusion applies only to flows in wind tunnels, and not to free-flight flows where stream characteristics are quite different.

Experiments aimed at clarifying the mechanism of transition are currently being carried out in the JPL 20-in. supersonic wind tunnel at Mach numbers $M = 2.2$, 3.0, and 4.5. Except where noted, the tunnel is operated with turbulent wall boundary layers. These layers radiate pressure disturbances. In most tests to date, the laminar boundary layer on a 14-in. long flat plate which spans the 18-in. width of the tunnel is studied. A 6-in. wide, 8-in. long flat plate was used in a directionality experiment. The leading edge thickness of each plate t was of the order of 5×10^{-4} in. This case is denoted by $t = 0$. In some tests, t was temporarily increased by affixing a strip of 0.010-in. thick metal to the bevelled bottom side of the plate, flush with the leading edge. Following each set of measurements with the strip in place, the strip was remotely detached without stoppage of the tunnel for a direct comparison with the $t = 0$ case. Constant-current hot-wire anemometers are used to measure the flow fluctuations. Auxiliary equipment includes wave analyzers, time-delay correlation instruments, an amplitude probability analyzer, and tape recording equipment.

⁸Mack, L. M., *Boundary-Layer Stability Theory*, Nov. 1969 (JPL internal document).

Results obtained at $M = 4.5$ are described first. For the case $t = 0$, the profile of fluctuation energy recorded by a hot wire traversed through the laminar layer at any station ahead of transition showed a single relatively sharp peak at $y \cong 0.8\delta$, where δ is the thickness of the layer. Such has been reported previously, and data on the location of the peak for several values of M are given in Ref. 1. The same result is found here to hold for each spectral component of the energy. For the case $t = 0.01$ in., near the leading edge the energy reached a peak at two vertical locations. The amplitude of the inner peak diminished with x and disappeared. The location of the outer peak was similar to that of the single peak appearing in the $t = 0$ case. Only the outer peak is considered in subsequent discussion. Figure 14 presents the x -variation of the peak value of fluctuation energy for four particular frequencies and for $t = 0$ and 0.01 in. For both values of t , transition would end at about 15–18 in. if the plate were longer. The energy scale is arbitrary, but the several curves may be compared among themselves. It is seen that increasing t to 0.01 in. drastically reduces the fluctuation level for all but the lowest frequencies. Such is consistent with the well-known delay of transition caused by modest increases in leading edge bluntness. More importantly, in contrast to expectations based upon stability theory, fluctuations of all frequencies grow monotonically.

Before estimating the magnitude of the fluctuations and comparing their growth with theory, it will be demonstrated in three ways that the fluctuations are the result of forcing by the tunnel stream sound field. First, the tun-

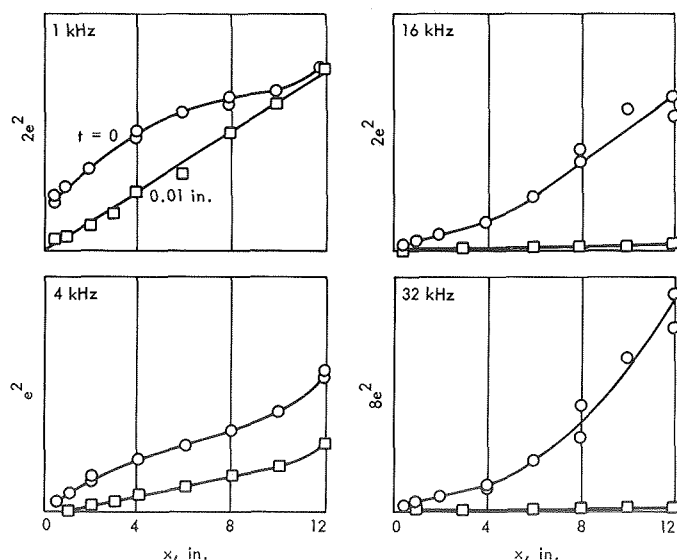


Fig. 14. Boundary layer fluctuation growth ($M = 4.5$, $Re = 95,000/in.$)

nel was operated at a sufficiently low supply pressure that the wall boundary layers were laminar. One of the four layers was then tripped to turbulence by means of air jets. A hot-wire probe was affixed to the small test plate such as to sense the peak fluctuation energy at $x = 7$ in. The plate was rotated about the roll axis. The boundary layer response, shown in Fig. 15, was largest when the plate faced the turbulent layer and smallest when it faced away from the turbulent layer.

Second, the spectral characteristics of the fluctuations in the boundary layer near the plate leading edge bear such close resemblance to those of the free stream that the former must surely be the consequence of the latter. Figure 16 presents one-dimensional energy spectra measured in the stream and in the layer at the energy peak for two values of x . It is seen that the three spectra are generally similar in shape. It was originally intended to determine the two-dimensional spectra, which would give wave-obliqueness information as well. For that purpose, measurements were made of the correlation $R(z)$ of fluctuations of various frequencies at two points separated by a lateral distance z . The Fourier cosine transform of $R(z)$ would then yield the spectrum of waves in the z direction. However, as can be seen for the two cases given in Fig. 17, R did not always vanish at the largest values of z attainable, so that the transform could not be computed without further assumption. Nevertheless, the close similarity of the two sets of data, together with the results of Fig. 16, indicates that the fluctuations in the boundary layer initially possess the same two-dimensional spectrum as those of the stream even though they are much larger in energy.

Third, fluctuations in the boundary layer were found to be strongly correlated with those of the free stream. One hot-wire probe was mounted such that the wire was about

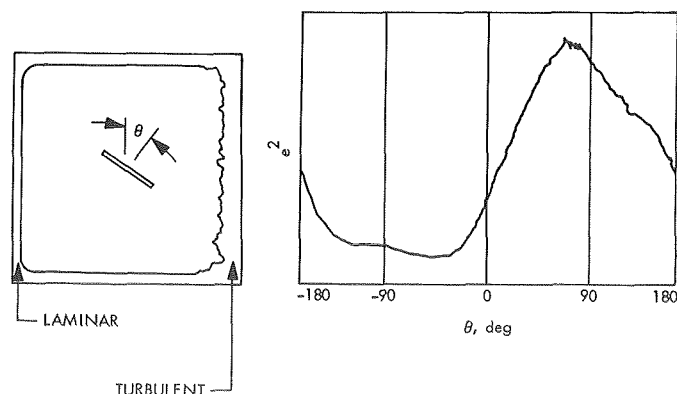


Fig. 15. Directionality experiment ($M = 4.5$)

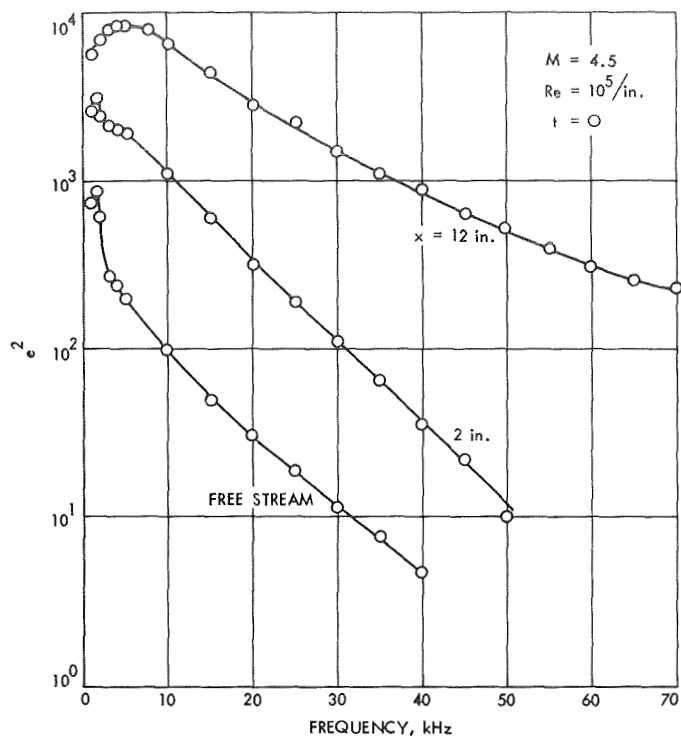


Fig. 16. Energy spectra

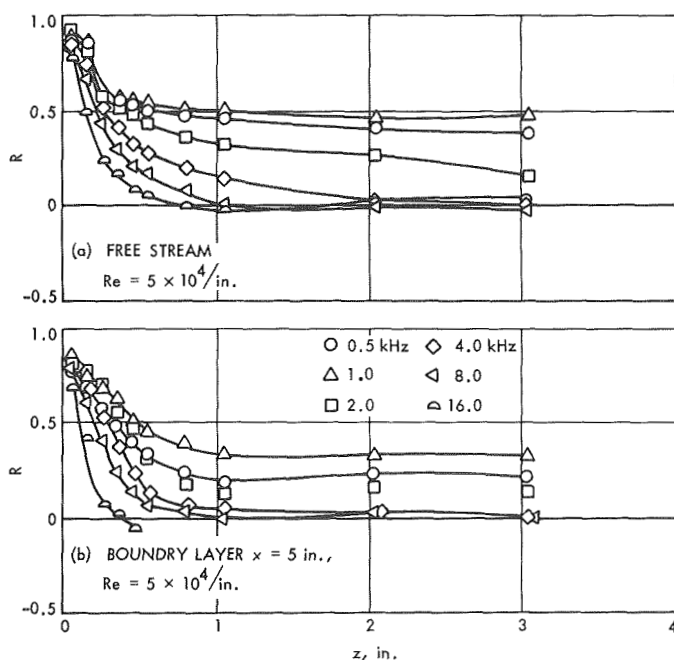


Fig. 17. Correlation coefficient

0.05 in. below the plate leading edge. The wake of this probe was carried below the plate. Another hot wire was positioned above the plate in lateral alignment with the first, at the y location of the energy peak, and at various

values of x . The cross correlation of the two signals as a function of time delay clearly showed that free-stream disturbances incident upon the leading edge persisted downstream into the transition region. It is not to be inferred that energy enters the layer only at the leading edge. From these space-time correlation measurements, the propagation speed of the boundary layer fluctuations was found to be approximately $0.64 u_\infty$, where u_∞ is the free-stream speed. This value nearly matches that of the empty-tunnel sound field. The sound field propagation speed was measured in this test by the two-wire technique, and in earlier tests performed by Laufer by using an altogether different technique (Ref. 2). It can be inferred that the boundary layer disturbances are not instability waves, which propagate at $0.7-0.9 u_\infty$, depending on wave length, at this Mach number.

Mack has presented calculated values of the disturbance amplification A/A_1 as a function of frequency (Footnote 8). A is the amplitude of a constant frequency disturbance at any value of R , and A_1 is the amplitude at the start of the unstable region; R is the square root of the x Reynolds number. The present data have been analyzed to obtain an amplification ratio A/A_∞ , where A and A_∞ are taken to be the square root of the fluctuation energy in the boundary layer and in the free stream ahead of the test plate at any dimensionless frequency $\omega v/u_\infty^2$. The theoretical and experimental amplification ratios are presented in Fig. 18a for the case $M = 4.5$. Comparison between the two can only be made on a qualitative basis because the equality of A_1 and A_∞ was not determined. It is seen that the measured amplification ratios do not exhibit the peaked response of the theoretical curves, and are substantially larger than the theoretical values to be expected at comparable values of R .

The magnitude of the flow fluctuations in the boundary layer preceding transition cannot be determined accurately. The fluctuations are so large that the hot-wire response must certainly be nonlinear. Moreover, the usual technique for the quantitative determination of the mass flow and temperature fluctuations from hot-wire measurements fails because it is assumed that the fluctuations are small. However, an order-of-magnitude estimate of the fluctuation level can be obtained in two ways. First, it is seen from Fig. 18a that the boundary layer fluctuations become at least 20 times larger than the free-stream fluctuations. It can be determined from Laufer's measurements (Ref. 2) that the free-stream mass-flow fluctuation is about 1.1% rms at this Mach number. The ratio of the peak-to-peak to rms fluctuation is of the order of 3 or 4. Thus, the peak-to-peak fluctuation in the layer is of the

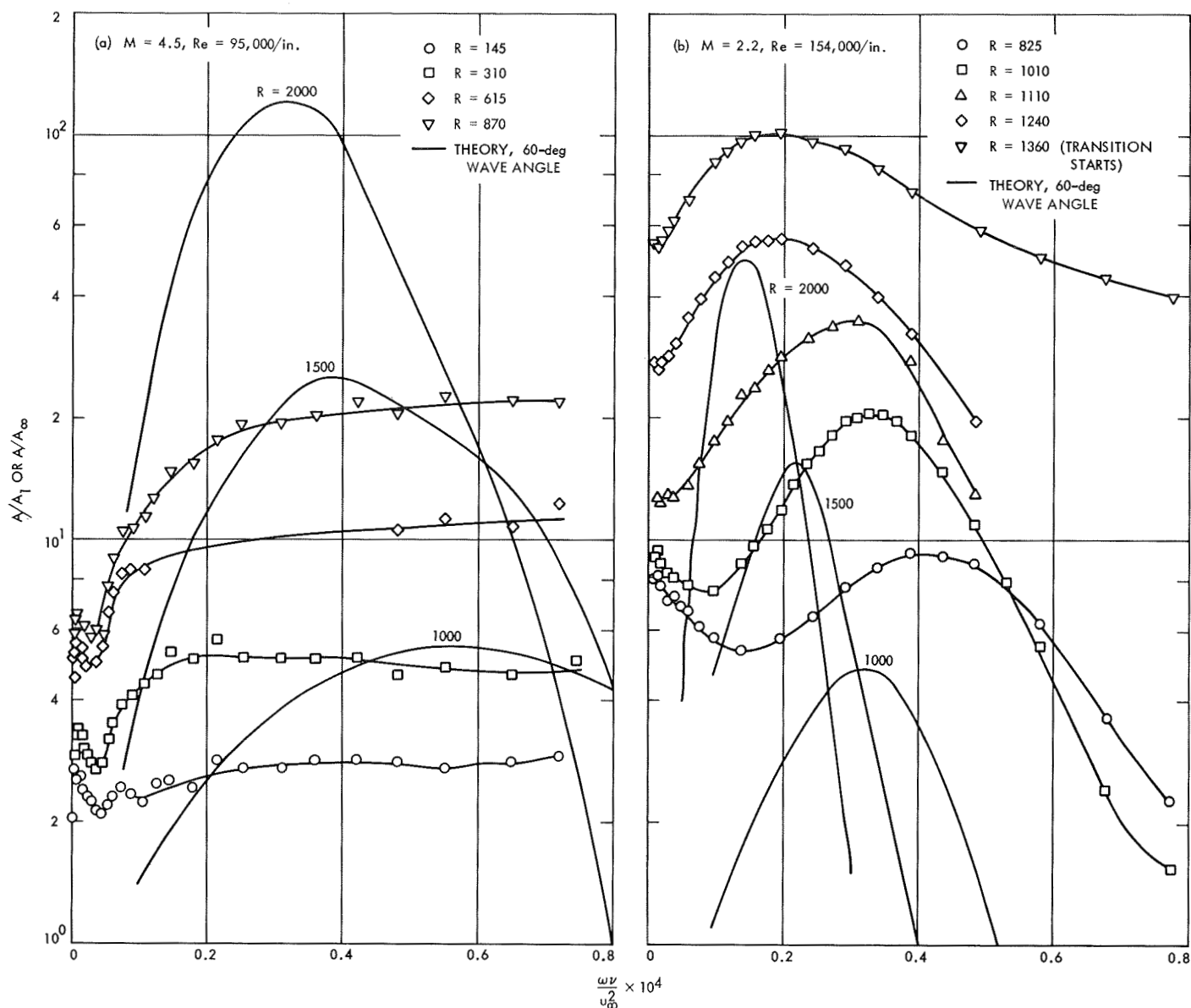


Fig. 18. Fluctuation amplitude ratio

order of 60–80% of the mean value. Second, for the case Reynolds number/in. = 1.7×10^5 , $x = 8$ in., the average dc voltage drop across the hot wire operated at constant current changed by 40 mV as the wire was traversed across the boundary layer. The voltage fluctuations observed at the fluctuation energy peak were more than 30 mV peak-to-peak.

Results obtained at $M = 3.0$ and 2.2 are considered now. At $M = 3.0$ and $t = 0$, the profile of fluctuation energy through the boundary layer showed two peaks of approximately equal amplitude for all values of x preced-

ing transition. Curves of A/A_∞ , where A is the amplitude at either peak, showed a clear similarity with regard to shape and numerical value to those of Fig. 18a. Thus, stability theory is apparently not more successful in accounting for the fluctuation growth at $M = 3.0$ than at 4.5 .

The situation at $M = 2.2$ was somewhat different. As reported by Laufer and Vrebalovich (Ref. 3), some of the higher frequency waves were found to exhibit regions of damping as predicted by theory. However, low-frequency waves which should have been damped were found to

grow rapidly, so that the total energy growth exceeded the predicted value. Amplification ratio data are compared with theory in Fig. 18b.

For $M = 45$, it was shown that the boundary layer fluctuations were initiated by the tunnel sound field. It was also shown that the fluctuation energy growths for $2.2 < M < 4.5$ are larger than can be accounted for by instability amplification. It may be speculated that these growths are attributable to an energy transfer from the stream to the boundary layer along its entire length. No entirely relevant theoretical study of this problem appears to have been made yet. It is believed that an essential feature that must be incorporated in an analysis is that the pressure field travels with a speed substantially less than u_∞ . Under that circumstance, there exists one layer of fluid in the boundary layer whose speed equals that of the pressure field, and which is, therefore, most strongly driven.

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D. Electron Impact Excitation of N_2 , R. T. Brinkmann and S. Trajmar⁹

1. Introduction

Nitrogen molecules comprise some 80% of our lower atmosphere and are the dominant constituent over much of the auroral and airglow regions of the upper atmosphere. Nitrogen emissions are prominent features of the aurora (Refs. 1-4) and, to a lesser extent, the airglow (Refs. 4-6). Most auroral emissions are caused by fast electrons (1 to 10 keV) dumped into the atmosphere in some not yet understood manner by the geoelectric and geomagnetic fields. Many airglow emissions result from the slowdown of photoelectrons possessing a few tens of eV, created in the earth's upper atmosphere by solar extreme ultraviolet radiation. One of the principal aims of auroral and airglow research is to understand the relation-

ship between emission morphology and spectroscopy, on the one hand, and particle precipitation or fluxes on the other. To do this quantitatively, however, one should have a complete and reliable set of the relevant cross sections for all the important atmospheric species. Unfortunately, there has existed no such set of cross sections for any species, whether of atmospheric interest or not. There have been a fairly large number of experimental studies which have lacked the completeness necessary for energy degradation studies (Refs. 6-8). And few of these results have been put on an absolute scale. Stolarski and Green (Ref. 9) and Green and Barth (Ref. 10) have done calculations with a complete set of cross sections, but the calculations are of dubious reliability because of the lack of adequate laboratory data for many important transitions. The cross sections of Takayanagi and Takahashi (Ref. 11) suffer the same drawback. In this article, we present and utilize a set of experimental cross sections for N_2 which we believe to be complete enough and reliable enough to be of use in atmospheric calculations.

2. Experimental

The electron impact spectrometer, consisting basically of an electron gun, hemispherical electrostatic monochromator, scattering chamber, hemispherical analyzer and detector, has been described elsewhere (Refs. 12 and 13). The present survey covered the angular range of 0-80 deg, the incident energies 15, 20, 30, 60, and 80 eV, and all possible energy losses. Some measurements were also carried out at 10-, 12-, 40-, and 100-eV electron impact energies but the data have not yet been fully reduced. Details of the experimental procedure, calculation of the effective scattering volume correction as a function of scattering angle, and estimation of the uncertainty introduced by the extrapolation beyond 80-deg scattering angle were discussed in Refs. 14 and 15 and are not reproduced here.

Figure 19 shows an energy-loss scan for 40-eV incident energy and a 10-deg scattering angle. The $A^3\Sigma_u^+$ state barely shows up at $\times 100$ magnification. Three-quarters of this state lies beneath the $B^3\Pi_g$ state, however, and the integrated $B^3\Pi_g$ intensity is not usually much greater than that of $A^3\Sigma_u^+$. The separation of the two states has been made by measuring the $A^3\Sigma_u^+$ intensity below the threshold of the $B^3\Pi_g$ state and applying corrections calculated from the Franck-Condon factors (Ref. 16). Similar corrections were applied to the other triplet and the $a^1\Pi_g$ states. The $a^1\Pi_g$ state gives rise to the important Lyman-Birge-Hopfield bands. Three vibrational levels of the C^3 are visible while a single peak marks the E^3 state. The $a''^1\Sigma_g^+$

⁹G. R. Steffensen and W. H. Williams helped with the experimental aspects of this investigation. We are also indebted to D. C. Cartwright, L. Broadfoot, and R. C. Carlson for useful discussions.

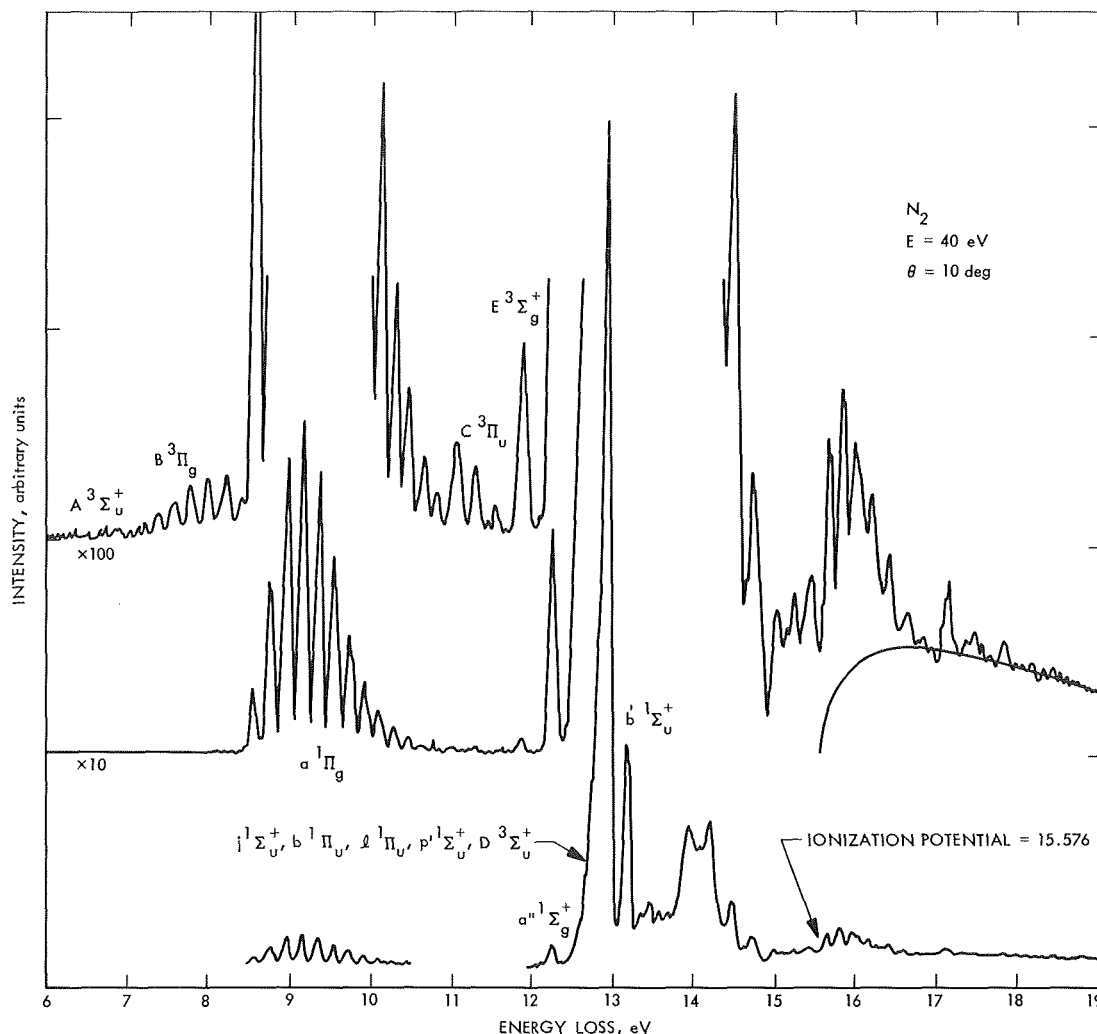


Fig. 19. Energy loss spectrum obtained at 40-eV incident energy E and 10-deg scattering angle θ (some of the various transitions from the ground state are indicated by their upper level designations)

is the only other symmetry-forbidden singlet of measurable intensity. Between this state and the first ionization potential, there is a complex maze of states, which have been arbitrarily assigned to designated energy-loss intervals. Above the ionization limit is seen, in addition to the continuum, a number of discrete states which have been identified as Rydberg levels (Ref. 17). Although the spectrum in Fig. 19 ends at 19-eV energy loss, data were taken at least out to one-half the sum of the incident energy and the ionization potential, the maximum possible energy loss under most conditions. Scans of the secondary electrons—that is, at greater energy losses than this—were frequently made, but these data were not used in the present analysis. Figure 20 is an energy level diagram for N_2 .

Figures 21 and 22 present the dependence on incident energy of the integral cross sections for electron impact

excitation of some typical states or groups of states from the ground state. The behavior of the various groups—spin-forbidden and/or symmetry-forbidden transitions, optically allowed transitions, and regions of the ionization continuum—are quite distinctive and could be used to assign the character of unknown states, were any present in sufficient intensity. The angular dependencies of the differential cross sections are also quite distinctive, but this will not be considered here. We observed no evidence for any spin-allowed, symmetry-forbidden state between the a'' $1\Sigma_g^+$ and the first ionization limit.

The normalization of the cross sections was made to the absolute measurements of the C $3\Pi_u$ by Burns et al. (Ref. 18) at 15 eV. At higher energies, however, their cross sections do not fall off rapidly enough to approach the Born behavior, strongly suggesting the presence of a

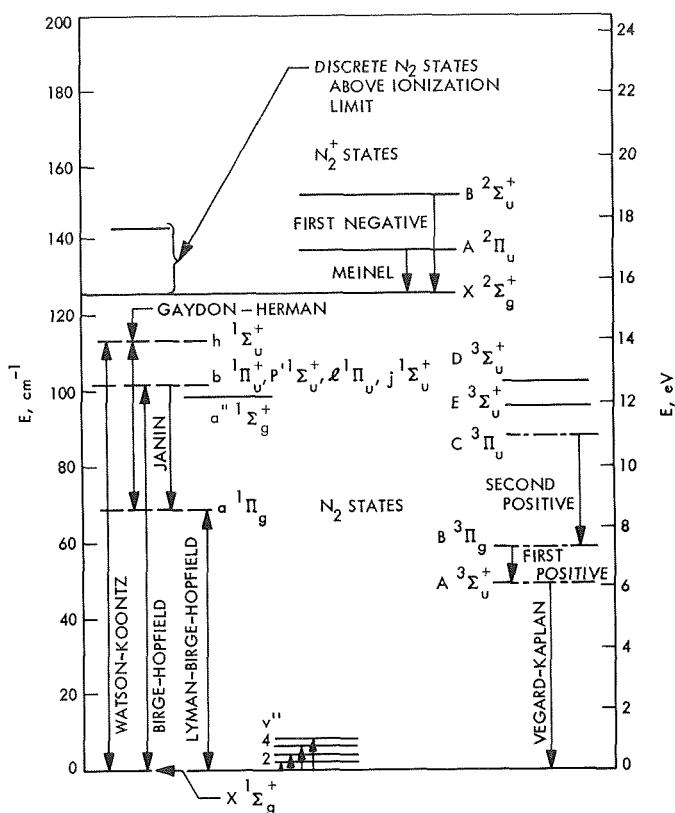


Fig. 20. Energy level diagram for N₂

spurious effect, possibly excitation produced by secondary electrons. At 60 and 80 eV, therefore, we have normalized the cross sections to the total ionization cross sections of Rapp and Englander-Golden (Ref. 19). Normalization at 20 and 30 eV was a compromise between the two approaches. The uncertainty in the normalization is estimated to be 20% at 15 eV, 30% at 20 eV, 20% at 30 eV, and 10% at higher incident energies.

3. Excitation Calculation: Monte Carlo Routine

A computer program, using a Monte Carlo procedure, was written to calculate the excitation frequency of the various energy-loss channels, which occur as fast electrons are slowed down by collision in nitrogen gas. The energy of the incident electron is specified and, one by one, such electrons are injected into the top of an N₂ atmosphere. The electron impact excitation and ionization cross sections (both differential and integral) described above were read into the computer, and pseudo-random numbers were generated to determine distances between collisions, scattering angles, the channels to be activated, etc. Fifty channels were used, most of them listed in Table 1, of which 19 were nonionizing and the rest ionizing. At incident energies less than 15 eV and greater than 80 eV, the

Table 1. Number of excitations caused by 1000 electrons of 100-eV energy

Designation	Feature or energy loss interval ^a	Number of primary excitations	Number of secondary excitations	Sum
el	elastic	20778	9983	30761
A ³	A ³ Σ _u ⁺	54	27	81
B ³	B ³ Π _g	68	57	125
a ¹	a ¹ Π _g	160	36	196
C ³	C ³ Π _u	52	25	77
E ³	E ³ Σ _g ⁺	6	1	7
1	a'' ¹ Σ _g ⁺	22		22
2	12.40-13.05	454	10	464
3	13.05-13.30	127	2	129
4	13.30-13.80	151	4	155
5	13.80-14.10	169	2	171
6	14.10-14.40	164	1	165
7	14.40-14.62	69	3	72
8	14.62-14.90	41	2	43
9	14.90-15.58	66	2	68
a	15.58-16.00 i	43	2	45
b	16.00-16.80 i	88	2	90
10	15.58-17.02 d	74	2	76
c	16.80-17.79 i	94	1	95
11	17.02-17.30 d	9	1	10
12	17.30-17.70 d	8		8
d	17.79-18.79 i	93	3	96
13	17.70-18.00 d	2		2
e	18.79-19.79 i	73	1	74
f	19.79-20.79 i	55		55
g	20.79-22.79 i	131		131
h	22.79-24.79 i	71		71
i	24.79-27.79 i	91		91
j	27.79-30.79 i	66		66
k	30.79-33.79 i	40		40
l	33.79-37.79 i	36		36
m	37.79-42.79 i	24		24
n	42.79-47.79 i	14		14
S1	47.79-70.00 i	16		16

^ai = ionization continuum, d = discrete features.

angular distributions were taken to be the same as at 15 eV and 80 eV, respectively. Below 15 eV and above 80 eV, the integral cross sections were extrapolated. Below 15 eV, optical excitation data (i.e., optical observation of electron impact excitation) such as that of Burns et al. (Ref. 18) and McConkey and Simpson (Ref. 20) were used as a guide for the extrapolation, in conjunction with electron impact data obtained by us at 12-eV and 10-eV incident energy. Above 80 eV, the cross sections were made to gradually approach the appropriate Born dependence—log (cE)/E, 1/E, or 1/E³ (where c = 0.070 eV⁻¹), depending on the nature of the transition.

The scattering angle of the secondary electron (relative to the line of approach of the incident electron) is obtained

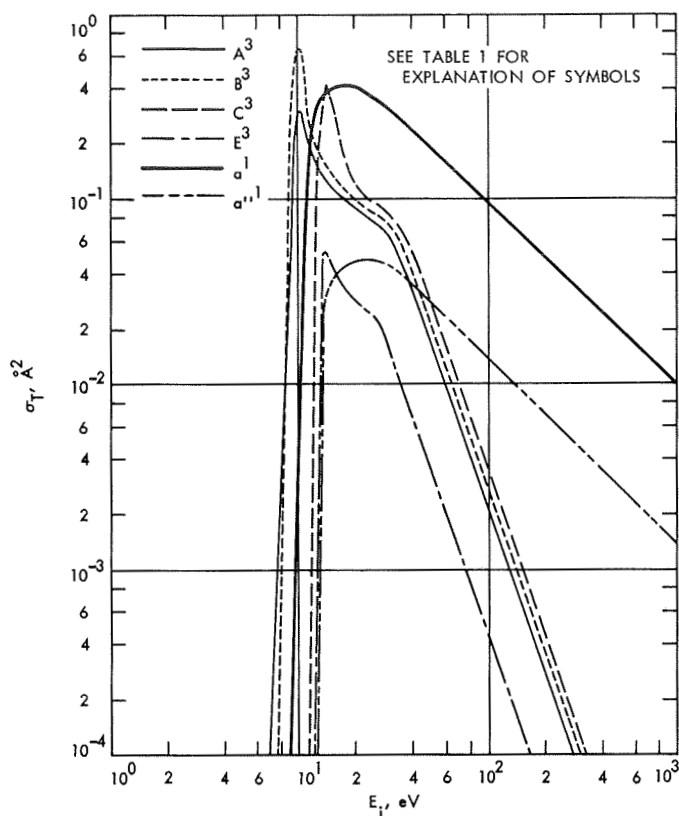


Fig. 21. Dependence of integral electron impact cross section σ_T on incident energy E_i for optically forbidden transitions considered

from an empirical simplification of the coincidence data obtained by Ehrhardt (Ref. 21): half of the secondary electrons are presumed to scatter at 4 times the primary scattering angle and the other half at π radians plus 4 times the scattering angle.

4. Results

To illustrate the use of the cross sections and the operation of the computer program, we have chosen to present results for calculations for 100-eV incident electrons. This energy is intermediate between the electron energies of greatest interest in connection with airglow and aurora. Ten groups of 100 electrons (100 eV each) were followed by the Monte Carlo method as they entered an N_2 atmosphere from above and either lost their energy in the gas or escaped out the top. For the entire calculation (1000 incident electrons), 984 electrons escaped out the top of the atmosphere. They carried 59,342 eV. Many of these were secondary electrons. The statistical accuracy (rms deviation among the 10 runs) of these results is 6% and 5%, respectively. The thickness of atmosphere was great enough that only one electron, with 6.7 eV, penetrated to

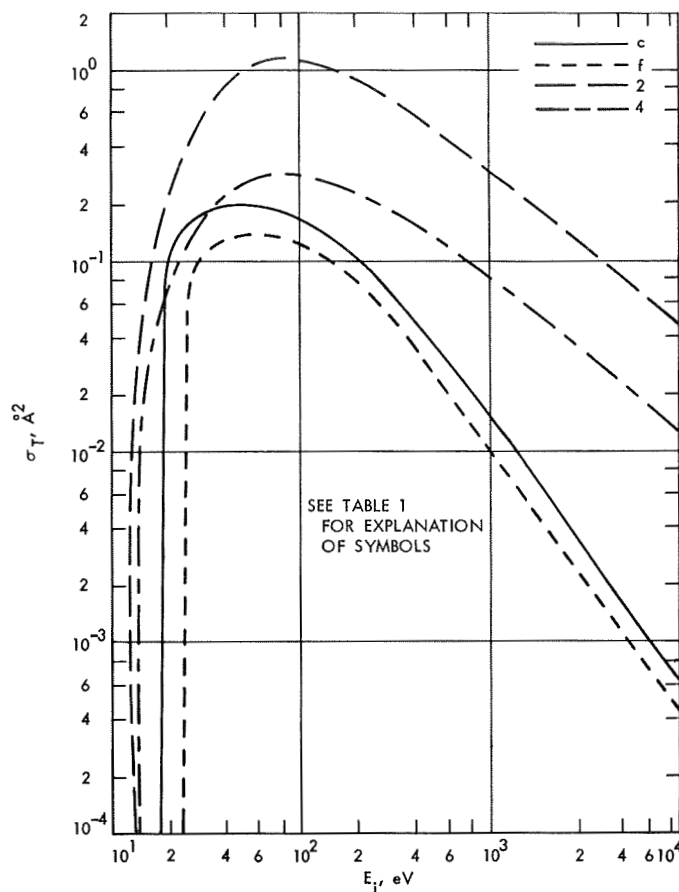


Fig. 22. Dependence of integral electron impact cross section σ_T on incident energy E_i for two representative groups of optically allowed bound state transitions (2 and 4) and two representative regions of the ionization continuum (c and f)

its base. We neglect this electron. There were a total of 944 ionizing collisions. The energy associated with an ion pair is then

$$\frac{(1000 \times 100 \text{ eV}) - 59,342 \text{ eV}}{944 \text{ ion pairs}} = 43.07 \text{ eV/ion pair}$$

Electrons which did not escape the atmosphere at the top or bottom interacted with the N_2 medium until they had less energy than the threshold of the lowest inelastic electronic state. This slow-electron production was kept track of but will not be discussed further here. In Table 1 are collected the number of excitations—primary, secondary, and total—for each of the channels which were accessible to 100-eV electrons. No tertiary or higher order excitation occurred because of the low energy of the incident electrons. Primary excitations, as used here, are those caused

by primary electrons; secondary excitations are those caused by secondary electrons.

We see from Table 1 that more than 90% of the collisions were elastic. Two-thirds of the elastic collisions were primary. The importance of secondary excitation decreases with increasing energy loss, as a general rule, and is much greater for triplets than for singlets. Almost half of the B^3 excitation is by secondary electrons. Only about one-fiftieth of the b' state (which is really a composite of several singlet states) excitation is by secondary electrons.

These numbers can be readily converted into emission rates. We assume, for purposes of normalization to the intensity of an IBC III aurora (which is defined by this normalization), that 6½% of the total ionization cross section is due to the 3914-Å vibrational band of the $B^2\Sigma_u^+ \rightarrow X^2\Sigma_g^+$ transition (the first negative system) (Refs. 22–28). Older measurements (Refs. 27–30) give somewhat lower values. The total intensity of the first negative system is about 50% greater than the 3914-Å band. The total excitation cross section for the first negative system is then about 9.8% of the total ionization cross section. Normalizing this system to 165 kR (IBC III), we produce Table 2, which shows the calculated auroral emission rates for various features, some comparison values from the calculation of Stolarski and Green (Ref. 9), and some observed values. The following comments pertain to and help explain Table 2.

First positive. We have assumed that 50% of the weak E^3 excitations result in cascading to the C^3 and 50% to the A^3 . C^3 cascades 100% to the B^3 , as does B^3 to A^3 . The ob-

served value of 2000 kR, given by Chamberlain (Ref. 1) seems too high. Harrison's infrared auroral observations (Ref. 31) of the (0,0) band of the first positive system can be combined with the laboratory measurements of, for example, Stanton and St. John (Ref. 32) to give an estimate of about 500 kR for the whole system. Shemansky and Vallance-Jones (Ref. 33) do obtain higher values, but the laboratory measurements of O'Neil and Davidson (Ref. 34) lead to low intensities (100–1000 kR). Shemansky and Broadfoot¹⁰ deduce a ratio first positive/second positive of 4. For the case represented in Table 2, the first positive system would have an intensity of 400 kR. If indeed the lower values are correct, then our calculations are in excellent agreement with observation.

Vegard-Kaplan. Neither of the calculated values has been corrected for quenching. About 28% of the excitation in the present calculation is direct. Stolarski and Green calculated that only ⅓ of the excitations were unquenched (the altitude assumed was about 170 km), using a lifetime for the A^3 state of 10 s. According to recent papers by Shemansky (Ref. 35) and Shemansky and Carleton (Ref. 36), the lifetime is about 2 s rather than 10 s. Using this figure, about 40% of the excitations will then lead to radiation. Applying this figure to our calculated value, we obtain excellent agreement with the "observed" value.

Lyman-Birge-Hopfield. Cascade contributions are not included in this figure, except for 50% from the weak $a''^1\Sigma_g^+$. All of the allowed singlets can cascade through the Lyman-Birge-Hopfield bands, but the branching ratios are uncertain. The observed value, of course, includes all the cascade contributions, not only from the b state and h state but from those not listed in Table 2 (see Table 1).

b. This designation refers to a combination of states, including the $b^1\Pi_u, p'^1\Sigma_u^+, l^1\Pi_u$ and $j^1\Sigma_u^+$ states.

h. This designation refers to energy-loss range numbers 5 and 6 in Table 1.

In the present calculation, excitation of O_2 and O have not been taken into account, whereas they have been included in the calculations of Stolarski and Green (Ref. 9). A comparison of the results is thus not strictly justifiable. However, we do not believe that the difference due to this effect will be large. The normalization of all emissions to the first negative system of N_2 , of course, removes the first-order discrepancy.

¹⁰Shemansky, D. E., and Broadfoot, L., private communication (to be published in *J. Chem. Phys.*)

Table 2. Calculated intensities for an IBC III aurora caused by 100-eV electrons

Feature	Present results, kR	Stolarski and Green (1967), kR	Observed, kR	Notes
First negative	165	165	165	Normalization Includes 50% of E^3 by cascade
First positive	367	2560	500, 2000 ^a	
Second positive	144	523	100	
Vegard-Kaplan	517 ^b	4950 ^b	200–300	Includes 50% of a''^1 by cascade
Lyman-Birge-Hopfield	369 ^c	660 ^c	450–1850 ^d	
b	828	2010		
h	599	1050		
eV/ion pair	43.07	41.5		

^aSee text for explanation.
^bUnquenched.
^cWithout cascading.
^dWith cascading from all optically allowed states.

5. Conclusion

We have obtained a fairly complete set of electron impact cross sections for N_2 molecules. The most important energy range—10 to 100 eV—is well covered. These cross sections have been used to calculate emission rates from the aurora, using a Monte Carlo technique. Results obtained from the calculations are in very good agreement with the meager observational data.

Future improvements which may be incorporated at some later date include direct absolute calibration of the spectrometer, inclusion of pure rotational and vibrational excitation, inclusion of O_2 and/or O excitation, and the possible inclusion of weak triplet states (e.g., $D^3\Sigma_u^+$ and $W^3\Delta_u$) which are experimentally inaccessible by virtue of the intense overlying singlet states but which may be amenable to theoretical calculation.

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E. Magnetospheric Flows and Their Time

Dependence, T. Unti and G. Atkinson

A two-dimensional model of the magnetosphere has been used to study magnetospheric flows. The derivation of the two-dimensional model, which incorporates a magnetic tail and perfect neutral sheet, has been described in SPS 37-51, Vol. III, pp. 200-201, and SPS 37-59, Vol. III, pp. 27-28. In this article, the model is used to determine the response of the magnetosphere to fluctuations in the solar wind.

Dimensional analysis shows that the magnetic field lines in the magnetosphere assume a particular topology, dependent on the solar wind momentum flux and the amount of magnetic flux in the geomagnetic tail. If the solar wind momentum flux is enhanced or diminished, the magnetosphere will flow toward a new equilibrium configuration. Similarly, the erosive action of the solar wind, peeling field lines from the nose of the magnetosphere and carrying them into the tail, will cause flows to change the topology of the magnetosphere. To study the flows produced by these effects, the two-dimensional model is used to determine two equilibrium configurations of the magnetosphere: (1) the configuration before the solar wind change, and (2) the configuration long after the change has taken place. The flows that must occur between one equilibrium configuration and the next can then be deduced.

The result of the study indicates that the magnetosphere-wide convection is, most of the time, determined by the rate at which magnetic field lines are transported into the tail. Thus, the erosive action of the solar wind is the primary agent that drives large-scale flows in the magnetosphere. This erosive action will change in response to changes in the "frozen in field" of the solar wind and to changes in plasma properties. Changes in the solar wind momentum flux produce significant magnetosphere-wide flows only if they are sudden (≤ 10 min). The convective flow velocity could also be controlled by the

reconnection of field lines at the neutral sheet, and subsequent collapse of these field lines to a dipole-like form. To affect the large-scale flow significantly, the reconnection would have to operate on a much shorter time scale than transport of field lines into the tail, i.e., it would have to be burst-like in nature.

The magnetosphere can be visualized as a giant transducer to "signals," or variations in the solar wind. The type of response depends on the abruptness of the signal. If the state of the solar wind varies gradually, the resulting magnetospheric flow reproduces the signal accurately. For rapid changes in the solar wind, 10 min or less, the response is determined by time constants of the magnetosphere. There are two important time constants. The first depends on the characteristic time for propagation of hydromagnetic waves and is on the order of 10 min. The second depends on ionospheric conductivity; it can vary from a few minutes to perhaps an hour. The results of this investigation compare favorably with existing observations of flows and the effects of flows. In particular, the results have been used to explain properties of such phenomena as magnetospheric substorms, sudden storm commencements, DP2's, and the steady convection.¹¹

F. Vibrational Excitation of N_2 and CO by Electron Impact, W. Williams and S. Trajmar

1. Introduction

N_2 , comprising 80% of the earth's atmosphere, and CO, a minor component, are of continued interest in atmospheric physics. There has been a great deal of work on N_2 and CO vibrational excitation in the low-energy range (0-10 eV), where the scattering process is dominated by a resonance (Refs. 1-3). Practically nothing has been done in the intermediate range, where potential scattering governs the vibrational excitation.

The vibrational excitation of H_2 and a quantum mechanical theory of vibrational excitation for homonuclear diatomic molecules were presented in Section B. The experimental results concerning the vibrational excitation of CO and N_2 in the 10-50 eV energy range and 0-80 deg scattering angle range will be presented here. The experimental apparatus is an electron impact spectrometer which consists basically of an electron gun, hemispherical analyzer, scattering chamber, and detector. It has been described in detail in Ref. 4.

¹¹The DP2 event is a world-wide magnetic disturbance with a period from 10 min to 1 h.

2. Results and Discussion

Due to the long time required to obtain a spectrum with adequate signal-to-noise ratio and the accompanying instrumental changes, measurements taken at different angles cannot be directly compared with one another. The most reliable information that can be obtained from a single experimental spectrum is the ratio of the intensity of the vibrational feature to the intensity of the elastic peak. All factors relating the scattered signal intensity to the differential cross section (DCS) cancel when intensity ratios are taken. Therefore, the intensity ratios are equal to the ratios of the corresponding DCSs. The elastic intensity distribution can be obtained under the same instrumental conditions. This intensity distribution, corrected for changes in scattering path length with angle, yields the elastic DCS in arbitrary units. The path length correction has been discussed by Trajmar et al. (Ref. 5). The product of the inelastic-to-elastic ratios and the elastic DCS then yields the vibrational excitation cross sections in the same arbitrary units as the elastic DCS.

The normalization of the CO cross sections at 20 eV to the absolute scale has been carried out by utilizing the experimental total e -CO cross section (Ref. 6), as described below. A similar normalization procedure for N_2 and CO at other energies is under way. From experimental spectra at a given energy, we have determined the angular behavior of cross section ratios (inelastic-to-elastic) corresponding to different electronic transitions. These transitions were brought to the same arbitrary scale as the elastic cross section by the method discussed above. Integral cross sections were obtained for each vibrational and electronic transition in the same arbitrary units by extrapolation of the DCS to 0 and 180 deg and integrating over all scattering angles (Ref. 17). The total cross section is equal to the sum of all elastic, inelastic, and ionization cross sections. Since we have the total e -CO cross section and the total ionization cross section available (Ref. 8) in absolute units and we determined the elastic and all inelastic cross sections in the same arbitrary units, we were able to put all inelastic cross sections on the absolute scale. This normalization procedure is only approximate due to the error associated with the measurements and normalization procedure; however, they are believed to be correct to within a factor of two.

Typical energy-loss spectra are represented by Figs. 23 and 24, where the elastic and the accompanying vibrational peaks are shown. The N_2 elastic peak is shown with the first four vibrational transitions at 20-eV impact energy and a scattering angle of 83.5 deg (Fig. 23). Three vibra-

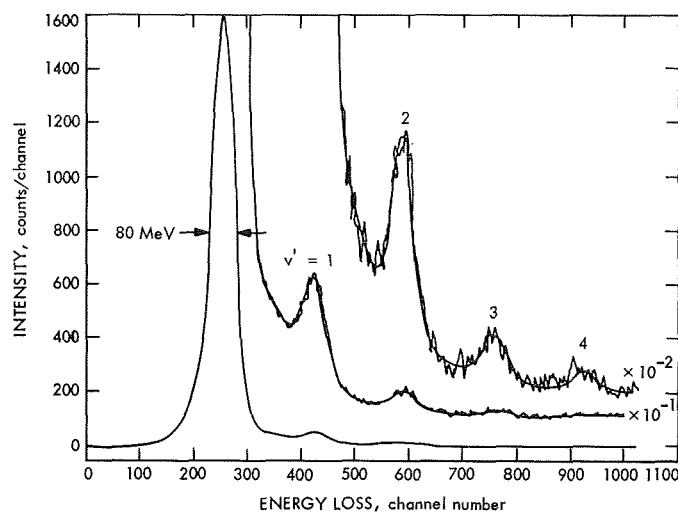


Fig. 23. Energy-loss vibrational excitation spectrum of N_2 at 83.5-deg scattering angle and 20-eV impact energy

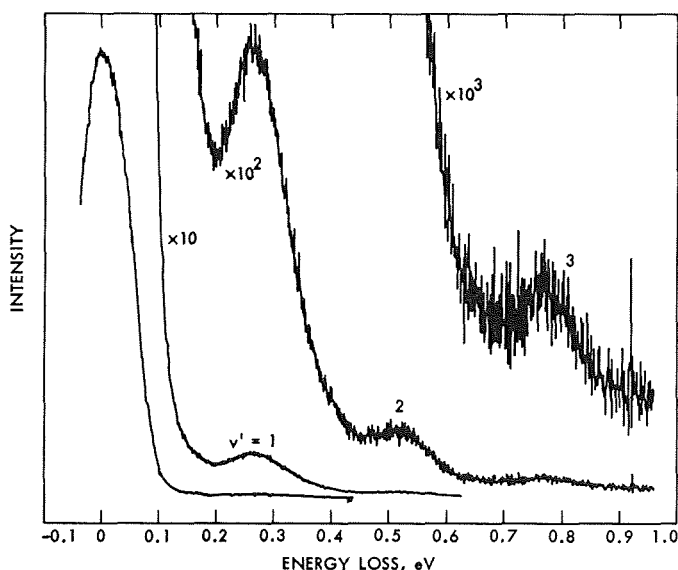


Fig. 24. Energy-loss vibrational excitation spectrum of CO at 25-deg scattering angle and 20-eV impact energy

tional transitions in CO are shown at 25 deg and 20 eV (Fig. 24).

The vibrational-to-elastic intensity ratios for the $v' = 1$, 2, and 3 levels in CO and N_2 at 20-eV impact energy versus angle are given in Fig. 25. In all cases, the CO ratios lie above the corresponding ratios of N_2 , but both have the same angular shape. N_2 and CO are isoelectronic

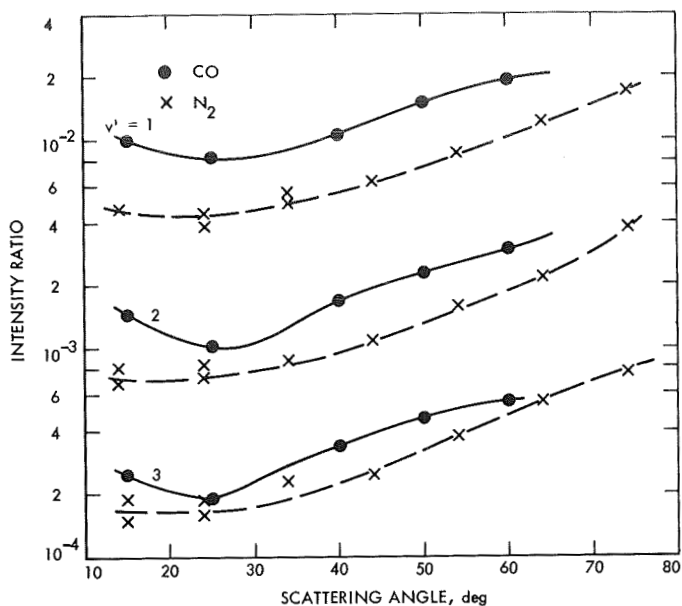


Fig. 25. Ratios of $v' = 1, 2$, and 3 vibrational transition intensities to the elastic scattering intensity for N_2 and CO at 20-eV impact energy as a function of scattering angle

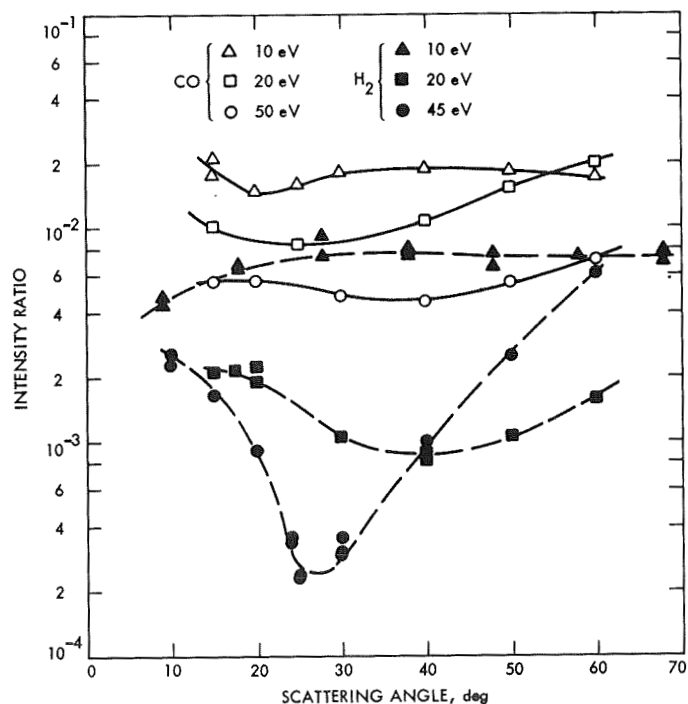


Fig. 26. Ratios of $v' = 1$ vibrational transition intensities to the elastic scattering intensity as a function of scattering angle at several energies for CO and H_2

and show many similarities in their electron impact spectra. One would expect similarities in the angular behavior of the vibrational DCSs also. Note, however, that CO has a permanent dipole moment which should influence the CO vibrational excitation, if not in shape, at least in magnitude. For photon excitation, the vibrational excitation of CO is allowed while that of N_2 to the same state is forbidden. In electron impact excitation, the CO case differs in that an extra term corresponding to the permanent dipole should be included in the interaction potential (the P_1 term in the multipole expansion of the potential—see Section B for details). Since the N_2 data are not yet normalized, it is not yet possible to compare the relative magnitudes of CO and N_2 vibrational excitation. However, since $R_{CO} > R_{N_2}$ and the elastic scattering is expected to be nearly the same, we can say that the $DCS_{vib.CO} > DCS_{vib.N_2}$.

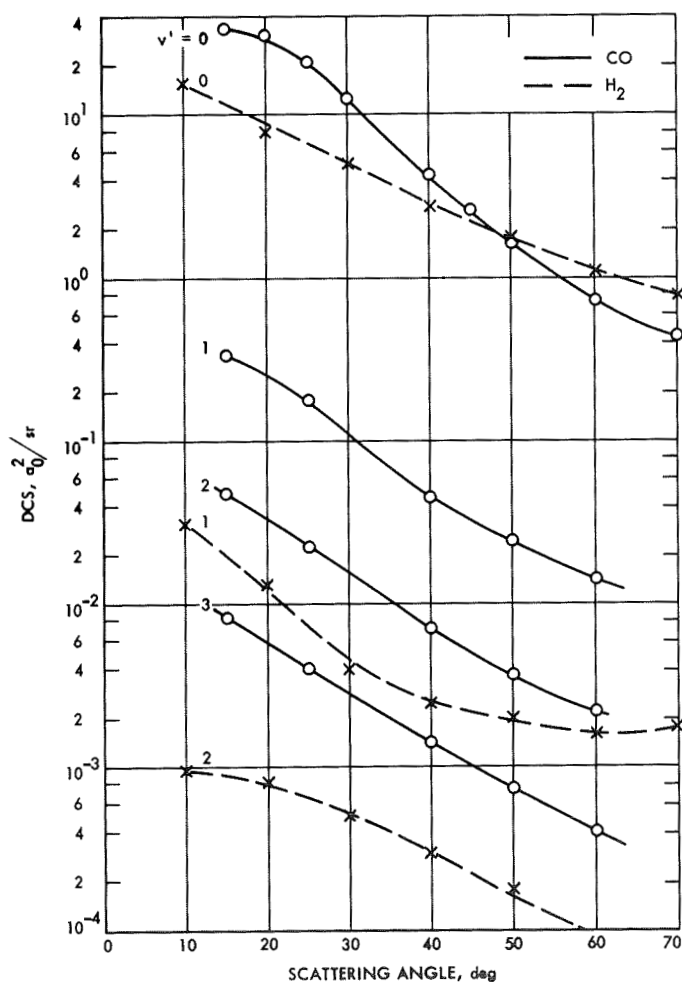


Fig. 27. Absolute differential cross sections for H_2 and CO as a function of angle at 20-eV impact energy

A comparison was made between the intensity ratio of the $v' = 1$ to elastic at several energies for H_2 and CO. The results are shown in Fig. 26. The deep minimum that appears in the H_2 intensity ratio at higher energies does not appear in the case of CO; furthermore, the minimum which is present in the CO curve moves to higher angles with increasing impact energy while the reverse is true for H_2 . Since we have determined the elastic cross sections for H_2 and CO, a quantitative comparison can be made between the vibrational excitation cross sections. This is done in Fig. 27. We find that the differential cross sections have similar shapes, but the H_2 vibrational cross sections are lower than those for CO by approximately a factor of ten, while the elastic cross sections have approximately the same values. Although this is an indication that the permanent dipole plays a significant role in the vibrational excitation process at 20-eV impact energy, more reliable conclusions can be drawn only after the calculations with and without the inclusion of the dipole term in the interaction potential have been carried out

and a comparison of the theoretical and experimental results has been completed.

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VI. Communications Systems Research

TELECOMMUNICATIONS DIVISION

A. Coding and Synchronization Research: Interplex—An Efficient Two-Channel Telemetry System for Space Exploration,

S. Butman and U. Timor

1. Introduction

Consider the two-channel phase-modulated coherent communication system used in deep space telemetry. The transmitted signal is

$$x(t) = 2P^{1/2} \sin [\omega_0 t + \theta_1 a(t) sq(\omega_1 t) + \theta_2 b(t) sq(\omega_2 t)]$$

where $a(t)$, $b(t)$ are binary data streams; $sq(\omega_i t)$ $i = 1, 2$, are square waves of frequency ω_i ; and θ_1 , θ_2 are modulation angles. The total power is

$$P = P_1 + P_2 + P_{RF} + P_L$$

where

$$P_1 = P (\sin \theta_1 \cos \theta_2)^2 = \text{power in first data channel}$$

$$P_2 = P (\sin \theta_2 \cos \theta_1)^2 = \text{power in second data channel}$$

$$P_{RF} = P (\cos \theta_1 \cos \theta_2)^2 = \text{power in carrier}$$

$$P_L = P (\sin \theta_1 \sin \theta_2)^2 = \text{power lost due to intermodulation}$$

In most cases we allocate more power to P_1 than to both P_2 and P_{RF} . This implies $\theta_1 > \theta_2$ and $\theta_1 > 45$ deg. In this case, the power loss is greater than the power available to the second channel:

$$\frac{P_L}{P_2} = \frac{(\sin \theta_1 \sin \theta_2)^2}{(\cos \theta_1 \sin \theta_2)^2} = (\tan \theta_1)^2 > 1$$

For example, in the *Mariner* Mars 1969 spacecraft we had

$$\theta_1 = 65 \text{ deg}$$

$$\frac{P_L}{P_2} = 6.63 \text{ dB}$$

To reduce this loss we propose a new modulation-demodulation scheme, which will interchange the role of P_2 and P_L , i.e., will utilize the power previously lost to transmit the low-rate data, and the only lost power will be P_2 (usually less than 1% of the total power).

The possible gain in the low-rate channel can be easily transferred to the high-rate channel by increasing θ_1 and decreasing θ_2 . The new scheme requires only minor changes in the modulator and demodulator.

2. New System

Let $a(t)$ and $b(t)$ be the data streams for the two channels, where for each t , $a(t)$ and $b(t)$ take the values $+1$ or -1 . The first subcarrier is modulated by $a(t)$ to yield

$$y(t) = a(t) sq(\omega_1 t)$$

and the second subcarrier is modulated by $b(t)y(t)$. The transmitted waveform will be

$$x(t) = 2^{1/2} \sin[\omega_0 t + \theta_1 a(t) sq(\omega_1 t) + \theta_2 b(t) a(t) sq(\omega_1 t) sq(\omega_2 t)]$$

If we modulate the second subcarrier by $a(t)b(t)$ instead of $y(t)b(t)$ we get a modified version of the scheme, with the same performance.

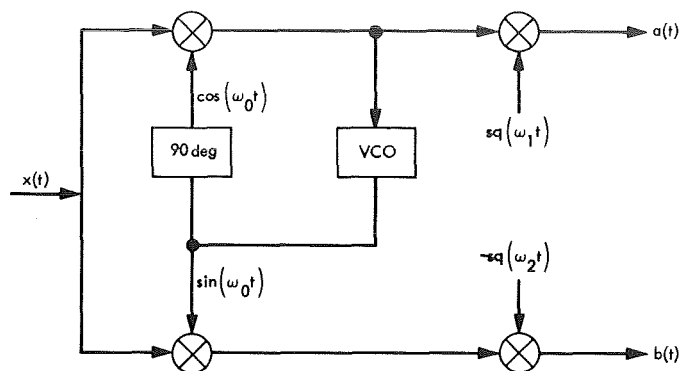


Fig. 1. Receiver for new scheme block diagram

The receiver (Fig. 1) demodulates the high-rate channel as in the usual scheme and multiplies the received waveform by $2^{1/2} \sin(\omega_0 t)$ —the output of the VCO—before demodulating the second subcarrier.

Disregarding terms with frequency of $2\omega_0$, we have

$$\begin{aligned} x(t) 2^{1/2} \sin(\omega_0 t) &= \cos[\theta_1 a(t) sq(\omega_1 t) + \theta_2 a(t) b(t) sq(\omega_1 t) sq(\omega_2 t)] \\ &= \cos[\theta_1 a(t) sq(\omega_1 t)] \cos[\theta_2 a(t) b(t) sq(\omega_1 t) sq(\omega_2 t)] \\ &\quad - \sin[\theta_1 a(t) sq(\omega_1 t)] \sin[\theta_2 a(t) b(t) sq(\omega_1 t) sq(\omega_2 t)] \\ &= \cos \theta_1 \cos \theta_2 - \sin \theta_1 \sin \theta_2 \underbrace{[a(t) sq(\omega_1 t)]^2}_{=1} b(t) sq(\omega_2 t) \\ x(t) 2^{1/2} \sin(\omega_0 t) &= \cos \theta_1 \cos \theta_2 - \sin \theta_1 \sin \theta_2 b(t) sq(\omega_2 t) \end{aligned}$$

The first term is dc, and the second term yields $b(t)$ when demodulated with $-sq(\omega_2 t)$. The power of this channel is

$$\tilde{P}_2 = (\sin \theta_1 \sin \theta_2)^2$$

while the power in the carrier and the high-rate channel remain unchanged. Thus we have

$$\begin{aligned} \tilde{P}_1 &= P_1 = (\sin \theta_1 \cos \theta_2)^2 \\ \tilde{P}_2 &= P_L = (\sin \theta_1 \sin \theta_2)^2 \\ \tilde{P}_L &= P_2 = (\cos \theta_1 \sin \theta_2)^2 \\ \tilde{P}_{RF} &= P_{RF} = (\cos \theta_1 \cos \theta_2)^2 \end{aligned}$$

The lost power is reduced (compare to the previous scheme) by a factor of

$$\frac{\tilde{P}_L}{P_L} = \frac{(\cos \theta_1 \sin \theta_2)^2}{(\sin \theta_1 \sin \theta_2)^2} = (\cot \theta_1)^2 < 1$$

In most practical cases P_L will be negligible (less than 1% of P).

3. Comparison of the Performances of the Two Schemes

Let $\alpha \triangleq P_2/P_1$ be the ratio of the power in the two channels. Then in the previous scheme

$$\alpha = \left(\frac{\tan \theta_2}{\tan \theta_1} \right)^2$$

If the carrier power requirements are negligible, this yields $\theta_1 + \theta_2 = 90$ deg. In all cases we will have $\theta_1 + \theta_2 \leq 90$ deg. As we shall presently see, this condition is not necessarily optimal in the new method.

a. Case I: The carrier requirements are negligible.

In this case

$$\tan^4 \theta_1 = \frac{1}{\alpha}$$

$$P_{1 \max} = \sin^4 \theta_1 = \frac{1}{(1 + \alpha^{1/2})^2}$$

$$P_2 = \alpha P_{1 \max}$$

Maximize $\tilde{P}_1$ keeping the ratio $\tilde{P}_2/\tilde{P}_1 \triangleq \tilde{\alpha} = \alpha$

$$\left. \begin{aligned} \tilde{P}_2 &= (\sin \theta_1 \sin \theta_2)^2 \\ \tilde{P}_1 &= (\sin \theta_1 \cos \theta_2)^2 \end{aligned} \right\} \quad \tilde{\alpha} = \tan^2 \theta_2 = \alpha$$

Clearly, $\tilde{P}_1$ will be maximized by taking $\theta_1 = 90$ deg. Thus

$$\tilde{P}_{1 \max} = \cos^2 \theta_2 = \frac{1}{1 + \alpha}$$

The improvement gained by the new method is

$$\Delta \triangleq \frac{\tilde{P}_{1 \max}}{P_{1 \max}} = \frac{\tilde{P}_2}{P_2} = \frac{(1 + \alpha^{1/2})^2}{1 + \alpha} = 1 + \frac{2(\alpha^{1/2})}{1 + \alpha}$$

where $\Delta > 1$ for all $\alpha > 0$.

The maximum gain will be achieved for $\alpha = 1$ (both channels have the same power) and is 3 dB. The gain for other values of α is given in Fig. 2. Note that the optimum modulation angles are $\theta_1 = 90$ deg, $\theta_2 \leq 45$ deg.

Maximize $\tilde{P}_1$, keeping $\tilde{P}_2 = P_2$

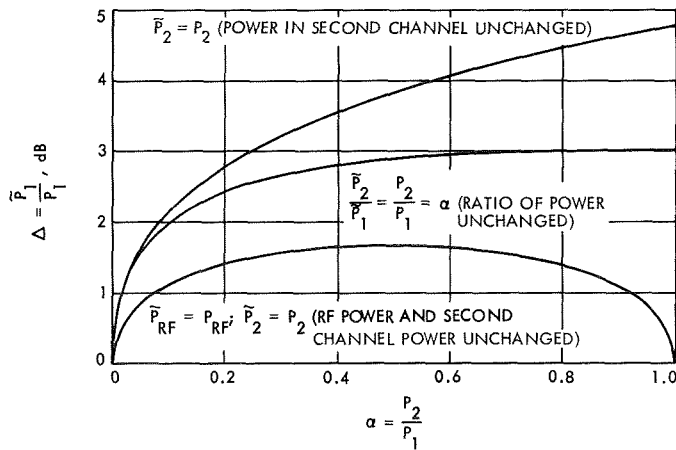


Fig. 2. Improvement of power in first data (high rate) channel in three cases

In this case we want to transfer all the gain in data power to the high-rate channel.

We set

$$\tilde{P}_2 = P_2 = \alpha P_1 = \frac{\alpha}{(1 + \alpha^{1/2})^2}$$

Again $\tilde{P}_1$ will be maximized by choosing $\theta_1 = 90$ deg.

Hence

$$\tilde{P}_1 = \cos^2 \theta_2$$

But

$$\tilde{P}_2 = \sin^2 \theta_2 = \frac{\alpha}{(1 + \alpha^{1/2})^2}$$

Therefore

$$\tilde{P}_{1 \max} = 1 - \frac{\alpha}{(1 + \alpha^{1/2})^2} = \frac{1 + 2(\alpha^{1/2})}{(1 + \alpha^{1/2})^2}$$

so Δ , the ratio of $\tilde{P}_{1 \max}$ to $P_{1 \max}$, satisfies

$$\Delta = 1 + 2(\alpha^{1/2}) > 1 \quad \text{for all } \alpha > 0$$

All the gain is in the first channel and can reach 4.77 dB (for $\alpha = 1$). Complete results are given in Fig. 2.

b. Case II: The carrier requirements are taken into consideration.

The modulation angles for the new scheme can be found from the conditions:

$$\begin{aligned} (\cos \theta_1 \cos \theta_2)^2 &= \tilde{P}_{\text{RF}} \\ \tan^2 \theta_2 &= \tilde{\alpha} \end{aligned}$$

We will no longer have $\theta_1 = 90$ deg, but $\theta_1 < 90$ deg.

As an example, consider the case where the existing P_{RF} in Case I was sufficient, i.e.,

$$P_{\text{RF}} = \frac{\alpha^{1/2}}{(1 + \alpha^{1/2})^2}$$

If we require

$$\tilde{P}_2 = P_2$$

$$\tilde{P}_{\text{RF}} = P_{\text{RF}}$$

then

$$(\cos \theta_1 \cos \theta_2)^2 = \frac{\alpha^{1/2}}{(1 + \alpha^{1/2})^2}$$

$$(\sin \theta_1 \sin \theta_2)^2 = \frac{\alpha}{(1 + \alpha^{1/2})^2}$$

which yield

$$\Delta = \frac{\tilde{P}_1}{P_1} = \frac{1}{2} (1 + \alpha^{1/2} + [(1 + \alpha^{1/2})^2 - 4\alpha(\alpha^{1/2})]^{1/2})$$

The results are described in Fig. 2.

4. Conclusion

The Interplex modulation-demodulation scheme performs better than the current method used (except in extreme cases where the performances are the same), with only minor changes in the system. The new modulation angles satisfy $\theta_1 < 90$ deg and $\theta_2 < 45$ deg. The power lost in most practical cases will be negligible, whereas it is far from negligible in the present system.

B. Coding and Synchronization Research: Optimization of Acquisition Time for Sequential Ranging System, S. Butman and J. Molinder

1. Introduction

The JPL sequential ranging system (SPS 37-52, Vol. II, pp. 46-49) gives correct results only if the acquisition of all but the highest frequency component is correct. The highest frequency component is used to obtain a phase estimate which is then used as a reference for the remaining lower frequency components. The probability of a correct decision on each component depends on the signal-to-noise ratio at the output of the correlator, and hence on the accuracy of the phase reference. The accuracy of the phase reference, in turn, depends on the signal-to-noise ratio of the correlators used for this purpose. For a fixed signal power and total integration time it is of interest to determine the optimal allocation of time between the phase estimation operation and the decisions on the remaining components.

In this article the best allocation of time between phase estimation and component acquisition is determined numerically under the constraint that equal integration times are used for all but the high-frequency component. The optimum performance under these con-

straints is approximately 1 dB worse than the performance if the exact phase reference were known at the start.

2. Probability of Being Correct versus Signal-to-Noise Ratio

Since the noise into the system is assumed white gaussian with two-sided spectral density of height $N_0/2$, the output of the i th component correlator using a unit power correlating signal is given by

$$V_i = A \left[1 - \frac{2}{\pi} \frac{|\tilde{\theta}|}{2^i} \right] + n_i \quad (1)$$

where A is the amplitude of return signal $\tilde{\theta} = \hat{\theta} - \theta$, $-\pi \leq \tilde{\theta} \leq \pi$, $\hat{\theta}$ is the estimate of the true phase delay θ , $|\tilde{\theta}|$ is the absolute value of θ , n_i is an independent gaussian random variable of zero mean and variance $\sigma^2 = N_0/2T$ for each $i = 1, 2, \dots, 9$, and T is the component integration time. Thus the probability that the i th decision is correct given $\tilde{\theta}$ is

$$P_i(C|\tilde{\theta}) = \frac{1}{(2\pi)^{1/2} \sigma} \int_{-A[1 - (2/\pi)(|\tilde{\theta}|/2^i)]}^{\infty} e^{-x^2/2\sigma^2} dx \quad (2)$$

$$\begin{aligned} &= Q \left[\frac{-A}{2^{1/2} \sigma} \left(1 - \frac{2}{\pi} \frac{|\tilde{\theta}|}{2^i} \right) \right] \\ &= Q \left[- \left(\frac{ST}{2N_0} \right)^{1/2} (1 - |\tilde{\theta}|/\pi 2^{i-1}) \right] \end{aligned} \quad (3)$$

where

$$S = A^2$$

and

$$Q(\alpha) = \frac{1}{\pi^{1/2}} \int_{\alpha}^{\infty} e^{-t^2} dt = \text{erf}(-\alpha)$$

Since the noise in each component is uncorrelated with, and thus independent of, the noise in every other component, the probability of being correct given $\tilde{\theta}$ is

$$P(C|\tilde{\theta}) = \prod_{i=1}^{N-1} Q \left[- \left(\frac{ST}{N_0} \right)^{1/2} \left(1 - \frac{2}{\pi} \frac{|\tilde{\theta}|}{2^i} \right) \right] \quad (4)$$

where N is the total number of components. Finally, the average probability of being correct is

$$P(C) = \int_{-\pi}^{\pi} \int_{\theta-\pi}^{\theta+\pi} p(C|\tilde{\theta}) p(\hat{\theta}, \theta) d\hat{\theta} d\theta \quad (5)$$

where $p(\hat{\theta}, \theta)$, as given in SPS 37-61, Vol. III, pp. 31-33, is

$$p(\hat{\theta}, \theta) = \frac{1}{2\pi} \frac{2}{\pi^2 \|g\|^2} \exp \left[-\frac{\|f\|^2}{2\sigma^2} \right] \times \left\{ \frac{1}{2} + \left(\frac{\pi}{2} \right)^{1/2} \frac{\langle f, g \rangle}{\sigma \|g\|} Q \left[-\frac{\langle f, g \rangle}{2^{1/2} \sigma \|g\|} \right] \right. \\ \left. \times \exp \left[\frac{\langle f, g \rangle^2}{2\sigma^2 \|g\|^2} \right] \right\} \quad (6)$$

where

$$f_1(\theta) \triangleq \frac{8}{\pi^2} [\cos \theta + (1/9) \cos 3\theta]$$

$$f_2(\theta) = f_1(\theta + \pi/2), \sigma_\tau^2 = \left(\frac{2S\tau}{N_0} \right)^{-1}$$

$g_1(\hat{\theta})$ and $g_2(\hat{\theta})$ are defined as shown in Fig. 3, S is the power in the equivalent *square wave* received (the bandwidth is such that only the first and third harmonics of the high-frequency square wave are transmitted), and τ is

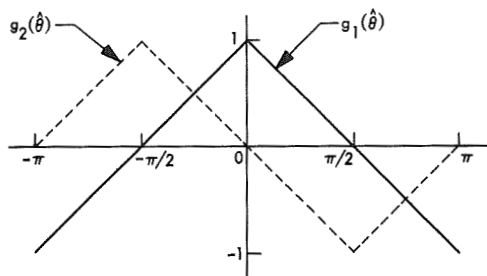


Fig. 3. $g_1(\hat{\theta})$ and $g_2(\hat{\theta})$ versus $\hat{\theta}$

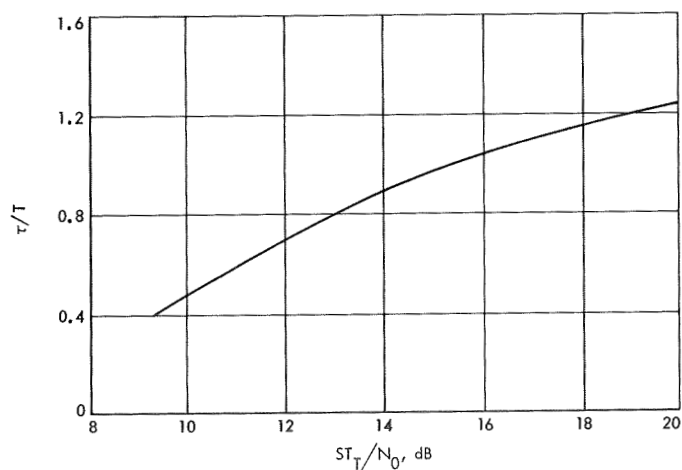


Fig. 4. Optimum τ/T versus ST_T/N_0 , $N = 10$

Table 1. Optimal τ/T versus ST_T/N_0 , dB (see Fig. 4)

ST_T/N_0	τ/T (accuracy ± 0.1)	ST_T/N_0	τ/T (accuracy ± 0.1)
10	0.5	16	1.0
11	0.6	17	1.1
12	0.7	18	1.1
13	0.8	19	1.2
14	0.9	20	1.3
15	1.0		

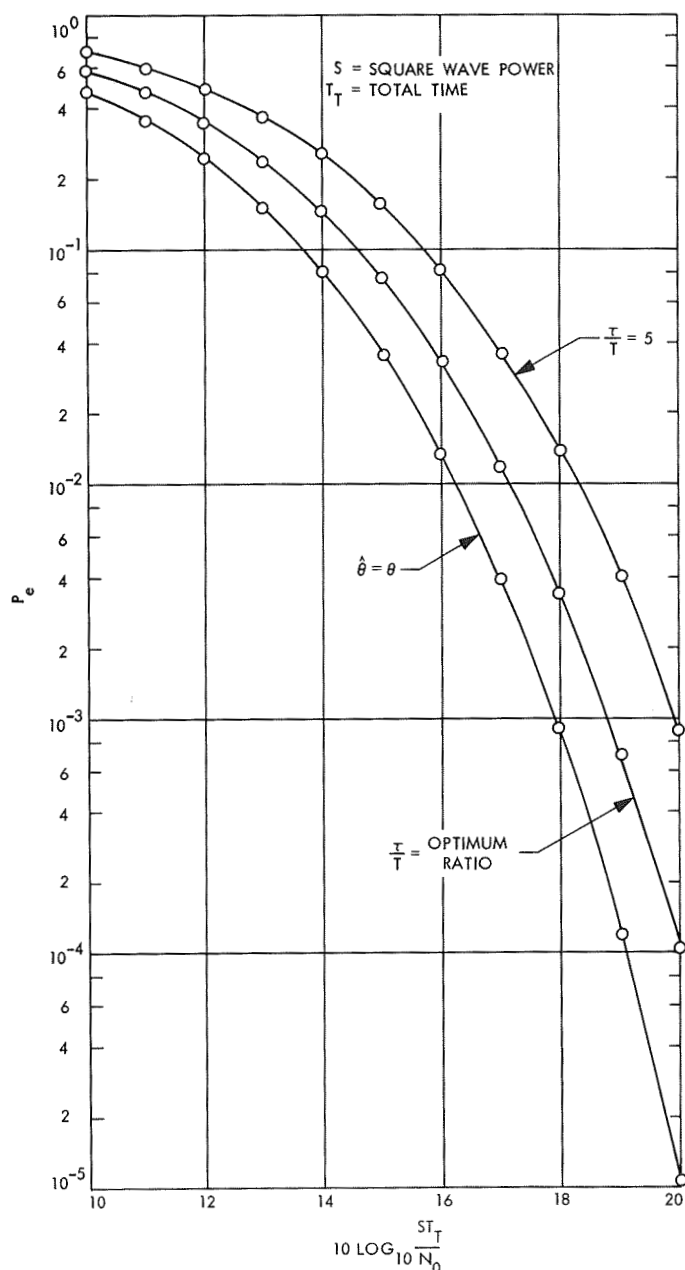


Fig. 5. P_e versus ST_T/N_0 for optimum ratio τ/T , $N = 10$

the integration time. Since $p(\hat{\theta}, \theta)$ has symmetry in θ with period $\pi/4$, Eq. (5) may be written as

$$P(C) = 8 \int_0^{\pi/4} \int_{\theta-\pi}^{\theta+\pi} p(C|\tilde{\theta}) p(\tilde{\theta}, \theta) d\tilde{\theta} d\theta \quad (7)$$

3. Results

Table 1 shows the optimal ratio of τ/T versus ST_T/N_0 where T_T is the total integration time allotted for phase estimation and all components for $N = 10$. The plot of the resulting probability of error $P(E) = 1 - P(C)$ is given in Fig. 5 along with curves showing $P(E)$, assuming the exact phase reference was known at the start ($\hat{\theta} = \theta$, thus eliminating the need for phase estimation) and $P(E)$ for $\tau/T = 5$.

It is approximately 1 dB worse than the case for $\hat{\theta} = \theta$. Plots of P_e versus τ/T at $ST_T/N_0 = 10, 15$, and 20 dB are

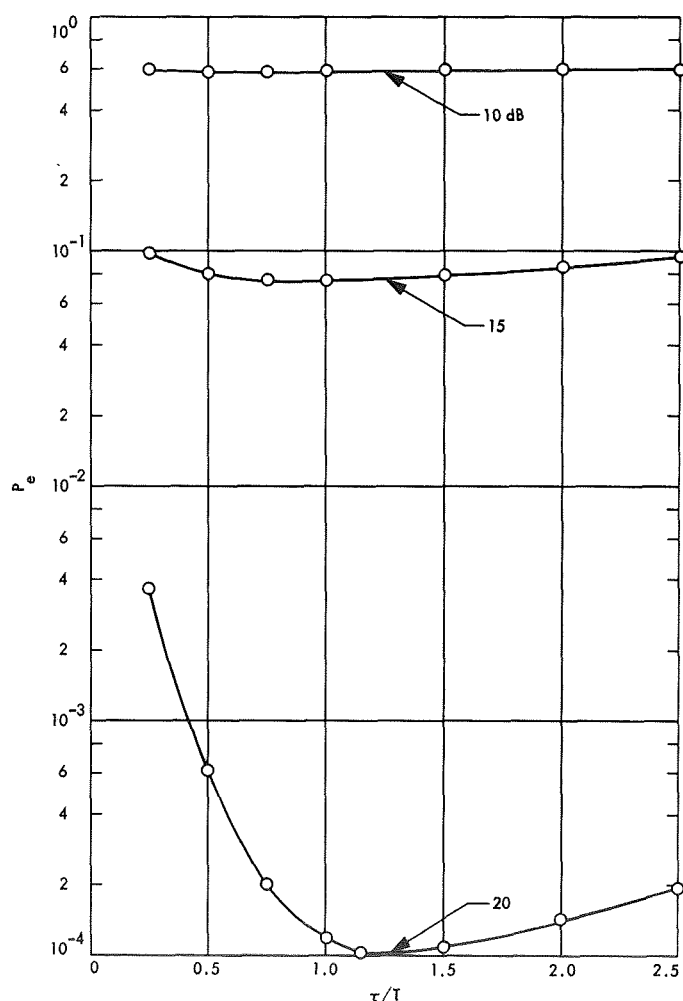


Fig. 6. P_e versus τ/T for fixed ST_T/N_0 , $N = 10$

given in Fig. 6. At low signal-to-noise ratios, P_e is quite insensitive to the value of τ/T , while at high signal-to-noise ratios the sensitivity increases.

C. Combinatorial Communication: Ramsey Bounds for the Coefficients of Internal Stability of Graph Products, R. J. McEliece and H. Taylor

1. Introduction

In SPS 37-61, Vol. III, pp. 51-54, McEliece et al. discussed the relationship between the zero-error capacity of a discrete memoryless channel and the coefficient of internal stability of powers of the adjacency graph of the channel. In this article we shall present the best possible upper bound for the coefficient of internal stability of product graphs that depends only on the coefficients of the factors. This result, when translated into the language of coding, gives the best possible upper bound on the size of the largest zero-error probability code of block length n , say, in terms of the largest such codes for block lengths less than n . It is important to emphasize that these bounds do not depend on the number of inputs and outputs of the channel, but only on the size of the largest zero-error probability codes.

2. Preliminaries

The Ramsey Number $M(k_1, k_2, \dots, k_n)$ is the smallest integer m with the property that if the edges of the complete m -graph are colored with n distinct colors, there will be at least one index i for which there will be a complete k_i -subgraph all of whose edges are of the i th color. (The fact that these numbers exist and are finite is known as *Ramsey's Theorem* (Ref. 1, Theorem 6.1.1 and Exercise 6.5).

The coefficient of internal stability (or independence number) of a graph G is the largest possible number of vertices of G which may be chosen so that no two are adjacent. This coefficient is denoted $\alpha(G)$.

The product $G_1 \times G_2 \times \dots \times G_m$ of the graphs $G_1, \dots, G_m$ is the graph whose vertex set is the Cartesian product of the vertex sets of the factors, and with distinct m -tuples $(x_1, \dots, x_m)$ and $(y_1, \dots, y_m)$ adjacent in the product, if and only, if x_i and y_i are either connected or identical for all i .

It is trivial that

$$\alpha(G_1 \times \dots \times G_m) \geq \alpha(G_1) \alpha(G_2) \dots \alpha(G_m)$$

The goal of this note is to present an upper bound on $\alpha(G_1 \times \cdots \times G_m)$ which depends only on $\alpha(G_1), \cdots, \alpha(G_m)$. In the next section we shall present the best possible such upper bound and show that the bound remains sharp when the factors G_i are all isomorphic. This is the case of importance for the study of zero-error probability codes.

3. Main Results

THEOREM 1

$\alpha(G_1 \times \cdots \times G_m) \leq M(\alpha_1 + 1, \alpha_2 + 1, \cdots, \alpha_m + 1) - 1$
where $\alpha_i = \alpha(G_i)$.

Proof: Let X be a subset of $G_1 \times \cdots \times G_m$ of size $\alpha(G_1 \times \cdots \times G_m)$ such that no two vertices of X are connected in $G_1 \times \cdots \times G_m$. Thus, if $x = (x_1, \cdots, x_m)$ and $y = (y_1, \cdots, y_m)$ are vertices from X , there will be at least one index i for which $x_i \neq y_i$ and x_i and y_i are not connected in G_i . Let us color the complete graph whose vertex set is X as follows: the edge joining x and y is colored with the i th color if i is the *smallest* index for which $x_i \neq y_i$ and x_i and y_i are not connected in G_i . Now if $\alpha(G_1 \times \cdots \times G_m) \geq M(\alpha_1 + 1, \cdots, \alpha_m + 1)$, then by Ramsey's theorem, we conclude that for some index i , there exists a complete $(\alpha_i + 1)$ -subgraph of the i th color. But this would imply the existence of $\alpha_i + 1$ vertices in G_i , no two of which were connected, in conflict with the definition of $\alpha(G_i)$. Theorem 1 is proved.

THEOREM 2. Given integers $\alpha_1, \alpha_2, \cdots, \alpha_m$, there exist graphs $G_1, G_2, \cdots, G_m$ such that $\alpha(G_i) = \alpha_i$ and $\alpha(G_1 \times \cdots \times G_m) = M(\alpha_1 + 1, \cdots, \alpha_m + 1) - 1$.

Proof. From the definition of the Ramsey Number $M(\alpha_1 + 1, \cdots, \alpha_m + 1) = M$ we know that there is a coloring of the complete $M - 1$ graph in m colors such that the largest complete i -colored subgraph is on α_i nodes, for all i . For each i , let G_i be the graph on these $M - 1$ vertices which includes all edges which are, in fact, not colored with the i th color; thus $\alpha(G_i) = \alpha_i$. But it is easy to see that in $G_1 \times \cdots \times G_m$, the diagonal [i.e., the set of vertices of the form $(x, x, \cdots, x)$] is completely disconnected; if the edge connecting x and y is of the i th color, then $(x, x, \cdots, x)$ and $(y, y, \cdots, y)$ are not connected in the i th component and so not connected in $G_1 \times \cdots \times G_m$. Thus $\alpha(G_1 \times \cdots \times G_m) \geq M - 1$. But Theorem 1 forces $\alpha(G_1 \times \cdots \times G_m) \leq M - 1$, and so equality holds. This proves Theorem 2.

THEOREM 3. Given integers α and m , there exists a graph G with $\alpha(G) = \alpha$ and $\alpha(G^m) = M(\alpha + 1, \cdots, \alpha + 1) - 1$.

Proof. We begin with the graphs G_i as constructed in the proof of Theorem 2. Let G be the graph which consists of

one copy each of $G_1, G_2, \cdots, G_m$, with every vertex from G_i connected to every vertex in G_j if $i \neq j$. Then clearly $\alpha(G) = \alpha$. Also, $G \times G \times \cdots \times G$ contains a subgraph isomorphic to $G_1 \times \cdots \times G_m$, and so $\alpha(G^m) \geq \alpha(G_1 \times \cdots \times G_m) = M - 1$. Again, $\alpha(G^m) = M - 1$ follows from Theorem 1. Theorem 3 is proved.

4. Discussion

While in one sense these theorems completely settle the question of giving bounds for $\alpha(G_1 \times \cdots \times G_m)$ in terms of the $\alpha(G_i)$, in another sense much is left to be desired, since most of the Ramsey Numbers are unknown. Thus, the only known nontrivial Ramsey Numbers are

$$M(3, 3) = 6, \quad M(3, 4) = 9, \quad M(3, 5) = 14, \quad M(3, 6) = 18, \\ M(3, 7) = 23$$

$$M(4, 4) = 18$$

$$M(3, 3, 3) = 17$$

However, there are a variety of weak upper bounds known, the easiest to work with being $M(\alpha, \beta) \leq \binom{\alpha+\beta-2}{\alpha-1}$.

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D. Combinatorial Communication: An Upper Bound on the Free Distance of a Tree Code,

J. Layland and R. McEliece

1. Introduction

In SPS 37-50, Vol. III, pp. 248-252, McEliece and Rumsey obtained an upper bound on the free distance of a systematic convolutional code of rate $1/v$, where v is a positive integer. In this note we will modify that argument slightly and obtain an upper bound on the free distance of an arbitrary tree code of finite memory, systematic or not. The bounds obtained are exactly the same as those for convolutional codes with the same parameters. These upper bounds provide strong evidence that nonsystematic codes have considerably larger free distance than systematic codes. This is one reason why nonsystematic codes will be used in upcoming projects. In *Subsection 3*, we point out that the so-called *Griesmer bound* for linear codes can sometimes be used to tighten these bounds for convolutional codes. Finally in *Subsection 4* we present a short table of the bounds for particular values of the parameters.

2. The Bound

A rate k/n , constraint length K (memory $K - 1$) binary tree code can be described as follows: If the information stream is partitioned into blocks of k bits each, say $I = (I_1^{(1)}, I_1^{(2)}, \dots, I_1^{(k)}, I_2^{(1)}, I_2^{(2)}, \dots, I_2^{(k)}, I_3^{(1)}, \dots)$, then the output of the encoder is a sequence of blocks of n bits each, $C = (C_1^{(1)}, C_1^{(2)}, \dots, C_1^{(n)}, C_2^{(1)}, C_2^{(2)}, \dots, C_2^{(n)}, C_3^{(1)}, \dots)$.

Here $C_i^{(j)}$ depends only on the information symbols $I_{i-s}^{(t)}$ for $s = 0, 1, \dots, K - 1, t = 1, 2, \dots, k$. The free distance of this code is defined as the minimum Hamming distance between two of the (infinite) code words C .

Let T represent the set of all code words in the tree code, and T_h represent those code words corresponding to information streams which have $I_i^{(j)} = 0$ for $i > h$. Clearly $d_{\min}(T) \leq d_{\min}(T_h)$ for all $h \geq 1$. On the other hand T_h can be thought of as a block code of length $n(h + K - 1)$, since for $i \geq h + K, C_i^{(j)}$ depends only on $I_\ell^{(t)}$ for $\ell \geq h + 1$, and these information bits are all zero.

Now according to Plotkin's bound (Ref. 1, Theorem 13.49), a code of length n with M code words has minimum distance $\leq n/(2(1/M - 1))$. Since T_h has block length $n(h + K - 1)$ and 2^{hk} code words, we arrive at

THEOREM 1

$$d_{\text{free}}(T) \leq \frac{n(h + K - 1)}{2} \left(\frac{2^{hk}}{2^{hk} - 1} \right) \quad \text{all } h \geq 1$$

We conclude this section by discussing systematic tree codes. The code is said to be systematic if $C_i^{(j)} = I_i^{(j)}$ for $1 \leq j \leq k$. Notice that in T_h , this means that $C_i^{(j)} = 0$ for $i \geq h + 1, 1 \leq j \leq k$, so that $k(K - 1)$ of the $n(h + K - 1)$ code word positions are identically zero. This reduces the effective block length of T_h , and we obtain

THEOREM 2. If T is systematic

$$d_{\text{free}}(T) \leq \frac{n(h + K - 1) - k(K - 1)}{2} \left(\frac{2^{hk}}{2^{hk} - 1} \right) \quad \text{all } h \geq 1$$

3. Improvement for Linear (Convolutional) Codes

If the code is linear, then the codes T_h are also, and instead of Plotkin's bound, we may use Griesmer's bound (Ref. 2), which improves Plotkin's slightly. Griesmer's bound says that if $d = d_0$ is the minimum distance of an (n, k) binary linear code, and if $d_i = [(d_{i-1} + 1)/2]$, then $d_0 + d_1 + \dots + d_{k-1} \leq n$. For example, consider a rate

$1/2, K = 6$ convolutional code. Here T_3 is a $(16, 3)$ block code which has $d_{\min} \leq 9$ by Plotkin's bound. But if $d_0 = 9$, then $d_1 = 5, d_2 = 3$ and $9 + 5 + 3 = 17 > 16$, so that no linear $(16, 3)$ code can have $d_{\min} = 9$. Thus a rate $1/2, K = 6$ convolutional code must have $d_{\text{free}} \leq 8$. Incidentally, this example suggests the interesting possibility that a nonlinear tree code might have $d_{\text{free}} = 9$, which would then be superior to any linear tree code.

4. A Short Table

We present in this section a short table of the upper bounds obtained by the arguments in this paper for non-systematic tree codes (Table 2). The only rates considered are $1/2, 1/3$, and $1/4$. Column P is the Plotkin upper bound of Subsection 2, and column G lists the Griesmer upper bound of Subsection 3, whenever it is superior to the Plotkin Bound. The last column L gives the best d_{free} known to be achievable by a convolutional code.

References

1. Berlekamp, E. R., *Algebraic Coding Theory*, McGraw-Hill Publishing Co., New York, 1968.
2. Solomon, G., and Stiffler, J. J., "Algebraically Punctured Cyclic Codes," *Information and Control*, Vol. 8, pp. 170-179, 1965.

Table 2. Upper bounds

K	Rate 1/2			Rate 1/3			Rate 1/4		
	P	G	L	P	G	L	P	G	L
2	4		3	6		5	8		7
3	5		5	8		8	10		10
4	6		6	10		10	13		13
5	8		7	12		12	16		16
6	9	8	8	13		13	18		18
7	10		10	15		15	20		20
8	11		10	17	16	16	22		
9	12		12	18		18	25	24	24
10	13		12	20		20	27		
11	14		12	22			29		
12	16		13	24		22	32		
13	17	16		25	24		34	33	
14	18	17		27	26		36		
15	19	18		28			38		

E. Combinatorial Communications: Epsilon Entropy and Data Compression,

E. Posner and E. Rodemich

1. Introduction

The theory of efficient handling of data for transmission or storage is presently known as "data compression." As space exploration progresses to more and more distant

missions, an efficient spacecraft-to-earth communication link becomes more important. As our technology gets more computer-based, the economic importance of data compression increases.

A mathematical theory is needed. It is the purpose of this article to provide such a theory. Thus, we start off by abstracting the notion of the source of the data to be transmitted as a probabilistic metric space. Such a space is a triple (X, d, μ) with the following two properties:

First, (X, d) is a complete separable metric space, consisting of points of the set X under the metric d . Second, (X, B, μ) is a probability space, where B is the Borel field generated by the open sets of X ; i.e., B is the field of Borel sets of X . (However, we will speak of subsets of X as measurable if they belong to the completion of B under μ .)

The points of X represent the possible experimental outcomes, i.e., the data; the metric d is a "fidelity criterion," such that $d(x, y)$ is the loss of fidelity when one outcome x occurs but another outcome y is thought to have occurred.

We were given an $\epsilon \geq 0$ as the allowed loss of fidelity. But we are also given a $\delta \geq 0$, such that the loss of fidelity can exceed ϵ , but only with probability δ or less. Any physical data transmission system has a probability $\delta > 0$ associated with it, such that, with probability δ , the system does not work as specified. For example, an analog-to-digital converter has as its δ the off-scale probability.

Given a probabilistic metric space (X, d, μ) , which we abbreviate as X , we wish to transmit outcomes within ϵ or less with probability $1 - \delta$ or more. One reasonable interpretation of this is the following: If, given a received message, we know the actual outcome within ϵ , then we know that the actual outcome falls within an ϵ -set, i.e., within a (measurable) set of diameter at most ϵ . However, with probability δ or less, there is no transmission and no such set of diameter at most ϵ . We shall sometimes also use a definition involving spheres of radius $\epsilon/2$ or less.

Now it might appear that mixed strategies should be considered, in which the set of diameter ϵ depends on other factors, or random variables, not depending on past or future samples from X , in addition to depending on the actual outcome. However, it is easily shown that there is a pure strategy comparable to any mixed strategy, insofar as reducing the load on the communication channel is

concerned. A pure strategy assigns a given outcome to a fixed ϵ -set. Thus, in a pure strategy, these ϵ -sets are to be disjoint. That is, we have a collection of disjoint ϵ -sets, such that with probability $1 - \delta$ or more, a point of the space X lies in one of the ϵ -sets. This leads to the definition of $\epsilon; \delta$ partition.

DEFINITION. Given $\epsilon \geq 0$, $\delta \geq 0$, an $\epsilon; \delta$ partition of the probabilistic metric space X is a finite or denumerably infinite partition of part of X by (disjoint) ϵ -sets, such that the union of the sets in the partition has probability $1 - \delta$ or more.

A data compression system based on a given $\epsilon; \delta$ partition U of X works as follows: observe an outcome, see into which set of the partition U the outcome falls, and merely transmit information which uniquely determines that set. In fact, any (pure strategic) data transmission system which yields fidelities of ϵ or better with probability $1 - \delta$ or more is of this type.

Other authors (Refs. 1, 2) usually consider restricting some average distance to be ϵ or better, instead of the actual distance between actual outcome and transmitted outcome. The actual distance is considered to be the appropriate criterion in judging most data systems.

So we now have an $\epsilon; \delta$ partition U of X . It must be determined how great a load this implies for the communication system. A measure of the load is the entropy of the partition, $H(U)$. This entropy is the number of bits necessary to transmit information as to which set of U the outcome fell, given that the outcome was in some set of U . Thus, using Shannon's formula (Ref. 3), we define $H(U)$ as follows:

Let

$$U = \{U_i\}$$

with

$$\mu(U_i) = p_i$$

$$\sum p_i = p > 1 - \delta$$

Let $q_i = p_i/p$, so that $\{q_i\}$ is a probability distribution. Then the entropy $H(U)$ is defined as the entropy of $\{q_i\}$:

$$H(U) = \sum q_i \log \frac{1}{q_i} \quad (1)$$

Thus, $H(U)$ is non-negative, and can be equal to $+\infty$.

The interpretation of $H(U)$ is the number of bits per sample (we use base e logarithms, even though we speak of bits instead of "nats") necessary to describe into which set of U the outcome falls, when minimum expected-length binary encoding is used for this purpose, but storage between experiments is not allowed.

However, the partition U may not have been especially well chosen with a view to minimizing $H(U)$. Thus, let $U_{\epsilon;\delta}$ denote the class of $\epsilon;\delta$ partitions of X (when $\epsilon > 0$, this class is non-empty). Then we define $H_{\epsilon;\delta}(X)$, the *epsilon; delta entropy* of X , as

$$H_{\epsilon;\delta}(X) = \inf_{U \in U_{\epsilon;\delta}} H(U) \quad (2)$$

(When $\delta = 0$, we write $H_\epsilon(X)$, the ϵ -entropy of X . If $\epsilon = 0$ and $U_{0;\delta}$ is empty, $H_{0;\delta}(X)$ is infinite.)

Thus, the $\epsilon;\delta$ entropy of X , which may equal $+\infty$ even when $\epsilon > 0$, provided that $\delta = 0$, measures the minimum number of bits necessary to describe at least $1 - \delta$ of X within accuracy ϵ . When $\delta = 0$, we can say that the ϵ -entropy of X is the number of bits necessary to describe elements of X within ϵ (without storage, but we will say more about this later) with probability 1. We are mainly concerned with ϵ -entropy; $\epsilon;\delta$ entropy enters chiefly as a device in proving theorems.

We have chosen to use sets of diameter ϵ in our definition of epsilon entropy to agree with the definition used in the theory of the epsilon entropy of compact metric spaces (Ref. 4). However, other workers in information theory use, in effect, sets of radius $\epsilon/2$ in the definition. We will treat both cases; when it is necessary to distinguish the two definitions, we will use the terms "diametric entropy" and "radial entropy," respectively. The original paper on the subject of data compression and epsilon entropy (Ref. 5) used diametric entropy, although Kolmogorov (Ref. 6) uses radial entropy.

The purpose of the remainder of this article is to make the statements of this section more precise. Various techniques will be given for obtaining upper and lower bounds for ϵ -entropy. The ϵ -entropy defined here will be related to the epsilon entropy of compact metric spaces and to Shannon's rate distortion theory. Finally, a very precise "channel coding theorem" and its converse will be stated and proved for ϵ -entropy, which makes the relevance of ϵ -entropy to data compression clear.¹

¹Summarized from the original paper submitted to the *Annals of Mathematical Statistics*. The proofs are omitted here.

2. Connection With ϵ -entropy for Compact Metric Spaces

A definition for the ϵ -entropy of a compact metric space finds wide use in various kinds of approximation theory. See Ref. 4 for a good expository treatment.

DEFINITION. An ϵ -partition ($\epsilon > 0$) of a compact metric space X with metric d is a partition of all of X by disjoint Borel sets of diameters at most ϵ (note that other workers use 2ϵ instead of ϵ).

DEFINITION. The ϵ -entropy $K_\epsilon(X)$ of a compact metric space X is the logarithm of the minimum number of sets in any ϵ -partition of X (since X is compact, $K_\epsilon(X)$ is finite).

An information-theoretic interpretation of $K_\epsilon(X)$ can be given as follows: Suppose data is generated as elements of a compact metric space X . We wish to transmit outcomes of X , not allowing storage from one experiment to the next, outputting binary words of fixed length, and using as short a length as possible. The ϵ -entropy $K_\epsilon(X)$ is the shortest length, except for round-off in the logarithm. If, however, any probabilistic information is known about X (that is, if the data is selected according to a Borel distribution μ on X) the average word length may be able to be shortened by taking advantage of μ in assigning word lengths. That is

$$H_\epsilon(X) \leq K_\epsilon(X)$$

since the entropy of an ϵ -partition of X by n sets is bounded from above by $\log n$. In general, one does not consider $K_\epsilon(X)$ for non-compact metric spaces, since this number can be infinite. (In fact, if $K_\epsilon(X) < \infty$ and all $\epsilon > 0$, then X is totally bounded.) In the general (non-compact) case then, words of fixed length cannot be used if one wishes to communicate with probability 1.

Examples can be given (a pentagon with center not connected to one vertex) of a complete separable metric space in which $K_\epsilon(X)$ is not the supremum of $H_\epsilon(X)$ taken over all Borel probability measures μ on X , although equality is often obtained.

The construction of ϵ -entropy for certain product spaces, done in the next section for probabilistic metric spaces, can also be done for compact metric spaces. Thus, we can speak of the *absolute epsilon entropy* of a compact metric space, which is essentially the infimum of the number of bits necessary to describe all of X to within ϵ , when words of fixed length are used, but when a large number n of experiments from X are performed before transmission.

3. Epsilon Entropy of Product Spaces

The relation of ϵ -entropy to channel coding theorems and their converse is the main consideration. We first define the kind of finite products of probabilistic metric spaces we consider. This definition is motivated by the desire to know each outcome of a sequence of experiments to within ϵ .

DEFINITION. Let $X = (X, d, \mu)$ and $Y = (Y, e, \nu)$ be probabilistic metric spaces. Then the product (probabilistic metric) space

$$X \times Y = [X \times Y, \max(d, e), \mu \times \nu]$$

is the probabilistic metric space whose point set is the cartesian product $X \times Y$ of X and Y , whose metric f is defined by

$$f[(x, y), (x', y')] = \max[d(x, x'), e'(y, y')]$$

and whose measure is the product measure of μ and ν .

DEFINITION. Let X be a probabilistic metric space. Define

$$X^{(1)} = X, X^{(n+1)} = X^{(n)} \times X, n \geq 1$$

Thus $X^{(n)}$ is the n -fold product of X with itself.

Since each $X^{(n)}$ is a probabilistic metric space, we can study the sequence $H_\epsilon(X^{(n)})$. We have the following lemma:

LEMMA 1. The sequence $\{H_\epsilon(X^{(n)})\}$ is subadditive in n . That is,

$$H_\epsilon(X^{(m+n)}) \leq H_\epsilon(X^{(m)}) + H_\epsilon(X^{(n)}) \quad (3)$$

We observe that $H_\epsilon(X^{(m+n)})$ can be less than $H_\epsilon(X^{(m)}) + H_\epsilon(X^{(n)})$. For example, let $m = n = 1$; let X be the unit interval with Lebesgue measure and linear metric; and let $\epsilon = 1/4$. Then

$$2H_\epsilon(X) = 2[(3/4) \log(4/3) + (1/4) \log 4]$$

whereas

$$\begin{aligned} H_\epsilon(X \times X) &= (9/16) \log(16/9) + (5/16) \log(16/5) \\ &\quad + 2(1/16) \log 16 < 2H_\epsilon(X) \end{aligned}$$

In fact, it can be proved that the $3/4$ partition of $X \times X$ in Fig. 7 achieves $H_\epsilon(X \times X)$; diametric entropy or radial entropy can be used here.

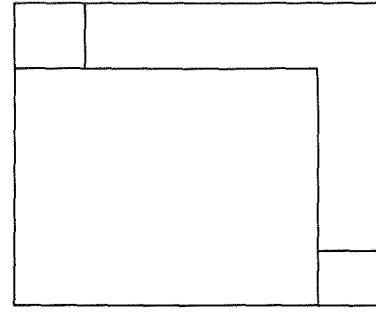


Fig. 7. $3/4$ -partition of $X \times X$

Another inequality which will be useful is

$$H_\epsilon(X^{(m)}) < H_\epsilon(X^{(m+n)}) \quad (4)$$

This is a special case of the following lemma:

LEMMA 2. Let X and Y be separable metric spaces with metrics d_1 and d_2 , and let $Z = X \times Y$ have the metric d_3 , such that if $z_j = (x_j, y_j)$, then

$$d_3(Z_1, Z_2) \geq \max[d_1(x_1, x_2), d_2(y_1, y_2)]$$

Let X, Y, Z be probabilistic metric spaces under some Borel probability distribution on Z and under the induced marginal distributions on X and Y . Thus $H_\epsilon(X) \leq H_\epsilon(Z)$.

We define the absolute ϵ -entropy $I_\epsilon(X)$ as

$$I_\epsilon(X) = \lim_{n \rightarrow \infty} \frac{1}{n} H_\epsilon(X^{(n)}) \quad (5)$$

After Lemma 1, a well-known result on subadditive sequences implies that this limit exists if the $H_\epsilon(X^{(n)})$ are all finite, and

$$I_\epsilon(X) \leq H_\epsilon(X), n = 1, 2, \dots \quad (6)$$

Furthermore, Lemma 1 and Eq. (4) ensure that if any $H_\epsilon(X^{(n)})$ is finite, they are all finite. If these entropies are all infinite, we interpret Eq. (5) to mean $I_\epsilon(X) = \infty$.

$I_\epsilon(X)$ is clearly non-negative, and the following result holds:

LEMMA 3. $I_\epsilon(X)$ is zero if, and only if, $H_\epsilon(X)$ is zero, and infinite if, and only if, $H_\epsilon(X)$ is itself infinite.

The following definition and lemma are needed before channel coding theorems can be formulated:

DEFINITION.

$$I_{\epsilon, [\delta]}(X) = \lim_{n \rightarrow \infty} \inf \frac{1}{n} H_{\epsilon, \delta}(X^{(n)}).$$

LEMMA 4. If $H_{\epsilon}(X) < \infty$, $I_{\epsilon}(X) = \lim_{\delta \rightarrow 0^+} I_{\epsilon, [\delta]}(X)$.

As a corollary to Lemma 4, we have the following lemma:

LEMMA 5. Let $H_{\epsilon}(X) < \infty$. Then

$$I_{\epsilon}(X) = \inf_{\{\delta_n\} \rightarrow 0} \liminf_{n \rightarrow \infty} \frac{1}{n} H_{\epsilon, \delta_n}(X^{(n)}) \quad (7)$$

4. Noisy Channel Coding Theorems

We can now relate our concept of epsilon entropy to channel coding. First note that Eq. (7) states that if $H_{\epsilon}(X)$ is finite, then $I_{\epsilon}(X)$ is the number of bits per sample necessary to describe X to within ϵ over a noiseless channel with probability approaching 1 when arbitrarily long storage is allowed.

At this juncture it can be observed that storage is necessary to achieve even $H_{\epsilon}(X)$. For the "single-use" number of bits necessary is actually not $H_{\epsilon}(X)$, but can be arbitrarily close to $\log 2$ more (Ref. 8). However, since we are interested chiefly in the case $H_{\epsilon}(X)$ large, we shall ignore this difference and refer to $H_{\epsilon}(X)$ itself as the number of bits necessary when storage between experiments is not allowed.

A noisy channel coding theorem for $I_{\epsilon}(X)$ can be stated and proved as in Shannon (Ref. 9). Before we can think of stating a precise result, we must review some definitions (Ref. 10, Chap. 2). Let X and Y be probability spaces with measures μ and ν , and let ρ be a probability distribution on $X \times Y$ defined on the product σ -field of μ and ν . Further, let

$$\rho(A \times Y) = \mu(A)$$

for A μ -measurable, and let

$$\rho(X \times B) = \nu(B)$$

for B ν -measurable. Then the *mutual information* of ρ (more precisely, of μ and ν with respect to ρ), $I(\rho)$, is defined as follows. Let $d\rho/d\mu d\nu$ denote the Radon-Nikodym derivative of ρ with respect to product measure $\mu \times \nu$. Then $I(\rho) = \infty$ if ρ is not absolutely continuous with respect to $\mu \times \nu$; that is, if $d\rho/d\mu d\nu$ is infinite on a set of

positive $(\mu \times \nu)$ -measure. If, on the other hand, ρ is absolutely continuous with respect to $\mu \times \nu$, then

$$\begin{aligned} I(\rho) &= E_{\rho} \left[\log \left(\frac{d\rho}{d\mu d\nu} \right) \right] = \int_{X \times Y} \left(\log \frac{d\rho}{d\mu d\nu} \right) d\rho \\ &= \int_{X \times Y} \left(\log \frac{d\rho}{d\mu d\nu} \right) \left(\frac{d\rho}{d\mu d\nu} \right) d\mu d\nu \end{aligned} \quad (8)$$

which may still be infinite. However, since we have

$$-\frac{1}{e} \leq t \log t, \quad t \geq 0 \quad (9)$$

$I(\rho)$ is well defined as a real number or $+\infty$. And since the function $t \log t$ is convex in $t \geq 0$, we have, from Eq. (8):

$$I(\rho) \geq \left(\log \left[\int_{X \times Y} \left(\frac{d\rho}{d\mu d\nu} \right) d\mu d\nu \right] \right) \left(\int_{X \times Y} \left(\frac{d\rho}{d\mu d\nu} \right) d\mu d\nu \right)$$

Since

$$\int_{X \times Y} d\rho = 1$$

we conclude that $I(\rho) \geq 0$.

It turns out to be easier to first consider the case of radial entropy. The diametric case will be considered afterward. It has a more complicated definition, which will be related to the radial definition. Let $R_{\epsilon}(X)$ be defined as that class of probability distributions ρ on $X \times X$ such that:

$$\rho(A \times X) = \mu(A) \text{ for } A \text{ a Borel set in } X$$

Define

$$\nu_{\rho}(B) = \rho(X \times B) \text{ for } B \text{ a Borel set in } X$$

and

$$\rho(\{(x, y) / d(x, y) \leq \epsilon/2\}) = 1$$

Then the noisy channel coding theorem in question turns out to be the statement that

$$I_{\epsilon}(X) = \inf_{\rho \in R_{\epsilon}(X)} I(\rho) = I'_{\epsilon}(X) \quad (11)$$

This result will now be shown to follow from Shannon's proof in (Ref. 9), coupled with our definitions and results. We will now explain what is going on, under the added restriction that $H_{\epsilon}(X)$ be finite. This restriction will be removed in the next section.

In his discussion on rate distortion (Ref. 9), Shannon studies coding for a source X with a fidelity criterion F which, for our purposes, is a non-negative Borel function of the distance d between two points of X , which is a non-decreasing function of distance. The question is considered of how much channel capacity is needed to send independent outcomes of X over a given noisy channel, if it is desired to keep the average distortion D between the actual outcome x and the decoded outcome y bounded by the constant A . The distortion D is defined as the expectation of $F(x, y)$, the expected loss of fidelity between actual outcome and decoded value. In our case,

$$\left. \begin{aligned} F(x, y) &= 0, & d(x, y) &\leq \epsilon/2 \\ F(x, y) &= 1, & d(x, y) &> \epsilon/2 \end{aligned} \right\} \quad (12)$$

It is desired to know if there is a value C_0 of channel capacity such that, over any channel with capacity $C > C_0$, outcomes of X can be transmitted over the channel in such a way that the distortion is zero, with probability approaching 1; whereas, if $C < C_0$, outcomes cannot be transmitted over the channel in such a way that the distortion is zero, with probability approaching 1, as the block length becomes infinite. Shannon's Theorems 1 and 2 of (Ref. 9) shows that this result is indeed true, where

$$C_0 = \inf_{\rho \in R_\epsilon(X)} I(\rho) \quad (13)$$

We state the desired result in a lemma. We mention that although the proof is adapted from Shannon's Theorem 1, the result does not follow from Shannon's statement.

LEMMA 6. *Let $H_\epsilon(X) < \infty$. Then*

$$I'_\epsilon(X) = I_\epsilon(X)$$

In any case

$$I'_\epsilon(X) \leq I_\epsilon(X)$$

The restriction that $H_\epsilon(X) < \infty$ will be removed when we prove that $H_\epsilon(X)$ is finite if $I_\epsilon(X)$ is finite.

To prove the channel coding result from Lemma 6, first we show that if K is a memoryless channel of capacity C , with

$$C > I_\epsilon(X)$$

then we can communicate over K , keeping the distance between every received outcome and its corresponding transmitted outcome at most $\epsilon/2$ with probability

approaching 1 as the block length increases. To do this, merely take a partition U of $X^{(n)}$ with

$$C > H(U) \geq I_\epsilon(X)$$

Use the centers of the U_i as a new source of entropy $H(U)$ and transmit over K with probability of (word) error approaching zero.

The proof of the converse is omitted here.

To do the diametric case involves a slight complication in the definition of $R_\epsilon(X)$. This happens because we can no longer choose a point y to send when we observe x , but instead must send an ϵ -set Y containing x when x occurs.

So we define matters as follows. Consider the complete separable metric space $Y(X)$ of closed subsets of X under the Hausdorff metric (Ref. 11, Sec. 1.4), or rather its closed subspace $Y_\epsilon(X)$ of sets of diameters at most ϵ . Then the diametric $R_\epsilon(X)$ is the class of Borel probability distributions ρ on $X \times Y_\epsilon(X)$ such that

$$\begin{aligned} \rho[W \times Y_\epsilon(X)] &= \mu(W) & \text{Borel } W \subset X \\ \rho(\{x, Y\} \mid x \in Y) &= 1 \end{aligned}$$

In other words, ρ sends points of X into closed ϵ -sets containing the point, with ν_ρ probability 1.

We then define

$$I'_\epsilon(X) = \inf_{\rho \in R_\epsilon(X)} I(\rho)$$

and the proof goes through as before, with

$$\nu_\rho(B) = \rho(X \times B) \quad \text{for Borel } B \subset Y_\epsilon(X)$$

The question can be raised as to what happens in the radial case if we define $R_\epsilon(X)$ as above, using the closed subset $Z_\epsilon(X)$ of $Y_\epsilon(X)$ of closed sets of radius $\epsilon/2$. The answer is that $I'_\epsilon(X)$ is unchanged. The main point is that unique centers of the $\epsilon/2$ spheres called out in the definition can be chosen in a measurable fashion.

5. Inequalities for I_ϵ and H_ϵ

This section gives some inequalities on I_ϵ and H_ϵ and, in particular, proves that $H_\epsilon(X) < \infty$ if $I_\epsilon(X) < \infty$, closing a gap in the last section. We remark that all previous lower bounds for $H_\epsilon(X)$ were lower bounds for $I_\epsilon(X)$, and, conversely, all previous upper bounds for $I_\epsilon(X)$ were actually upper bounds for $H_\epsilon(X)$ (Refs. 5 and 12). The reason

it is so hard to get bounds for one and not the other is that $H_\epsilon(X)$ and $I_\epsilon(X)$ are close in ratio if either is large. We shall have to distinguish between radial and diametric entropy some of the time; when necessary, we use ${}_D I_\epsilon(X)$, ${}_D H_\epsilon(X)$ for the diametric case, and ${}_R I_\epsilon(X)$, ${}_R H_\epsilon(X)$ for the radial case. Observe the trivial inequalities

$$\left. \begin{aligned} {}_R H_{2\epsilon}(X) &\leq {}_D H_\epsilon(X) \leq {}_R H_\epsilon(X) \\ {}_R I_{2\epsilon}(X) &\leq {}_D I_\epsilon(X) \leq {}_R I_\epsilon(X) \end{aligned} \right\} \quad (14)$$

When a result is true for either definition, we shall omit the presubscript. Our next lemma is quite useful, but will be strengthened later. It provides an upper bound on $I_\epsilon(X)$, but not on $H_\epsilon(X)$.

LEMMA 7.

$$I'_\epsilon(X) \leq E_\mu \left[\log \frac{1}{\mu[S_{\epsilon/2}(x)]} \right]$$

The prime on $I'_\epsilon(X)$ will be removed after the main theorem. The next lemma is sometimes a useful lower bound on $I_\epsilon(X)$, and, of course, on $H_\epsilon(X)$.

LEMMA 8. *Let*

$$\begin{aligned} \alpha &= \sup_{S \text{ an } \epsilon\text{-set}} \mu(S) \\ \beta &= \sup_{x \in X} \mu(S_{\epsilon/2}(x)) \end{aligned}$$

so that $\beta \leq \alpha$ (by Ref. 5, Theorem 3, these supremums are maximums). Then

$${}_D I_\epsilon(X) \geq \log \frac{1}{\alpha} \quad (15)$$

$${}_R I_\epsilon(X) \geq \log \frac{1}{\beta} \quad (16)$$

The next lemma is useful to bound the entropy of a union of spaces; it was implicitly used in Lemma 4. By abuse of language, we shall speak of the epsilon entropy of subsets of X even if they are merely measurable and not closed. The interpretation is quite natural: to send X , say whether you are in Y or Z , then send Y or Z .

LEMMA 9. *Let $X = \cup_k Y_k$, Y_k disjoint and measurable. Regard each Y_k as a probabilistic metric space in its own right, with measure and metric inherited from X . The renormalized probability measure on Y_k is $\mu_k = p_k^{-1} \mu$,*

where $\mu(Y_k) = p_k$. Define

$$\mathcal{P} = \{p_k\}$$

$$H(\mathcal{P}) = \sum p_k \log \frac{1}{p_k}$$

the entropy of $\mathcal{P}$. Then

$$H_\epsilon(X) \leq \sum p_k H_\epsilon(Y_k) + H(\mathcal{P})$$

$$I'_\epsilon(X) \leq \sum p_k I'_\epsilon(Y_k) + H(\mathcal{P}) \quad (17)$$

Furthermore, these inequalities become equalities if, for each k, l with $k \neq l$,

$$\inf_{y_k \in Y_k, y_l \in Y_l} d(y_k, y_l) > \epsilon \quad (18)$$

Remark. The primes will be removed from $I'_\epsilon(X)$ in Eq. (17) after we prove the main theorem.

6. Packings of Sets

This section prepares for the main theorem by introducing a new kind of random coding argument and using it to prove a result on "asymptotic close packing." The argument was first used in Lemma 6.

LEMMA 10. *Let V be the random ϵ -partition of X defined in the following way. Choose a sequence $\{x_i\}$ of points of X randomly d_μ and independently. Define*

$$W_i = S_{\epsilon/2}(x_i) \quad i \geq 1$$

Define the ϵ -partition $V = \{V_n\}$ by

$$V_n = W_n - \bigcup_{i=1}^{n-1} W_i \quad n \geq 1$$

Then

$$\mu \left(\bigcup_{n=1}^{\infty} V_n \right) = 1 \quad \text{with probability 1} \quad (19)$$

Also,

$$\begin{aligned} H_\epsilon(X) &\leq E[H(V)] \leq E \left(\log \frac{1}{\mu[S_{\epsilon/2}(x)]} \right) \\ &\quad + E \left(\log \frac{1}{1 - \mu[S_{\epsilon/2}(x)]} \right) \cdot E \left(\frac{1 - \mu[S_{\epsilon/2}(x)]}{\mu[S_{\epsilon/2}(x)]} \right) \end{aligned} \quad (20)$$

and

$$E[\mu(V_n)] = \int \mu[S_{\epsilon/2}(x)] (1 - \mu[S_{\epsilon/2}(x)])^{n-1} d\mu(x) \quad (21)$$

As a consequence of Lemma 10, we have the following result: Note that Lemma 7 yields

$$I_\epsilon(X) \leq \log \frac{1}{\alpha} < \infty$$

if $\mu[S_{\epsilon/2}(x)] > \alpha > 0$ with probability 1.

LEMMA 11. *Let*

$$\mu[S_{\epsilon/2}(x)] \geq \alpha > 0 \quad \text{with probability 1}$$

Then $H_\epsilon(X) < \infty$. In fact,

$$H_\epsilon(X) \leq \log \frac{1}{\alpha} + \frac{1-\alpha}{\alpha} \log \frac{1}{1-\alpha} < \log \frac{1}{\alpha} + 1 \quad (22)$$

We are now ready for the asymptotic close-packing result.

THEOREM 1. *Let X be a probabilistic metric space such that*

$$\mu[S_{\epsilon/2}(x)] = \alpha \quad \text{with probability 1} \quad (23)$$

Then

$${}_R I_\epsilon(X) = \log \frac{1}{\alpha} \quad (24)$$

and

$$0 \leq {}_R H_\epsilon(X) - {}_R I_\epsilon(X) < 1 \quad (25)$$

If, in addition,

$$\mu(A) \leq \alpha \quad (26)$$

for every measurable set A of diameter at most ϵ , then

$${}_D I_\epsilon(X) = \log \frac{1}{\alpha} \quad (27)$$

and

$$0 \leq {}_D H_\epsilon(X) - {}_D I_\epsilon(X) < 1 \quad (28)$$

Remarks. The probability α must be positive, since $\mu(X) = 1$, so that $H_\epsilon(X) < \infty$ by Lemma 11. The assumption that $\mu[S_{\epsilon/2}(x)] = \alpha$ with probability 1 and that no measurable set of diameter at most ϵ has probability exceeding α is satisfied by spheres, toruses, etc., under the natural measure given by surface hyper-area normalized to 1.

We have seen that

$$H_\epsilon(X) - I_\epsilon(X) < 1$$

if $\mu[S_{\epsilon/2}(x)]$ is constant with probability 1. It is not known whether this difference can be arbitrarily large, but we shall prove in the next section

$$H_\epsilon(X) - I_\epsilon(X) = 0 [H_\epsilon(X)]$$

for $H_\epsilon(X)$ large.

Discussion. Theorem 1 is called an "asymptotic close packing" result for the class of spaces to which it applies, for it says that the space can be partitioned by ϵ -sets in such a way that the resulting partition has entropy less than one more than the entropy of a partition consisting of $1/\alpha$ sets each of probability α . When $\alpha \rightarrow 0$, then, the space can be partitioned by a partition practically as "nice" as a close-packing one. Such nice partitions use up "most" of the probability with sets of probability "almost" equal to α . One can use Theorem 1 to state and prove apparently otherwise difficult theorems about partitions of n -spheres, regular graphs, etc. (Ref. 13).

7. The Main Theorem

In order to get the strongest possible form of the main theorem that we can, we first need the following lemma, a strengthening of the known result (Ref. 5, p. 54) that if $\{a_n\}$ is a non-negative sequence summing to 1, such that

$$\sum a_n \log n < \infty$$

then

$$\sum a_n \log \frac{1}{a_n} < \infty$$

(The converse is true, as well, but we do not need it here.)

LEMMA 12. *For $\alpha, B > 0$, consider all non-negative sequences $a_1, a_2, \dots$ with*

$$\sum_{n=1}^{\infty} a_n \leq 1 \quad (29)$$

$$\sum_{n=1}^{\infty} a_n (\alpha + \log n) \leq B \quad (30)$$

Let

$$M(B) = \sup \sum_{n=1}^{\infty} a_n \log \frac{1}{a_n} \quad (31)$$

over all such sequences.

Let λ be the unique solution on $(1, \infty)$ of the equation

$$e^{-\lambda\alpha} [-\zeta'(\lambda) + \alpha\zeta(\lambda)] = Be \quad (32)$$

where $\zeta(s)$ is the Riemann zeta-function. For $B > \alpha$, let σ be the unique solution on $(1, \infty)$ of

$$\frac{-\zeta'(\sigma)}{\zeta(\sigma)} = B - \alpha \quad (33)$$

Let B_0 be the value of B at which $e^{-\lambda\alpha}\zeta(\lambda) = e$. Then

$$M(B) = \begin{cases} \lambda B + e^{-1-\lambda\alpha}\zeta(\lambda), & B \leq B_0 \\ \sigma(B - \alpha) + \log \zeta(\sigma), & B > B_0 \end{cases}$$

As $B \rightarrow \infty$

$$M(B) = B + \log B + O(1)$$

as $B \rightarrow 0$

$$M(B) = \frac{B}{\alpha} \log \frac{\alpha}{B} + o(B)$$

The next lemma gives a lower bound to $I(\rho)$ for every ρ in $R_\epsilon(X)$.

LEMMA 13. Let $\rho \in R_\epsilon(X)$. Then

$$I(\rho) \geq E_\mu \left(\log \frac{1}{\nu_\rho [S_{\epsilon/2}(x)]} \right) = {}_R K(\rho) \quad (34)$$

Let $\rho \in D R_\epsilon(X)$. Then

$$I(\rho) \geq E_\mu \left(\log \frac{1}{\nu_\rho (A_x)} \right) = {}_D K(\rho) \quad (35)$$

where

$$A_x = \{Y \in Y_\epsilon(X) | x \in Y\}$$

The next lemma upper bounds $H_\epsilon(X)$ in terms of the right-hand side of Eqs. (34) or (35).

LEMMA 14. Let $\rho \in R_\epsilon(X)$. Then there exists an absolute constant C such that

$${}_R H_\epsilon(X) \leq {}_R K(\rho) + \log^+ {}_R K(\rho) + C \quad (36)$$

Let $\rho \in D R_\epsilon(X)$. Then, for the same absolute constant C ,

$${}_D H_\epsilon(X) \leq {}_D K(\rho) + \log^+ {}_D K(\rho) + C \quad (37)$$

The next lemma is not really needed for our main purposes, but is inserted to show that $H_\epsilon(X)$ must approach 0 if $I_\epsilon(X)$ does.

LEMMA 15. There exists a universal constant D such that, if

$$K(\rho) \leq \frac{1}{4} \left(1 - \frac{1}{e} \right)^2$$

then

$$H_\epsilon(X) \leq D [K(\rho)]^{1/2}$$

Note that D can be made arbitrarily close to 2 as $K(\rho) \rightarrow 0$.

We are now ready to state Theorem 2, the main result of this paper.

THEOREM 2. There exist universal constants C and D such that

$$H_\epsilon(X) \leq I_\epsilon(X) + \log^+ I_\epsilon(X) + C \quad (38)$$

and, for

$$I_\epsilon(X) \leq \frac{1}{8} \left(1 - \frac{1}{e} \right)^2$$

$$H_\epsilon(X) \leq D [I_\epsilon(X)]^{1/2} \quad (39)$$

8. Consequences of the Main Theorem

Various consequences of Theorem 2 are now discussed. The first result is of independent interest and also needed for the second result. We recall (Ref. 5, p. 1008) that $H_0(X)$ is defined as the infimum of the entropies of all partitions of X less a set of measure zero by atoms; when there is no such partition, $H_0(X)$ is infinite. With this definition, $H_\epsilon(X)$ is continuous from above in ϵ , even at $\epsilon = 0$, as was shown in the above cited reference.

Corollary 1. $I_\epsilon(X)$ is continuous from above in ϵ on $\epsilon \geq 0$.

Corollary 2. For any probabilistic metric space X , one has

$$H_\epsilon(X) \sim I_\epsilon(X) \quad \text{as } \epsilon \rightarrow 0 \quad (40)$$

The next group of corollaries pick up some loose ends involving $I'_\epsilon(X)$, defined in Eq. (11) as the infimum of $I(\rho)$ for ρ in $R_\epsilon(X)$.

Corollary 3. $I'_\epsilon(X) = I_\epsilon(X)$.

Corollary 4. Eq. (17) holds for $I_\epsilon(X)$ in place of I'_ϵ .

Now define for $0 \leq \delta \leq 1$

$$\bar{I}_{\epsilon, \delta}(X) = \inf_{\substack{A \text{ measurable} \\ \mu(A) \leq \delta}} I_{\epsilon}(X - A) \quad (41)$$

which is always finite for $\delta > 0$. Notice that

$$\bar{I}_{\epsilon, 0}(X) = I_{\epsilon}(X)$$

Also,

$$\bar{I}_{\epsilon, \delta}(X) \leq H_{\epsilon, \delta}(X) \quad (42)$$

The next lemma is needed in the proof of Corollary 5.

LEMMA 16. *Let X be a probabilistic metric space with $H_{\epsilon}(X)$ finite. Then for every $\alpha > 0$ there exists a finite β such that, if Y is a measurable subset of X ,*

$$\mu(Y) \geq \alpha$$

which implies

$$H_{\epsilon}(Y) \leq \beta$$

We then have the following result, proved for $H_{\epsilon}(X)$ in (Ref. 5), Theorem 5.

Corollary 5. $I_{\epsilon}(X) = \lim_{\delta \rightarrow 0^+} \bar{I}_{\epsilon, \delta}(X)$, and, in fact, $\bar{I}_{\epsilon, \delta}(X)$ is continuous from above in δ , $1 > \delta \geq 0$. Also, given $\eta > 0$, there is a $\delta_0 > 0$ such that, for A measurable of measure at most δ_0 ,

$$|I_{\epsilon}(X - A) - I_{\epsilon}(X)| < \eta \quad (43)$$

if $I_{\epsilon}(X)$ is finite. If $I_{\epsilon}(X)$ is infinite, then, given $N > 0$, there is a $\delta_0 > 0$ such that, for A measurable of measure at most δ_0 ,

$$I_{\epsilon}(X - A) > N \quad (44)$$

Finally, if $\delta > 0$, $\bar{I}_{\epsilon, \delta}(X)$ is continuous from below in δ if X is non-atomic.

To get a stronger "strong converse" define, as in Lemma 5,

$$I'_{\epsilon}(X) = \inf_{(\delta_n) \rightarrow 0} \liminf_{n \rightarrow \infty} \frac{1}{n} H_{\epsilon, \delta_n}(X^{(n)}) \quad (45)$$

so that

$$I'_{\epsilon}(X) = I_{\epsilon}(X) \quad (46)$$

if $H_{\epsilon}(X) < \infty$. As in Lemma 5, $I'_{\epsilon}(X)$ is the amount of capacity needed to transmit outcomes of X when it is

desired that an arbitrarily large fraction of a block of outcomes be known to within ϵ , with probability approaching 1. Alternatively, $I'_{\epsilon}(X)$ is the rate distortion function for zero distortion. The stronger converse is really just that $I_{\epsilon}(X) = I'_{\epsilon}(X)$. This means that if K is a memoryless channel of capacity $C < I_{\epsilon}(X)$, we cannot transmit outcomes of X over K such that, with probability approaching 1, an arbitrarily large fraction of a long block of outcomes is known to within ϵ .

First recall

$$I_{\epsilon, [\delta]}(X) = \liminf_{n \rightarrow \infty} \frac{1}{n} H_{\epsilon, \delta}(X^{(n)})$$

for $0 \leq \delta < 1$. We then have the following corollary:

Corollary 6

$$I_{\epsilon, [\delta]}(X) = \liminf_{n \rightarrow \infty} \frac{1}{n} \bar{I}_{\epsilon, \delta}(X^{(n)})$$

In order to prove the stronger converse, and strengthen Lemmas 4 and 5 by removing the requirement from them that $H_{\epsilon}(X)$ be finite, we need a lemma. We first need to define the class $R_{\epsilon, \eta}(X)$ of probability distributions for $0 \leq \eta$; note that

$$R_{\epsilon, 0}(X) = R_{\epsilon}(X)$$

DEFINITION. $R_{\epsilon, \eta}(X)$ is that class of Borel measures ρ on $X \times X$ whose marginal distribution on the first coordinate is μ , and such that

$$\rho(\{(x, y) \mid d(x, y) \leq \epsilon/2\}) \geq 1 - \eta$$

The class ${}_D R_{\epsilon, \eta}(X)$ is defined similarly for the diametric case. Then define

$$I_{\epsilon, \eta}(X) = \inf_{\rho \in R_{\epsilon, \eta}(X)} I(\rho)$$

so that $I_{\epsilon, \eta}(X)$ is the rate distortion function evaluated for distortion η . Therefore

$$I_{\epsilon, \eta}(X) \leq \bar{I}_{\epsilon, \eta}(X) + \eta \log \frac{1}{\eta}$$

and

$$I_{\epsilon, 0}(X) = I_{\epsilon}(X)$$

Finally, define

$$I'''_{\epsilon}(X) = \lim_{\eta \rightarrow 0^+} I_{\epsilon, \eta}(X)$$

We have been unable to determine, except when either is 0 or ∞ (in which case the answer is affirmative), whether

$$I''_{\epsilon}(X) = I_{\epsilon}(X)$$

The next lemma shows, however, that the two are asymptotic when either is large. A strengthening of Theorem 2, it is enough for the stronger converse. The result for $\eta > 0$ gives a result on the relation between $H_{\epsilon, \eta^{1/2}}(X)$ and $I_{\epsilon, \eta}(X)$.

LEMMA 17. Let

$$0 \leq \eta \leq \frac{1}{e^2}$$

Define

$$L_{\epsilon, \eta}(X) = \frac{1}{1-\eta} \left(I_{\epsilon, \eta}(X) + \frac{2}{e} \right) + \log \frac{1}{1-\eta}$$

Then for C , the same constant as in Theorem 2,

$$H_{\epsilon, \eta^{1/2}}(X) < (1 + 6\eta^{1/2}) \{L_{\epsilon, \eta}(X) + \log^+ L_{\epsilon, \eta}(X) + C\} + 3\eta^{1/2} \quad (47)$$

$$H_{\epsilon}(X) \leq I''_{\epsilon}(X) + \log^+ I''_{\epsilon}(X) + C + \frac{4}{3} \quad (48)$$

Now comes the stronger converse; define

$$I''_{\epsilon}(X) = \inf_{(\delta_n) \rightarrow 0} \liminf_{n \rightarrow \infty} \frac{1}{n} H_{\epsilon, \delta_n}(X^{(n)})$$

Corollary 7. $I''_{\epsilon}(X) = I_{\epsilon}(X)$.

The restriction that $H_{\epsilon}(X)$ be finite is removed from Lemma 5 by Corollary 6.

We can likewise show the following, which removes the assumption from Lemma 4 that $H_{\epsilon}(X)$ be finite.

Corollary 8

$$I_{\epsilon}(X) = \lim_{\delta \rightarrow 0^+} I_{\epsilon, [\delta]}(X)$$

We now summarize a combination of our principal results. If K is a memoryless channel with capacity Γ less than $H_{\epsilon}(X) - \log^+ H_{\epsilon}(X) - C$, C a universal constant (or of finite capacity if $H_{\epsilon}(X)$ is infinite) then it is not possible to transmit outcomes of X over K such that, with probability approaching 1, an arbitrarily large fraction of a sequence of outcomes are known within ϵ or even a little

more than ϵ . On the other hand, if Γ exceeds $H_{\epsilon}(X)$, then outcomes of X can be transmitted over K such that, with probability approaching 1, all of a sequence of outcomes are known within ϵ , and such that block coding of outcomes need not be done before the channel encoder. In other words, the "one-shot" epsilon entropy tells whether the maximum error in a long block can be kept as little more than ϵ as we please, with probability approaching 1.

The outstanding open problem in this theory is: Can

$$H_{\epsilon}(X) - I_{\epsilon}(X)$$

be arbitrarily large when either is finite? More generally, determine the function

$$f(x) = \sup_{H_{\epsilon}(X) \leq x} [H_{\epsilon}(X) - I_{\epsilon}(X)]$$

which is finite-valued if x is finite. It is clear that $f(x)$ is strictly increasing as x increases, that $f(0) = 0$, and that $f(x)$ is continuous in $x \geq 0$. The main problem is whether $f(x)$ is bounded in $x \geq 0$.

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F. Discretionary Research: A Simple Sequence-Permutation Method, L. Kleinrock

1. Introduction

Goldstein and Posner (SPS 37-61, Vol. III, pp. 77-78) describe a new method for disguising speech with no bandwidth expansion. They accomplish this by sampling the speech and then by permuting the sequence of speech samples. This report describes a very simple method for carrying out the permutation with the necessary properties. It is shown that a (useful) special case of this permutation code can be decoded (depermuted) by a device which is no more complex than the coder itself. Welch (SPS 37-61, Vol. III, pp. 78-81) describes another class of permutation devices.

2. Permutation Coder

Consider a sequence of speech samples $X_1, X_2, X_3, \dots$. We wish to generate a sequence $Y_1, Y_2, Y_3, \dots$ such that:

- (1) $Y_1, Y_2, \dots$ is a permutation of $X_1, X_2, \dots$.
- (2) For some fixed integer n , the sample X_i ($i = 1, 2, \dots$) is mapped into the set

$$\{Y_{i-n+1}, \dots, Y_{i+n-1}\}$$

The justification for condition (2) is that we are willing to store a data sample for, at most, $2n - 1$ time units at the transmitter before transmitting it in the permuted sequence (the same is true regarding the storage time at the receiver), thus restricting the storage size required.

The system as described by Goldstein and Posner, is to contain a register P which holds n samples in positions $(p_n, p_{n-1}, \dots, p_1)$ from the sequence $X_1, X_2, \dots$. Furthermore a "key" sequence is to be generated which creates a sequence of integers from the set $\{1, 2, \dots, n\}$. One element from the key sequence is generated for each data sample X_i . If the key element for X_i is generated as the integer k , then the new sample X_i is inserted into the k th position of the P register (i.e., into stage p_k), and the

old sample in this position is "bumped out" and transmitted, creating the sample Y_i .

A simple example is shown in Table 3 for $n = 3$. Observe that the first three keys place X_1 into stage p_1 , X_2 into stage p_2 , and X_3 into stage p_3 ; this primes the n -stage P register. The next key, 3, forces X_4 into stage p_3 , thereby ejecting the old sample X_3 which becomes the first transmitted sample, Y_1 , etc. The transmitted sequence is thus seen to be $X_3, X_2, X_1, X_6, X_4, X_5, X_8, \dots$

Note in the transmitted sequence $\{Y_i\}$ that two adjacent samples differ from their original position in the $\{X_i\}$ sequence by, at most, $2n - 1$ ($= 5$ in the example); this will be the case, if and only if, the key sequence $\{k_i\}$ is such that for

$$k_i = m, \quad 1 \leq m \leq n$$

then

$$e_m \leq 2n - 1 \quad (1)$$

where e_m is number of key symbols observed until the key symbol m first repeats. Eq. (1) is equivalent to requiring that each integer m ($1 \leq m \leq n$) occur at least once in every string of $2n - 1$ key symbols.

Table 3. Example of permutation sequence for $n = 3$

Step i	Input X_i	Key sequence k_i	Contents of n -stage P register			Transmitted Y_i
			p_3	p_2	p_1	
1	X_1	1			X_1	
2	X_2	2		X_2		
3	X_3	3	X_3			
4	X_4	3	X_4			$Y_1 = X_3$
5	X_5	2		X_5		$Y_2 = X_2$
6	X_6	1			X_6	$Y_3 = X_1$
7	X_7	1			X_7	$Y_4 = X_6$
8	X_8	3	X_8			$Y_5 = X_4$
9	X_9	2		X_9		$Y_6 = X_5$
10	X_{10}	3	X_{10}			$Y_7 = X_8$
11	X_{11}	1			X_{11}	$Y_8 = X_7$
12	X_{12}	2		X_{12}		$Y_9 = X_9$
13	X_{13}	1			X_{13}	$Y_{10} = X_{11}$
14	X_{14}	2		X_{14}		$Y_{11} = X_{12}$

We are now faced with the problem of finding a simple means for generating the key sequence $\{k_i\}$ with the property given in Eq. (1). This we do as follows. Consider that we have a shift register S with $2n$ positions and that we place two copies of each of the first n integers in the stages of this register (e.g., Fig. 8), where we label the j th

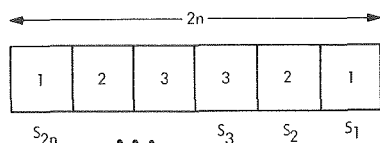


Fig. 8. Shift register of the encoder ($n = 3$)

stage as S_j . Assume also that we have available a pseudo-random binary number generator, which generates the sequence $\{c_i\}$ where

$$c_i \in \{0, 1\}, i = 2, 3, 4, \dots$$

The key sequence will be generated by shifting the contents of our $2n$ -stage shift register to the right and observing the sequence of values which appear in stage S_1 under the following operating rules:

For the i th shift, replace S_j by S_{j+1} for

$$j = 1, 2, \dots, 2n - 2$$

and

- (1) If $c_i = 0$, then replace S_{2n-1} by S_{2n} and replace S_{2n} by S_1 .
- (2) If $c_i = 1$ then replace S_{2n-1} by S_1 and leave S_{2n} untouched.

See Fig. 9. Thus, either we make a cyclic shift to the right among the $2n$ stages (if $c_i = 0$) or we make a cyclic shift to the right among the rightmost $2n - 1$ stages (if $c_i = 1$). As mentioned earlier, the key sequence $\{k_i\}$ is given by the sequence of integers appearing in stage S_1 . For example, the key sequence in Table 3 could have been generated from the random sequence $\{c_i\}$ and the initial configuration of S in Fig. 8 as shown in Table 4.

Let us now prove that the procedure described above generates a key sequence which has the property expressed in Eq. (1). Consider the values of S_j ($j = 1, \dots, 2n - 1$) at step i , and observe that these $2n - 1$ entries will shift

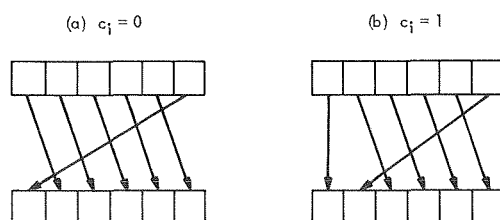


Fig. 9. Encoder shifting operation

Table 4. Generation of key sequence shown in Table 3

i	c_i	S_6	S_5	S_4	S_3	S_2	S_1	k_i
1		1	2	3	3	2	1	1
2	1	1	1	2	3	3	2	2
3	0	2	1	1	2	3	3	3
4	1	2	3	1	1	2	3	3
5	0	3	2	3	1	1	2	2
6	0	2	3	2	3	1	1	1
7	1	2	1	3	2	3	1	1
8	0	1	2	1	3	2	3	3
9	0	3	1	2	1	3	2	2
10	1	3	2	1	2	1	3	3
11	1	3	3	2	1	2	1	1
12	0	1	3	3	2	1	2	2
13	0	2	1	3	3	2	1	1
14	1	2	1	1	3	3	2	2

right one position at each step regardless of the random sequence $\{c_i\}$, since only the new values of S_{2n} and S_{2n-1} depend upon c_i . Thus, given $S_{2n-1}, \dots, S_1$ at step i , we must have that $k_i = S_1, k_{i+1} = S_2, \dots, k_{i+2n-2} = S_{2n-1}$. But, in this sequence $k_i, \dots, k_{i+2n-2}$, each integer from 1 to n is represented at least once (in fact, all but one integer are represented exactly twice, and this one integer is represented exactly once). This last is true, since at any step i only one integer is in stage S_{2n} ; the mix described above must, therefore, be in stages $S_{2n-1}, \dots, S_1$ which must then be the next $2n - 1$ key symbols. See Fig. 10 which shows the register at step 7 from Table 4. Thus we have proven that such keys obey the condition in Eq. (1).

3. Properties Required for Decoders

As mentioned earlier, the P register is used to hold the samples $\{X_i\}$ until they are "bumped out" and transmitted. Let us define, for our key sequence,

$$d_i = \text{number of generated key symbols until } k_i \text{ is first repeated} \quad (2)$$

Thus, from Table 3 we have $d_1 = 5, d_2 = 3, d_3 = 1, d_4 = 4, d_5 = 4, \dots, d_{12} = 2$.

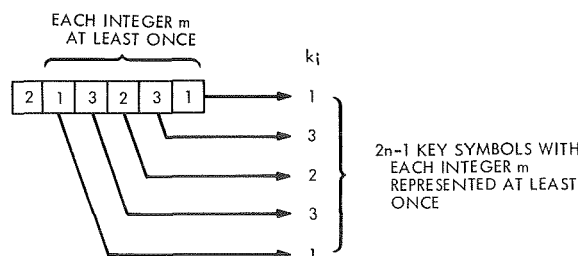


Fig. 10. History regardless of $\{c_i\}$

Our decoder (depermutter) is to operate in a similar fashion, making use of a second register Q , which holds n samples in positions $(q_n, q_{n-1}, \dots, q_1)$ from the received sequence $Y_1, Y_2, \dots$. A second key sequence $\{K_i\}$ (which must be derivable from the random sequence $\{c_i\}$) where $K_i \in \{1, 2, \dots, n\}$, determines the position q_{K_i} , where the next received sample Y_i is to be inserted, and the old sample in this position is "bumped out" and delivered as the next sample Z_i of the decoded speech.

In Table 5, we show how to decode the permuted speech of Table 3. Note that the inherent time delay between the generation of X_i at the source and delivery of X_i at the destination is $2n$ (neglecting propagation delays). *The critical observation is that every sample must see a total system delay of exactly $2n$.*

Let us define, as in Eq. (2),

$$\Delta_i = \text{number of generated key symbols (of } \{K_i\}) \text{ until } K_i \text{ is first repeated} \quad (3)$$

Thus, from Table 5 we require $\Delta_1 = 5$, $\Delta_2 = 3$, $\Delta_3 = 1$, $\Delta_4 = 5$, $\Delta_5 = 2$, $\dots$, $\Delta_8 = 2$. Now let us trace the history of sample X_i . First, it is delayed by d_i in the transmitter. Thus, recognizing that the first transmitted sample Y_1 occurs n time steps after the first input X_1 , then the sample X_i will be the $(i + d_i - n)$ th Y sample, namely

$$Y_{i+d_i-n} = X_i \quad (4)$$

Moreover our sample, which is the $(i + d_i - n)$ th, will be delayed by Δ_{i+d_i-n} time steps in the receiver. However, from the critical observation made earlier, the total delay must be $2n$; so we require, for any encoder-decoder using

the P and Q registers, that

$$d_i + \Delta_{i+d_i-n} = 2n \quad (5)$$

We have already described how to generate the key sequence $\{k_i\}$ from the random sequence $\{c_i\}$. We must now specify a simple way to create the second key sequence $\{K_i\}$ from $\{c_i\}$ so that we can carry out the decoding operations shown in Table 5; the procedures shown will guarantee that Eq. (5) is satisfied.

Operation would be simple if

$$K_i = k_i \quad (6)$$

Then we could generate $\{K_i\}$ exactly as we did in the encoder. In order for Eq. (6) to hold, we must have

$$d_i = \Delta_i \quad (7)$$

and this requires, therefore, that

$$\Delta_i + \Delta_{i+\Delta_i-n} = 2n \quad (8)$$

as can be seen from Eq. (5). Now, is Eq. (8) true in general? Certainly it is always true for $\Delta_i = n$. However, other values of Δ_i and n can be found for which it fails, using the key generator we have described (for example, from Table 5, $\Delta_1 = 5$, and $\Delta_{1+\Delta_1-3} = \Delta_6 = 2$ and $5 + 2 \neq 6 = 2n$). Note, however, that the special case $n = 2$ always satisfies Eq. (8), as we now show! We require, for $n = 2$

$$\Delta_i + \Delta_{i+\Delta_i-2} = 4 \quad (9)$$

Now $\Delta_i = 1, 2$, or 3 only:

- (1) Consider $\Delta_i = 1$, as exemplified by the following pair imbedded in the key sequence

$$\dots, \underset{i}{1}, \underset{i+1}{1}, \dots$$

Since each integer 1, 2 must occur at least once in every $2n - 1 = 3$ adjacent key symbols, then, the keys k_{i-1} and k_{i+2} must each be equal to 2. That is,

$$\dots, \underset{i-1}{2}, \underset{i}{1}, \underset{i+1}{1}, \underset{i+2}{2}, \dots$$

In this case,

$$\Delta_i = 1, \Delta_{i+\Delta_i-2} = \Delta_{i-1} = 3$$

Table 5. Decoding operations

Step i	Received samples Y_i	Key sequence K_i	Contents of n -stage Q register			Decoded speech Z_i
			q_3	q_2	q_1	
1	$Y_1 = X_3$	1			X_3	
2	$Y_2 = X_2$	2		X_2		
3	$Y_3 = X_1$	3	X_1			
4	$Y_4 = X_6$	3	X_6			$Z_1 = X_1$
5	$Y_5 = X_4$	2		X_4		$Z_2 = X_2$
6	$Y_6 = X_5$	1			X_5	$Z_3 = X_3$
7	$Y_7 = X_8$	2		X_8		$Z_4 = X_4$
8	$Y_8 = X_7$	1			X_7	$Z_5 = X_5$
9	$Y_9 = X_9$	3	X_9			$Z_6 = X_6$
10	$Y_{10} = X_{12}$	1			X_{11}	$Z_7 = X_7$
11	$Y_{11} = X_{12}$	2		X_{12}		$Z_8 = X_8$

and so

$$\Delta_i + \Delta_{i+\Delta_{i-2}} = 4 = 2n$$

which satisfies Eq. (9).

- (2) For $\Delta_i = 2$ we have the case $\Delta_i = n$, which always satisfies Eqs. (9) and (8).
- (3) For $\Delta_i = 3$, we have, for example,

$$\dots, 2, 1, 1, 2, \dots$$

$i \quad i+1 \quad i+2 \quad i+3$

In this case

$$\Delta_i = 3, \Delta_{i+\Delta_{i-2}} = \Delta_{i+1} = 1$$

and so

$$\Delta_i + \Delta_{i+\Delta_{i-2}} = 4$$

which satisfies Eq. (9).

Thus, $n = 2$ always satisfies Eq. (9) if Eq. (1) is satisfied. Our encoding procedure guarantees that Eq. (1) holds.

Now a permutation encoder-decoder system for $n = 2$ is extremely poor for voice disguising, since samples are not "mixed" well enough. However, we may concatenate (as did Welch) many $n = 2$ encoder stages and do likewise for the decoder to obtain the better mixing from larger n , while still maintaining the essential simplicity of encoding and decoding available from the $n = 2$ case. This we describe in *Subsection 4*:

4. Implementation for Concatenated $n = 2$ Coder-Decoders

As shown above, Eq. (8) always holds for $n = 2$. Thus the decoding key sequence $\{K_i\}$ is the same as the encoding key sequence $\{k_i\}$, and each is trivial!

The $n = 2$ encoder-decoder single-level system is shown in Tables 6 and 7, respectively, with suitably shifted index.

In Fig. 11 we show the input and output terminals for an encoder and a decoder.

Now, we wish to concatenate these systems. For this purpose we use superscripts to denote which level in the chain we refer to. Thus, consider M concatenated encoders, the m th feeding the $(m + 1)$ th, etc., as shown in Fig. 12.

Table 6. Encoder for $n = 2$

i	c_i	s_4	s_3	s_2	s_1	k_i	X_i	p_2	p_1	Y_i
1		1	2	2	1	1	X_1		X_1	
2	1	1	1	2	2	2	X_2	X_2		
3	0	2	1	1	2	2	X_3	X_3		X_2
4	1	2	2	1	1	1	X_4		X_4	X_1
5	1	2	1	2	1	1	X_5		X_5	X_4
6	0	1	2	1	2	2	X_6	X_6		X_3
7	0	2	1	2	1	1	X_7		X_7	X_5
8	1	2	1	1	2	2	X_8	X_8		X_6
9	0	2	2	1	1	1	X_9		X_9	X_7
10	1	2	1	2	1	1	X_{10}		X_{10}	X_9

Table 7. Decoder for $n = 2$

i	Y_i	$K_i = k_i$	q_2	q_1	Z_i
1	X_2	1		X_2	
2	X_1	2	X_1		
3	X_4	2	X_4		X_1
4	X_3	1		X_3	X_2
5	X_5	1		X_5	X_3
6	X_6	2	X_6		X_4
7	X_7	1		X_7	X_5
8	X_9	2	X_9		X_6

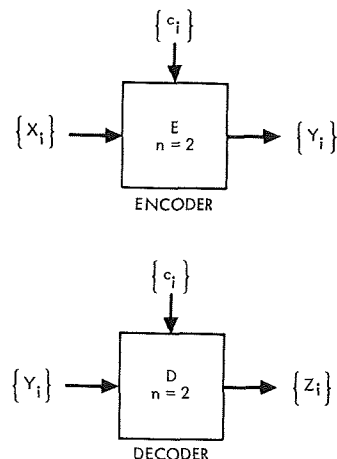


Fig. 11. Notation for encoder and decoder in Tables 6 and 7

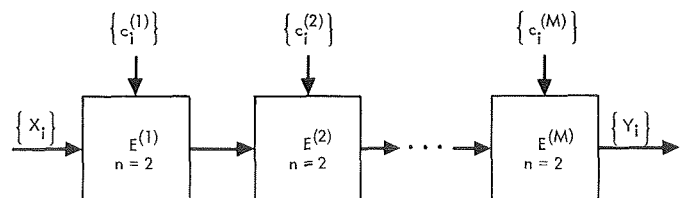


Fig. 12. M-level encoder concatenation

Note that the output from the M th level is the transmitted sequence. We assume that the random binary sequences $\{c_i^{(m)}\}$ are all independent of each other. The decoder which works for the system of Fig. 12 is shown in Fig. 13. Note that the random sequence used for $E^{(M-m+1)}$; thus $K_i^{(m)} = k_i^{(M-m+1)}$. Clearly then the input to $E^{(m)}$ is the same as the output

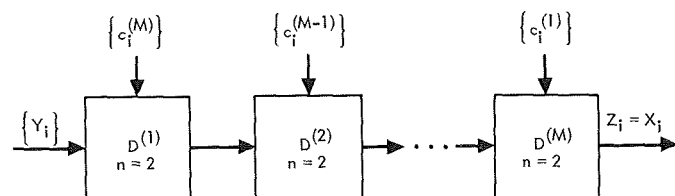


Fig. 13. M -level decoder

from $D^{(M-m+1)}$ and, most importantly $Z_i^{(M)} = X_i$ (with the obvious shift in indices of $4M$). We note that the "effective" length n_e for this concatenated system is $n_e = nM$.

The initial contents of the various S registers should be chosen so that a mixed speech sequence can easily be generated. Experimental work on this device is now being undertaken. Estimates of the randomness properties of sequences generated in this way will be made.

5. Conclusion

By using concatenation and a novel permutation encoder, we can easily implement an extremely simple voice disguiser where the encoder and decoder are essentially identical.

VII. Communications Elements Research

TELECOMMUNICATIONS DIVISION

A. Large Spacecraft Antennas: Performance Comparison of Focal Point and Cassegrain Antennas for Spacecraft Applications,

P. W. Cramer, Jr.

1. Introduction

The primary objective of this task was to establish which of the two high-gain antenna configurations, cassegrain or focal point feed, is optimum for spacecraft applications. The maximum gain for a given diameter was used as the criterion for comparison. It was anticipated that the focal point feed antennas would be more effective for the smaller diameter antennas and cassegrain for the larger. Therefore, the task reduced to establishing the diameter for which the two antenna types had the same gain performance. A byproduct of this task was the determination of optimum design parameters for the two configurations.

To provide a true comparison, the parameters for each antenna type were varied for each diameter, and the best design for each diameter was used in the final comparison. To facilitate the scaling needed for the investigation of a number of parametric values, and to give a common base

of comparison between the antenna types, a feed pattern that could be expressed simply as a mathematical function was necessary. A $\cos^4 \theta$ power pattern was selected as a reasonable representation of typical feeds and an easily handled function for scaling purposes.

Some of the design results would not be applicable to feeds that deviated substantially from the assumed pattern, so the design results should be considered as guidelines, and for critical systems a final optimization using the actual feed pattern would be necessary.

2. Focal Point Feed Configuration

Four different diameters were considered for the comparison. For each diameter, focal length-to-diameter (F/D) ratios ranging in value from 0.35 to 0.5 and reflector edge illumination tapers of 7 to 15 dB were selected. The overall antenna efficiency for all combinations of the above parameters was then calculated utilizing the Antenna Feed Efficiency Program.¹

¹Cramer, P., *Modifications of Rusch Scattering Program and Other Associated Programs for Univac 1108*, Jan. 26, 1970 (JPL internal document).

Antenna efficiency was not a function of the reflector diameter, since the efficiency analysis used does not use frequency as a parameter and the illumination was of the form $E = \cos^2(K_f\theta)$, where K_f is a constant. (In an actual design, however, it would be expected that as the diameter of the reflector approached a few wavelengths, the efficiency would begin to drop off. However, since the nominal frequency used for this task is 8434 MHz and the smallest diameter reflector considered is 96 in., we are well above the point where this would be an appreciable problem.)

The evaluation then reduced to obtaining the best efficiency for each F/D as a function of edge illumination. Figure 1 illustrates the results of this calculation. It should be noted that the peaks of the curves are quite broad. A variation in the edge taper of ± 0.5 dB from the best taper causes an efficiency reduction of less than 0.1% from the maximum antenna efficiency and a variation of ± 1 dB in the edge taper causes a reduction of no more than 0.3% in efficiency. A 0.3% change in the magnitude of the lowest efficiencies obtained in this study corresponds to a loss of

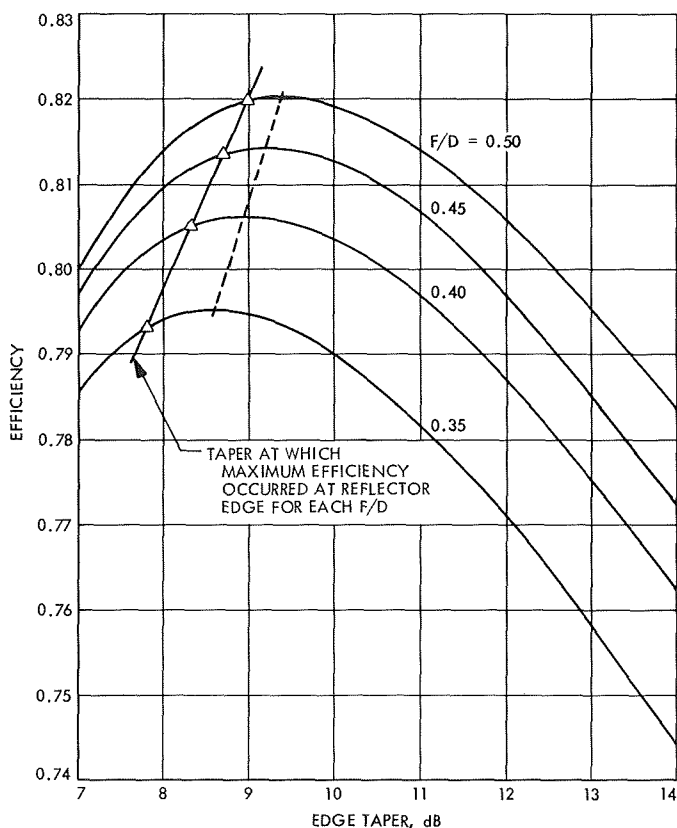


Fig. 1. Pattern efficiency vs edge taper for paraboloidal reflector

0.02 dB, which is negligible. Thus, there is some flexibility in the choice of edge illumination that can be used. The data of Fig. 1 are shown in Fig. 2 as maximum pattern efficiency plotted against F/D .

It is interesting to note that if the edge taper is adjusted for the overall highest efficiency, the maximum efficiency, as a function θ , actually falls at an angle less than the edge angle of the reflector. An interpretation of this result can be made from the following viewpoint: The maximum efficiency, when located at a smaller angle than the reflector edge angle, suggests that, to improve the efficiency, a smaller edge angle should be chosen. However, a small edge angle corresponds to a higher F/D . But we have already noticed that the efficiency increases with larger F/D ratios (see Fig. 2). Therefore, the smaller angular value of the maximum efficiency is just indicating that higher efficiencies are obtained with larger F/D ratios.

Figure 3 shows the data of the previous figures broken down into spillover and illumination efficiency as a function of F/D . The illumination efficiency is the most strongly affected and primarily accounts for the changes in pattern efficiency as a function of F/D .

Since the feed for this configuration is located at the focal point, a transmission line is required to connect the feed with an electronic package located behind the reflector. Since the input to a cassegrain feed can be located behind the reflector by appropriate adjustment of the cassegrain parameters and, therefore, near the electronic package, the transmission line losses associated with the focal point design must be taken into consideration for a fair comparison.

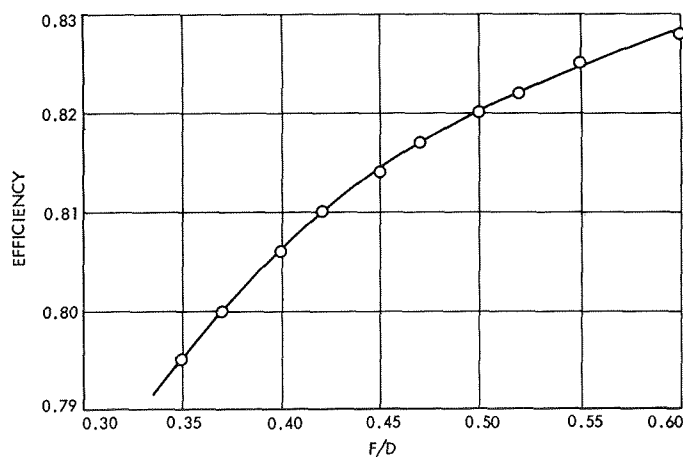


Fig. 2. Pattern efficiency vs F/D for focal point fed paraboloidal reflector

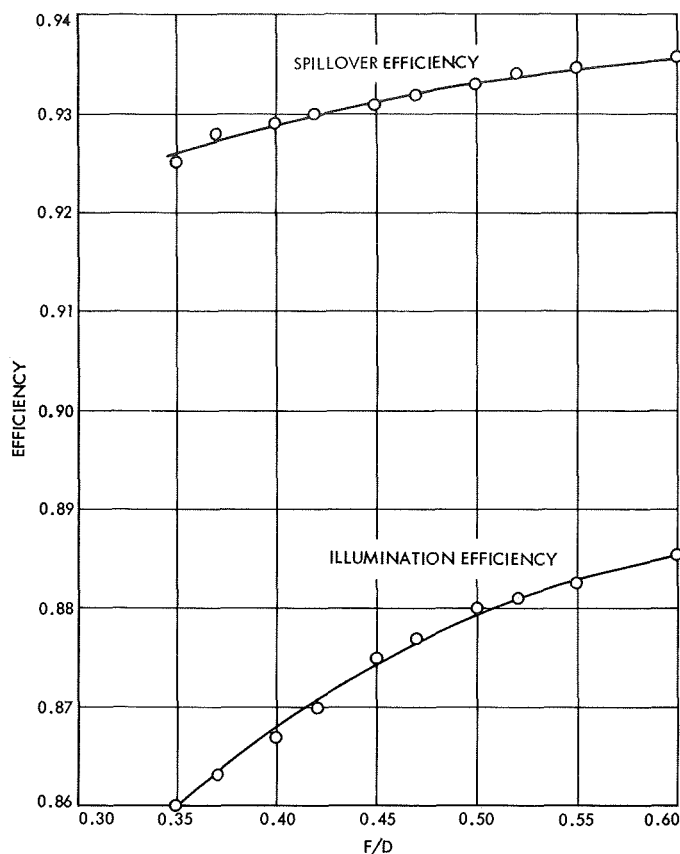


Fig. 3. Spillover and illumination efficiencies of focal point fed paraboloidal reflector

Since the primary frequency of interest is X-band, waveguide can be used without an excessive weight penalty. In this case, an aluminum waveguide with a transmission line loss of 0.032 dB/ft was assumed. A transmission line path from the focal point to the reflector, along a feed support leg making an angle of 30 deg with the antenna axis, was assumed and also a length from the leg end to the center of the antenna. Although some variations on this design are possible, as shown in *Subsection 5*, the potential improvements are small and do not strongly affect the results. Losses as a function of focal length were computed and values ranging from 0.16 to 0.83 dB were obtained.

To compute the highest overall efficiency for each antenna diameter, the efficiency, including line loss, was computed and plotted in Fig. 4. From these data the highest overall efficiency may be selected for each diameter. The overall efficiency includes the transmission line loss, which is a function of focal length, and the pattern efficiency, which is a function of F/D . The focal lengths were restricted to those resulting from F/D ratios between 0.35 and 0.5, since these are the most practical from antenna design consideration.

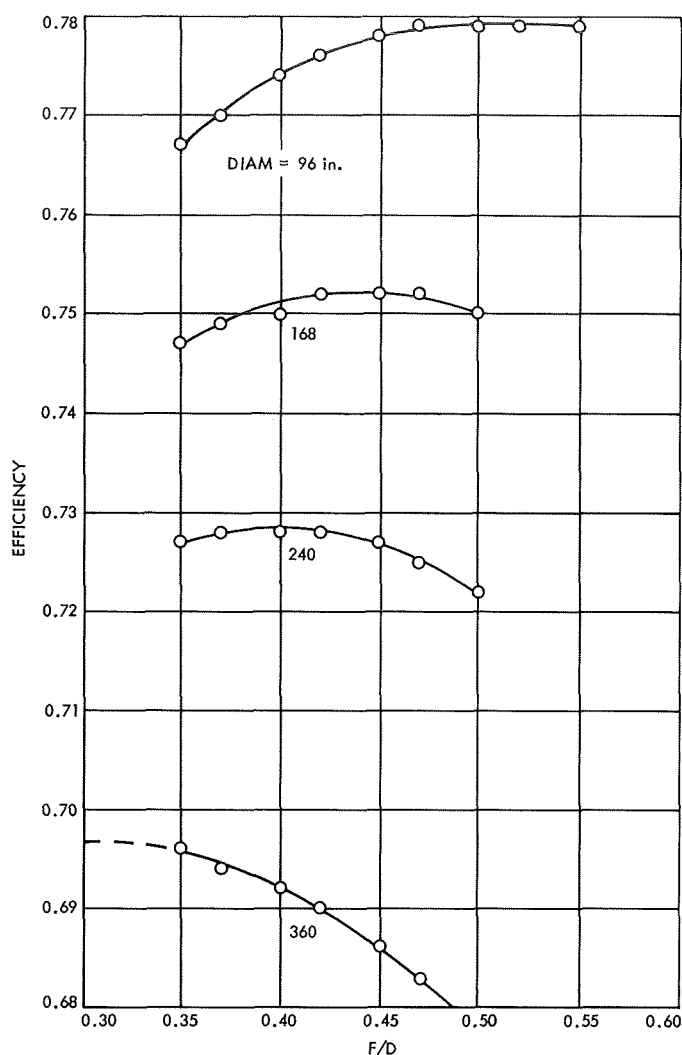


Fig. 4. Overall efficiency vs F/D for focal point fed paraboloidal reflector

3. Optimization Program

To establish the parameters for the cassegrain part of the evaluation, a computer program was written to determine the diameter, focal length, and the length of the transverse axis of the hyperbola, given the diameter and F/D of the main reflector, and the beamwidth, aperture diameter, and location of the phase center of the primary feed. A minimum blockage criterion was used in selecting the parameters where the blockage caused by the feed aperture must be equal to the blockage caused by the hyperbola, as shown in Fig. 5.

For large antennas used in the DSIF, the feeds are so small that this blockage is caused by the feed cone structure rather than the feed itself. However, for cassegrain

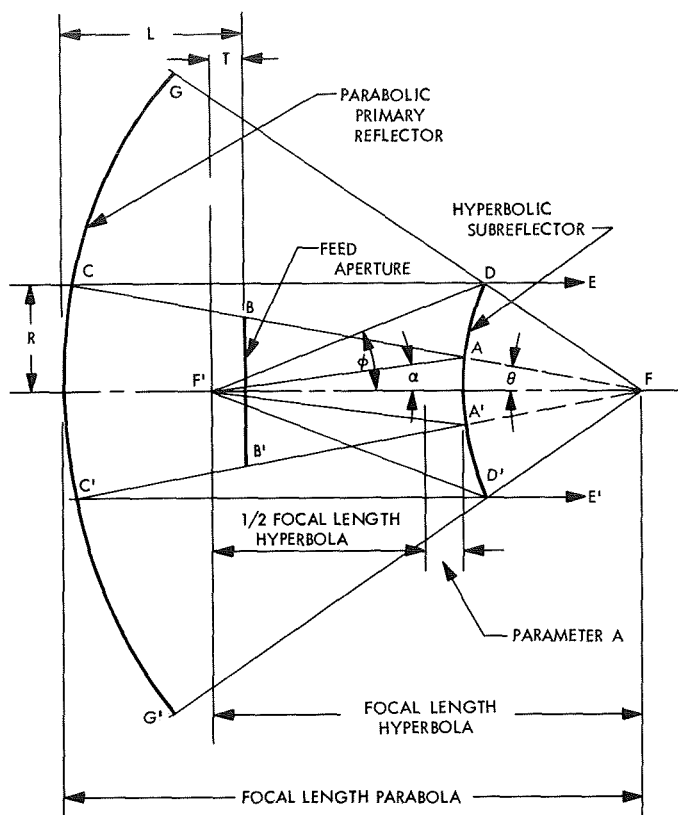


Fig. 5. Cassegrain configuration

antenna designed for spacecraft use, antenna diameters of 14 ft are considered typical. With feed diameters of 1.5 ft, blockage loss due to a feed is significant.

The only way to reduce the blockage loss, assuming an optimum configuration, is to design for the smallest feed aperture for a given beamwidth and beam shape. Since feeds that have shaped beams require larger than normal apertures, checks should be made to see if the additional efficiency gained by using a feed with an optimum beam shape are greater than the efficiencies lost due to increased blockage.

The feed configuration used for the analysis was derived from some early feed work done in support of the Thermo-electric Outer-Planet Spacecraft (TOPS) Program. The TOPS feed concept consisted of a horn antenna designed for monopulse tracking at S-band and telemetry at both X- and S-band. The aperture diameter used was 16 in. with a half beamwidth of 20 deg and a phase center located 4 in. inside the feed. Since the feed is designed to work at both frequencies, the aperture was sized for S-band and, therefore, the blockage loss is much higher than that which would otherwise result if the feed had been designed for X-band only. Although the TOPS feed design has changed

since the start of this study, it is not expected that these changes would cause appreciable differences in the efficiency performance; therefore, the above dimensions can be considered typical as far as this study is concerned.

Configurations evaluated consisted of four main reflector diameters: 96, 168, 240, and 360 in. with F/D ratios of 0.35, 0.4, and 0.5. The feed dimensions described above were scaled so that the configurations could be evaluated for a number of different subdish illumination angles for each case of diameter and F/D . The evaluation consisted of determining the following subdish parameters: diameter, focal length, location, and the parameter A under the minimum blockage criteria (see Fig. 5). It should be noted that the hyperbola's focal length uniquely determines the location of the primary feed since the phase center of the primary feed is located at one hyperbolic focus and the focus of the parabola is located at the other hyperbolic focus.

4. Cassegrain Antenna Calculations

The cassegrain antenna calculations consisted of computing the scattered field patterns from hyperbolic subreflectors, selected from the optimization studies, when illuminated by a primary feed. A modified form of the Rusch Scattering Program (Footnote 1) was used to perform the calculation. The total power in the primary feed patterns was also computed. The computed scattered pattern from the hyperbola was then fed into the Antenna Feed Efficiency Program (Footnote 1) along with the total power in the corresponding primary feed pattern. The efficiency of a parabolic reflector, when illuminated by this scattered field pattern, was calculated as a function of θ , the angle between the line from the focal point to the reflector edge and the axis of the parabola. Thus, given an angle θ , the efficiency can be determined if the parabola selected has an edge angle θ , and is illuminated by the given scattered field pattern.

The feed pattern shape selected for illuminating the hyperbola was $E = \cos^2(K_c\theta)$, the same as that for the focal point feed. This selection was made so as to have a common basis for comparing cassegrain and focal point antennas. Obviously, the cassegrain feeds will be more directive or $K_c > K_f$. Edge tapers for the hyperbola ranged from 7 dB to, in some cases, 17 dB in the study.

The efficiencies were computed for two primary feed illumination angles for each case of F/D and diameter used in the optimization program. The two illumination angle cases were selected as follows: the angle giving the least blockage loss and an angle locating the feed close to

the parabola vertex. It was found that the efficiency varied less than 0.5% between the two cases for $F/D = 0.40$ and all diameters considered, indicating that the primary feed axial location is not very important under the following conditions:

- (1) The feed properties, including pattern, are linearly scalable as a function of antenna beamwidths. This assumption was used when evaluating the feed at different axial locations that resulted from varying the beamwidth of the feed.
- (2) The hyperbola parameters are selected on a minimum blockage loss basis.

For the configurations having the lowest efficiency, the 0.5% efficiency change amounts to less than 0.03 dB. (For F/D ratios as large as 0.58, the variation in efficiency was on the order of 1.2%, which could be significant. However, applications of F/D ratios this large are not common.)

This result is important since it allows the feed location to be determined by physical considerations, such as the best location of the base of the feed, mounting requirements, and transmission line constraints. Another advantage is that the feed can be sized to give the best overall feed performance, without concern that the feed dimensions or beamwidths chosen will force locating the feed at a point where the antenna system efficiency will suffer. The largest half angle for the feed that was investigated was 40 deg. For angles greater than 40 deg, i.e., smaller feed-subdish spacings, the position of the feed may become a factor in the antenna efficiency. Therefore, special care may be necessary when considering configurations with feed illumination half angles greater than 40 deg.

The most interesting result of the efficiency data for the cassegrain antenna was that the best diameter for the primary (parabolic) reflector was smaller than that determined by the purely geometrical approach of the optimization study. In the purely geometrical design (Fig. 5), a ray from the feed phase center F' to a point D on the edge of the subreflector is reflected to a point G on the edge of the main reflector. However, when subreflector diffraction effects are taken into consideration, the optimum diameter (i.e., edge angle) of the main reflector is approximately 13% smaller than this geometrical case. Figure 6 illustrates this behavior in the case of a parabola that has a diameter of 168 in. and $F/D = 0.35$. Curve B shows the efficiency of the cassegrain antenna as a function of edge taper at the subreflector, where the parabola diameter is held constant at 168 in. The highest efficiency

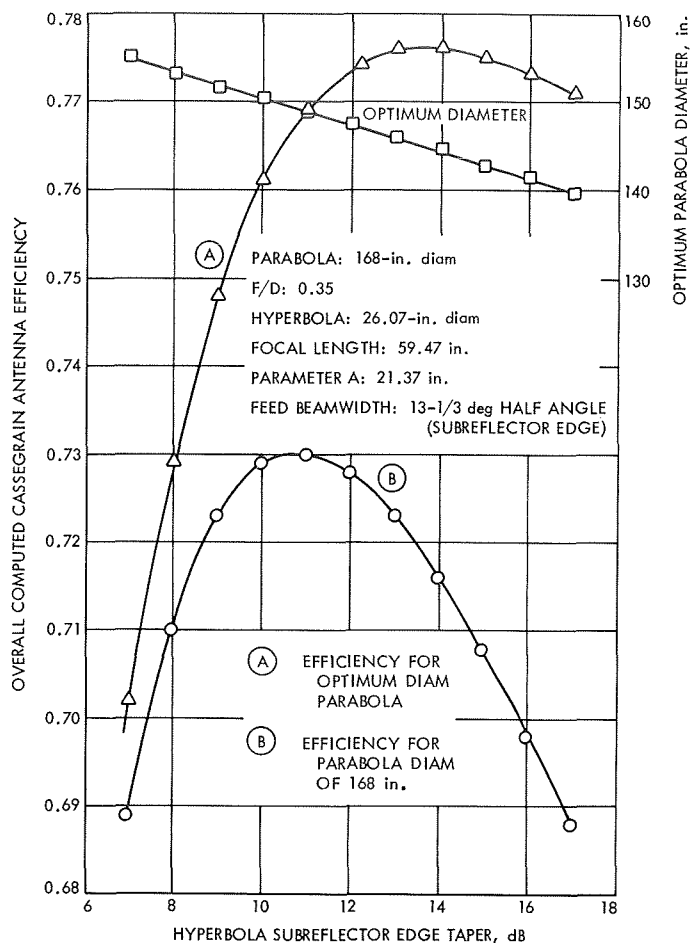


Fig. 6. Efficiency vs subreflector edge taper for cassegrain configuration

is 73% at an edge taper of 11 dB. Curve A was derived by reducing the parabola edge angle until the highest efficiency was obtained for each value of subreflector edge taper. The corresponding diameter is also plotted on Fig. 6. For an edge taper of 11 dB, the efficiency in this case is 76.9%, 3.9% above that for the geometric case. However, curve A does not peak out at 11 dB, but continues to increase to a maximum value of 77.6% at 13.5-dB edge taper for an improvement of 4.6%. Thus, for maximum performance, the primary reflector should always be smaller than the geometric diameter or, conversely, the subreflector should be larger. In actual practice the subreflector would be made larger, as the primary reflector diameter would be selected on the basis of the overall gain requirement.

Figure 7 defines some terms that will be used in discussing this point. The geometric primary reflector diameter and geometric subreflector diameter shown are those from the configuration optimization calculations. The optimum

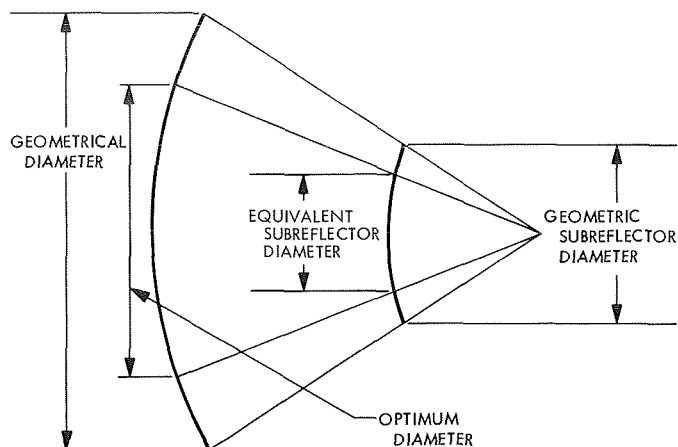


Fig. 7. Cassegrain design terminology

diameter is the primary reflector diameter for highest efficiency. If we select a geometric diameter for the subreflector based on the optimum primary reflector diameter, with all other parameters remaining constant, then the new subreflector diameter will be called the equivalent subreflector diameter. In Fig. 8, the data of Fig. 6 have been replotted in terms of the equivalent taper, where the equivalent taper is the level of the illuminating feed pattern at the subreflector's equivalent diameter. Also shown in Fig. 8 is the optimum (equivalent) parabola diameter associated with each taper. This equivalent taper is nearly constant, so a good rule of thumb approach to the design of a cassegrain antenna would be to determine the parameters based on a geometric configuration, adjust the edge taper on the subreflector to approximately 9.2 dB, and then extend the subreflector to a new diameter approximately 17% larger than that for the geometric case (see Fig. 9 for appropriate subreflector scale factors).

The scale factors shown in Fig. 9 are considered as a lower bound. For different feed locations they can be 2 to 7% larger for F/D ratios of 0.4 and 0.58, respectively. Therefore, the values displayed would be primarily a useful starting point for determining a subreflector diameter. The edge tapers at the subreflector equivalent diameters are quite stable at 9.1 to 9.2 dB for different cassegrain configurations and therefore, except for magnitude, the efficiency curve in Fig. 8 can be considered typical. The effect of the choice of F/D on the efficiency of an optimum cassegrain antenna is shown in Fig. 10.

The improvement in performance with the oversized subreflector results from the reduction of the scattering loss from the subreflector. Figure 11 shows the subreflector scattering efficiency as a function of the subreflector diameter in terms of wavelengths.

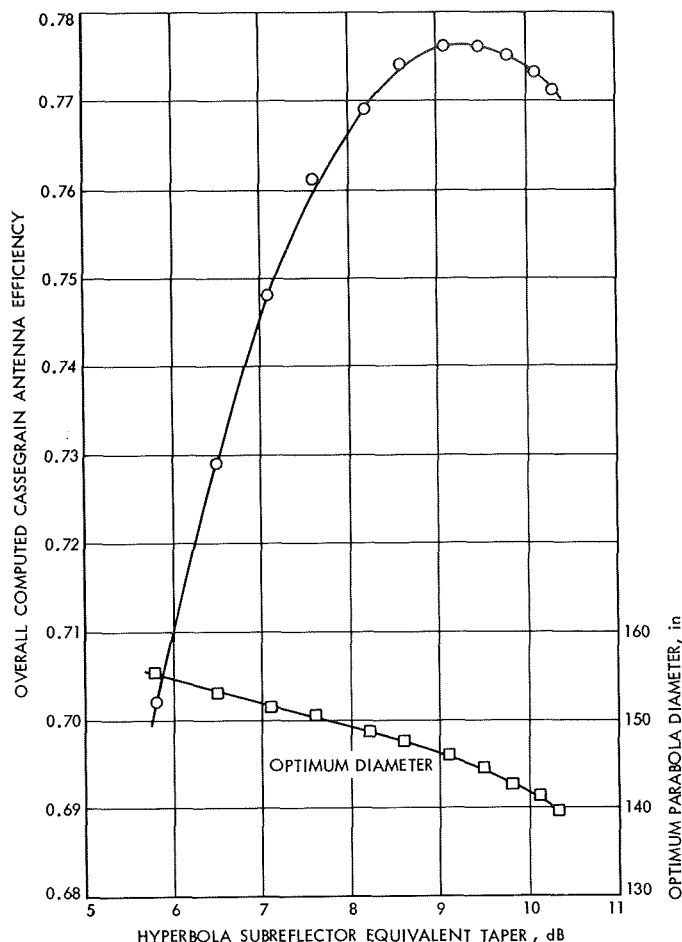


Fig. 8. Efficiency vs subreflector equivalent taper for cassegrain configuration (optimum diameter case)

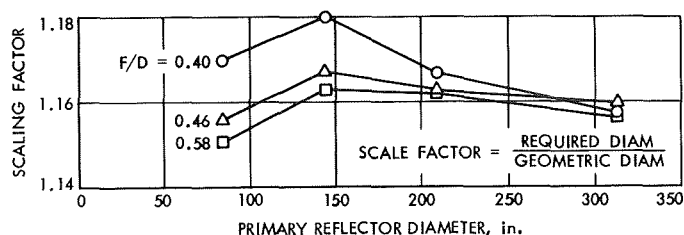


Fig. 9. Scaling factor for subreflector diameter

The scattering efficiency is defined as

$$\eta = \frac{\eta_c}{\eta_f}$$

where

η_c = efficiency of cassegrain antenna with subreflector diffraction effects

η_f = efficiency of the cassegrainian system if reflection occurred optically (efficiency of the illumination feed pattern at the edge of the subreflector)

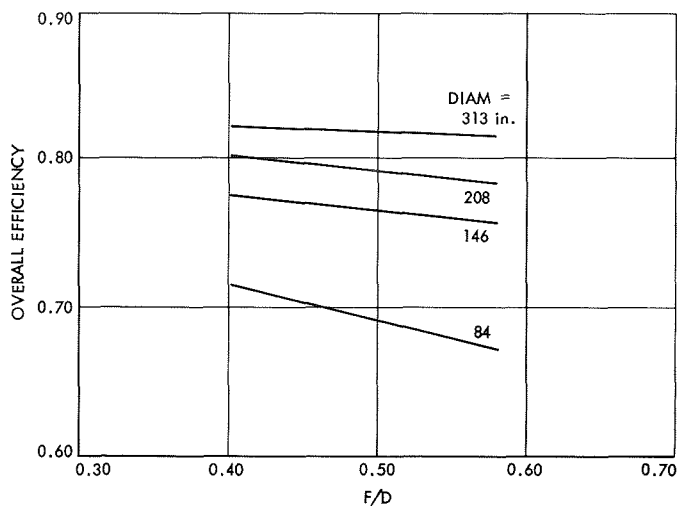


Fig. 10. Efficiency vs F/D for optimum case

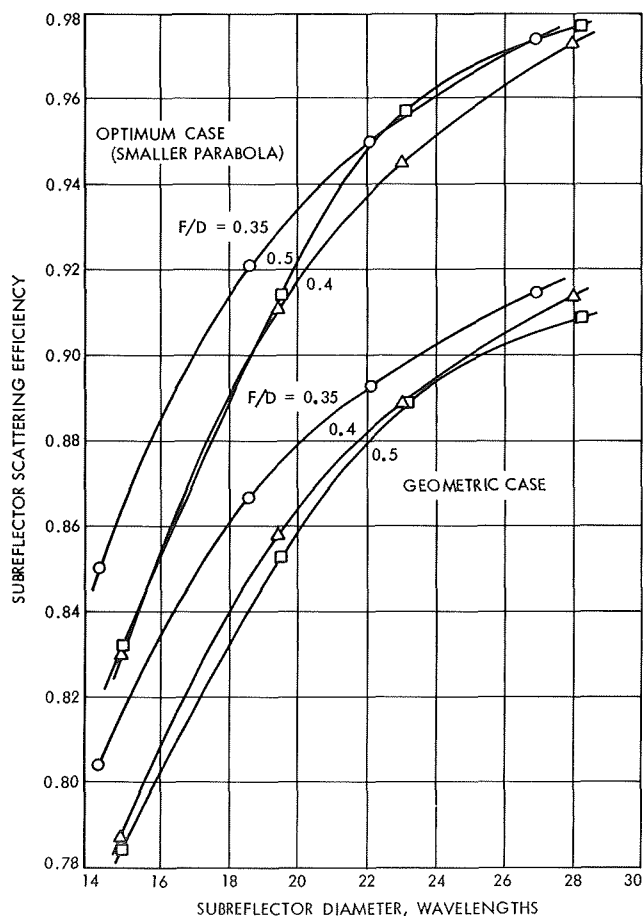


Fig. 11. Cassegrain subreflector scattering efficiency

Two cases are shown (Fig. 11). The first is based on the geometric configuration and is represented by the lower set of curves. The second is based on the optimum primary reflector diameter and includes the upper set of curves. In each case several F/D values were used. As can be seen, the scattering efficiency for an oversized subreflector is approximately 3% higher than for the geometric case. Also, it can be seen that as the cassegrain antenna gets larger, requiring a larger subreflector, subreflector scattering efficiency also increases. A further refinement of the cassegrainian design is possible by subreflector shaping (Ref. 1), which actually can result in scattering efficiencies greater than 1.0. This type of design will be studied in the future.

For determining the overall efficiency of a cassegrain antenna as a function of diameter (for comparison with the focal point configuration), the F/D and subreflector taper that gave the highest optimum cassegrain antenna efficiency for each primary reflector optimum diameter were selected.

5. Comparison Between Focal Point and Cassegrain Antenna Configurations

Figure 12 shows the results of the efficiency of the optimized cassegrainian antennas compared to results of the optimal focal point antennas derived from Fig. 4.

From these curves, it can be generally said that focal point antennas are preferable below diameters of 130 in. and cassegrain above 130 in. In the vicinity of 130-in.

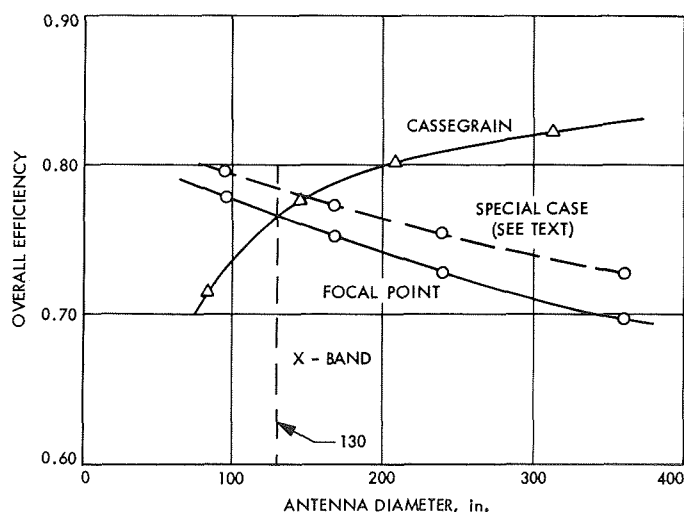


Fig. 12. Performance comparison between cassegrain and focal point fed paraboloid antennas

diameters, the choice would probably be based on configuration and/or mechanical considerations rather than efficiency.

As mentioned in *Subsection 2*, the focal point feed curve was based on providing the antenna input on the axis of the antenna just behind the primary reflector. If the input (electronic package) is moved to the base of one of the support legs, the increase in efficiency results in the dashed curve in Fig. 12. As can be seen, this moves the intersection of the cassegrain and focal point curves to approximately 150 in., or a change of only 20 in., which is not very significant considering the range of antenna sizes that are of interest. Therefore, the focal point design is not unfairly penalized by the on-axis assumption.

In summary it can be concluded that in general focal point antennas are more desirable for diameters less than approximately 130 in. and cassegrain configurations above 130 in. (if efficiency is the primary consideration). If the constraint of S-band/X-band operation was eliminated in favor of just X-band, the cassegrain antenna would have a higher efficiency and would become desirable at a smaller diameter due to a smaller illuminating feed causing a lower blockage loss, a factor that would be most pronounced at the smaller diameters. Also high-efficiency shaped cassegrainian antennas will favor a smaller crossover diameter. Thus, there are exceptions that might shift the crossover diameter either way, but for the present the selection of 130 in. appears to be a useful rule of thumb. It should also be remembered that to achieve an improvement over the focal point designs, particularly near this crossover point, it is necessary to optimize the subreflector diameter rather than simply using a geometrical design.

Reference

1. Galindo, V., "Design of Dual-Reflector Antennas with Arbitrary Phase and Amplitude Distributions," *IEEE Trans. Ant. Prop.*, Vol. AP-12, pp. 403-408, July 1964.

B. Large Spacecraft Antennas: Use of Efficiency Program to Calculate Feed Defocusing Loss,

A. Ludwig and J. Hardy

1. Introduction

When a feed is displaced a distance ΔZ from the focus of a parabolic reflector antenna (see Fig. 13), the resulting decrease in performance is called axial defocusing loss. Displacements transverse to the axis also cause loss, but the primary effect in this case is to shift the antenna beam, and the amount of gain loss for a given displacement is

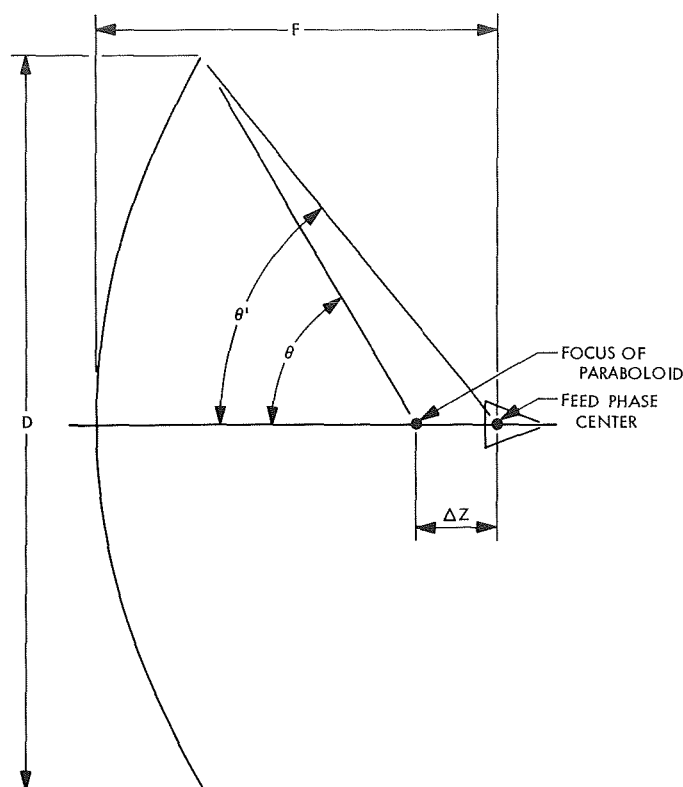


Fig. 13. Defocusing geometry

small compared to axial defocusing. Defocusing loss is well known, but surprisingly difficult to calculate unless gross simplifying assumptions are made. This article presents results of a relatively easy way to calculate axial defocusing loss. Results are computed in particular for the Thermoelectric Outer-Planet Spacecraft (TOPS) antenna, which is a case of current interest.

2. Method

Typically, part of a feed development program at JPL will be the evaluation of antenna feed efficiency using a computer program developed for this purpose (Ref. 1). Axial feed defocusing may be accurately simulated numerically by modifying the feed phase pattern data as follows:

$$\phi'(\theta) = \phi(\theta) + \frac{\Delta Z}{\lambda} (1 - \cos \theta) \times 360.0 \quad (1)$$

where

ΔZ = defocusing distance, in.

λ = RF wavelength, in.

$\phi'(\theta)$ = phase pattern of the defocused feed, deg

$\phi(\theta)$ = original phase pattern, deg

By evaluating the efficiency of the feed with the modified phase pattern and comparing with the efficiency of the original pattern, the defocusing loss may be determined. This method does not involve any further approximations over the physical optics technique used in the feed efficiency program; it does, however, make one of those approximations invalid if very large values of ΔZ are involved. The conventional physical optics technique assumes that the E and H fields are transverse to a ray from the origin of the coordinate system, which, in this case, is the focus of the paraboloid. If the feed is displaced a large distance from the focus, this assumption becomes invalid. In practice, this effect will be negligible except for extreme defocusing.

Using this technique, the computed defocusing loss will be symmetrical for positive or negative displacements, except for the geometrical edge angle effect shown in Fig. 13. If the original angle at which the efficiency is evaluated is θ , then the efficiency in the defocused case should be evaluated at θ' . This geometrical correction

introduces an odd symmetry for positive or negative displacements, but is normally a small factor as will be seen in the data presented below.

The advantages of using the efficiency program to compute defocusing loss are:

- (1) It is simple and involves little work.
- (2) It is not necessary to make gross assumptions about the feed pattern (such as uniform illumination) but rather an actual feed pattern may be used.
- (3) The program takes only a few seconds of computer time.
- (4) One computer run yields results for *all* values of the focal length-to-diameter (F/D) ratio (the F/D ratio has a one-to-one correspondence to an edge angle and the efficiency data is output for all edge angles).
- (5) The results are only dependent on D/λ rather than D and λ , except for the edge angle correction.

3. Results

Defocusing data for two F/D ratios are shown in Fig. 14 for two test case illuminations. These results illustrate that the loss is strongly dependent on the F/D ratio, but is only moderately sensitive to the illumination shape. The F/D dependence is shown explicitly in Fig. 15 for three fixed

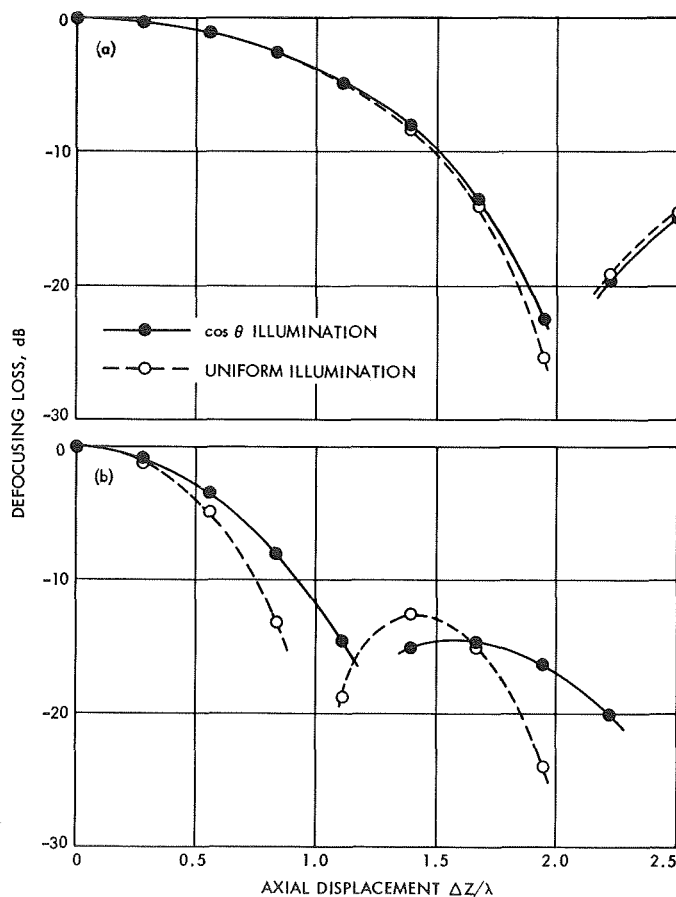


Fig. 14. Defocusing loss for: (a) $F/D = 0.433$,
(b) $F/D = 0.25$

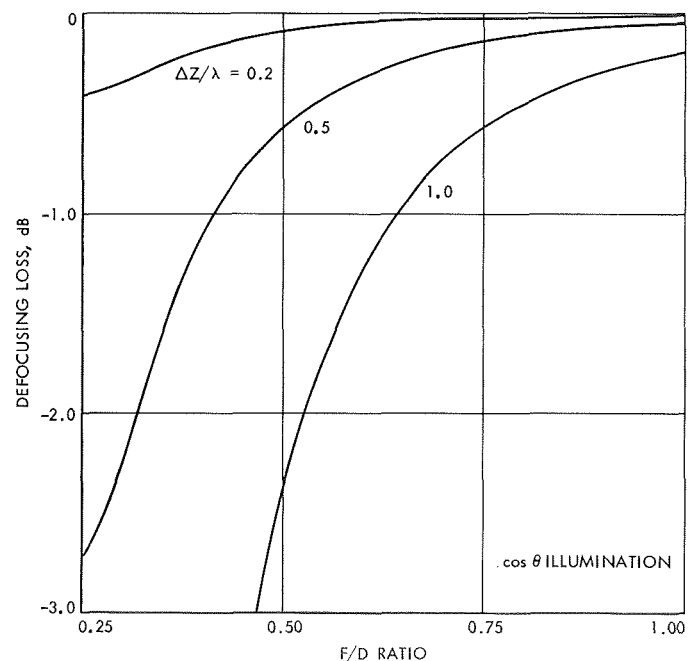


Fig. 15. Defocusing loss vs F/D ratio

displacements. In this data, the geometrical edge angle correction is not included. For the particular case of the TOPS antenna configuration, with $F/D = 0.420$ and $D = 168$ in., the results are compared to a highly sophisticated solution obtained by Rusch (Ref. 2) in Fig. 16. Even when the angle correction is neglected, the results agree to better than 0.1 dB for losses up to 4 dB. The difference between the exact (Rusch) solution and the approximate solution is shown in Fig. 17. Note that the full-scale difference is only 0.05 dB and that the defocusing loss is about 4 dB at $\Delta Z/\lambda = \pm 1.0$, so the differences we are dealing with are quite small. When the edge angle correction is included in the efficiency program results, somewhat surprisingly, the agreement is not improved. As shown in Fig. 17, although the *shape* of the correction is very similar to the difference between the exact and approximate solutions, the magnitude differs by about a factor of 2. The reason for this is being investigated, but since the differences involved are so small, it does not really affect the usefulness of the results.

References

1. *Computer Programs for Antenna Feed System Design and Analysis: Vol. I. Programs and Sample Cases*, Program X, Technical Report 32-979. Edited by A. Ludwig. Jet Propulsion Laboratory, Pasadena, Calif., Apr. 15, 1967.
2. Ingerson, P. G., and Rusch, W. V. T., "Studies of an Axially Defocused Paraboloid," *IEEE 1969 GAP International Symposium Program and Digest*, pp. 62-68, Dec. 1969.

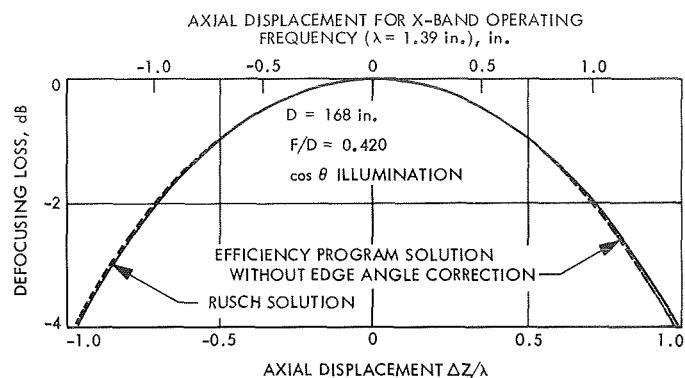


Fig. 16. Defocusing loss for the TOPS antenna

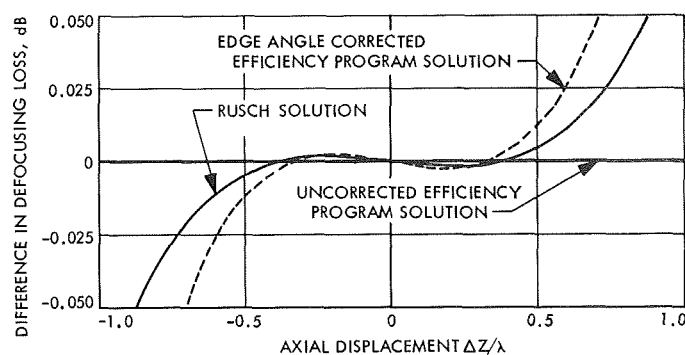


Fig. 17. Difference in defocusing loss between approximate and exact solutions

VIII. Spacecraft Radio

TELECOMMUNICATIONS DIVISION

A. Advanced Development of a Multiple-Cathode Electron Gun, *L. J. Derr*

1. Introduction

Planning for the outer-planet missions of the 1970s has shown that continuous operation of the spacecraft transmitter may be required for periods as great as 12 yr. The RF power levels, efficiencies, and frequency requirements for these missions are currently best satisfied with an electron tube as the final amplifier of these transmitting systems. However, the required life of such transmitters is beyond the present proven 3-4 yr life capability of available tube types. Therefore, it is necessary to develop electron tubes that not only satisfy the mission RF power, efficiency and frequency requirements, but also satisfy the advanced transmitter life capability requirement demanded by these exploratory missions.

The operating life of an electron tube is strongly dependent upon the emission capability of the tube's electron gun. Failures in this area are caused by cathode material depletion, gaseous poisoning of the emitting surface, and heater failures. After proper burn-in, the electron gun becomes the major life-determining component of the

transmitter tube. This, then, is the component that must be improved to obtain extremely long transmitter life.

Although present JPL life tests show that a 4-yr life capability is possible (Ref. 1), research has not yet produced a cathode that clearly demonstrates a substantial increase in tube life capability over older, more tested cathodes. A reasonable approach to extreme life appears to be one where multiple cathodes, of a proven design, are provided in a single electron tube. Each cathode would be mechanically moved, either automatically or by command, into the proper operating position as its predecessor reached the end of its operational life span. Mechanical models of this multiple-cathode electron gun (MEG) concept, originated at JPL, have demonstrated the ability to properly position the cathodes and operate satisfactorily in a vacuum environment.

2. Device Description

Mechanically, the device can be described as a rotating disc upon which multiple cathode/filament assemblies have been symmetrically positioned. A thermostatic motor is used to rotate the disc, which is supported on a

precision-bearing system. The rotation is indexed and locked by a thermostatically actuated arm that is also supported on a precision-bearing system. The tolerance required for cathode positioning in electron tubes is 0.001 in. in all possible directions of movement. (The JPL mechanical models have demonstrated positioning of the multiple cathodes to within a 0.0001-in. tolerance.)

The first mechanical model of the MEG (Fig. 1) utilized radially disposed cathodes, individual driving and locking systems, and radiant heating elements for actuating the thermostatic motors. Although this model clearly supports the feasibility of the MEG concept, some of its components were too fragile for spacecraft use. The required thermostatic motor heater power to cycle the unit was approximately 100 W.

Mechanical and electrical improvements made in the second model of the MEG (Fig. 2) included axially disposed cathodes to provide better position control between the electron tube's cathodes and its first electrode (the anode). (The driving and locking systems used are shown in Fig. 3.) In this improved design, the number of thermostatic motors was reduced to one and the motor used to create both the driving and locking forces required. This motor is heated directly through a novel electrical arrangement that eliminates the need for fragile, inefficient, radiant heating elements. The required thermostatic motor heater power to cycle this unit is 25 W.

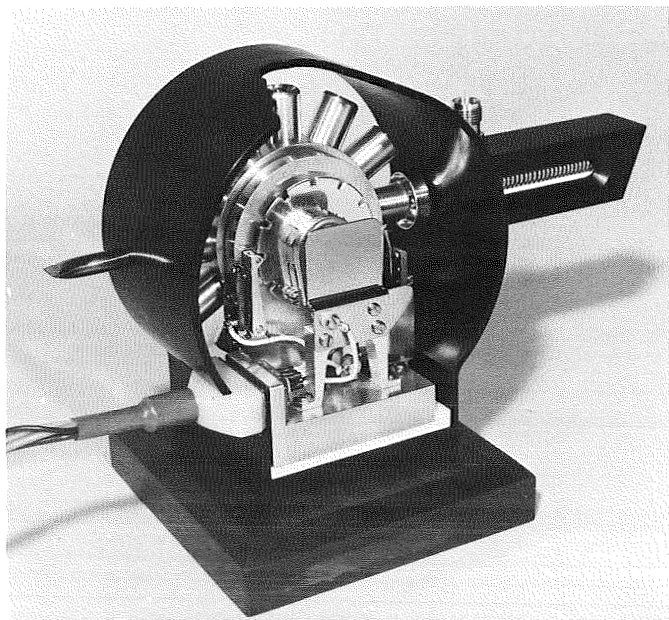


Fig. 1. First MEG mechanical model

3. Environmental Testing

Certain environmental tests have been performed to demonstrate the second model's capability to function as

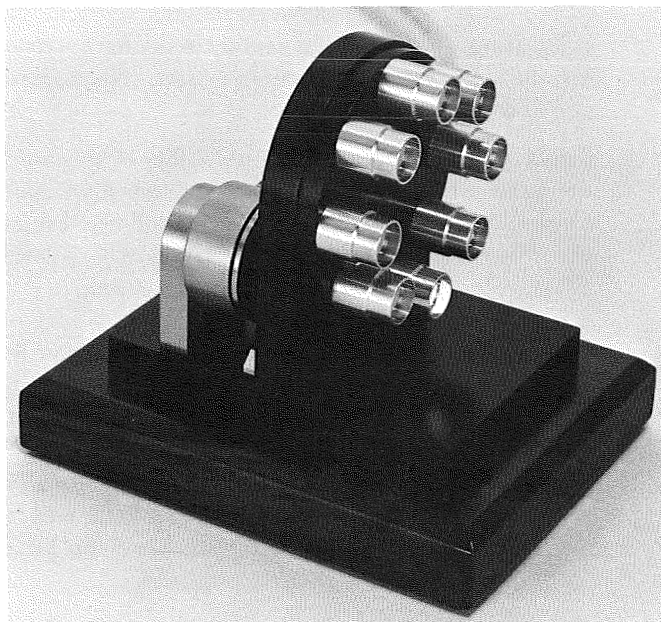


Fig. 2. Second MEG mechanical model

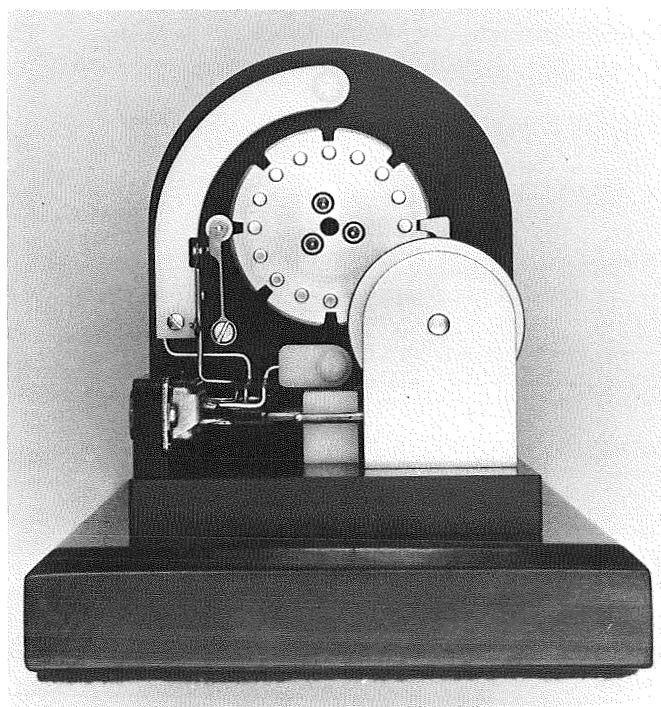


Fig. 3. Driving and locking systems used in the second MEG model

a component of an actual electron tube. These tests were as follows:

- (1) *Vacuum test.* The unit was subjected to a 5-day vacuum test at 10^{-6} torr. During this test, the cathodes were cycled daily to demonstrate the absence of molecular sticking between moving surfaces. No outgassing of the heated motor could be detected on the test station's vacuum gages. No failures occurred during this test.
- (2) *Cycle test.* To determine the reliability of this unit's mechanical design, a continuous cycle test was performed (in air) during which the unit was cycled every 5 min for a 30-day period. The unit completed 2180 cycles without a failure or malfunction.

A control unit to operate the MEG was developed for use on all test demonstrations. The rotating disc and the locking arm operate mechanical switches within the MEG that sense the various stages of completion of each cycle. The control unit utilizes this logic to automatically time its heating and cathode voltages. The same logic will be provided to the spacecraft power supply when the MEG unit is fully developed as a transmitting tube.

4. Conclusions and Future Plans

The mechanical models of the MEG have demonstrated the feasibility of moving, positioning, and operating a multiple-cathode disc within a vacuum environment. The

next developmental task is to fabricate working models of this design with actual, operating cathodes. These units will then be attached to a working linear beam tube to provide the final proof of the MEG concept.

A contract has been written with the Watkins-Johnson Company to perform the necessary development tasks. This program will require 18 mo and will consist of the following phases:

- (1) Development of a third mechanical model for environmental evaluation of its physical structure.
- (2) Design and fabrication of a vacuum-enclosed MEG diode, with actual cathodes, for electrical evaluation.
- (3) Design and fabrication of a final MEG, which will be attached to a 25-W S-band traveling-wave tube for RF evaluation.

Upon completion of this contract, the MEG-traveling-wave tube will be delivered to JPL and begin a multi-year life test. Should this electron-gun concept prove successful, the MEG will be applicable to all linear beam tubes and find use wherever automated cathode replacement is desired.

Reference

1. Hughes, R. S., "Spacecraft S-Band 10-300 Watt R.F. Amplifier Tubes," paper presented at the AIAA Third Communications Satellite Systems Conference, April, 1970.

IX. Spacecraft Communications Systems

TELECOMMUNICATIONS DIVISION

A. Preliminary System Testing of the Proposed TOPS All-Digital Command Detection Algorithm, J. C. Springett and W. J. Weber

1. Introduction

An all-digital command detection/synchronization algorithm has been proposed (SPS 37-59, Vol. III, pp. 68-70 and SPS 37-61, Vol. III, pp. 126-129) as a potential solution to the problems of long-life and high-reliability required by the TOPS and other outer-planet programs. Because the all-digital algorithm represents a departure from previous command systems, a short-term program was undertaken to experimentally evaluate portions of the algorithm in order to assess overall system performance. This article describes the results of that effort.

2. Rationale and Approach

The algorithm, as originally proposed, was the subject of the investigation described in this article. The signal-plus-noise process is input to a low-pass filter (LPF) with cutoff frequency W , in Hertz, and the output of the filter is sampled in an analog-to-digital converter (A/DC) at rate $2W$ samples/s. The samples are operated on by a digital processor in four distinct modes. Mode 1 begins with a

noise-only condition at the input, and functions to determine when a signal-plus-noise condition exists. In mode 2, the signal present is an unmodulated square wave at the subcarrier frequency, and the processor's function is to determine which of 16 possible subcarrier phase estimates is closest to being in-phase with the received signal. In mode 3, the square-wave subcarrier signal is replaced, on the basis of one code bit per subcarrier cycle, with a pseudo-noise (PN) sequence, and the processor decides, using a maximum likelihood criteria, as to which phase of the PN code is proper. This procedure produces the proper alignment of the bit sync clock. (The sequence of signals comprising unmodulated subcarrier signal followed by the PN sequence, etc., is prearranged; thus, the processor knows how long each component lasts and waits following the acquisition of each component as appropriate.) The last function of mode 3 is to detect a phase-reversal in the PN sequence ($\overline{PN}$), setting up the processor for mode 4, which performs data detection on the subcarrier modulated by the data bits. It is also the function of mode 4 to track any phase-drift between the received and local clocks and to continually check for the presence of signal.

In the simulation and experimental work that was performed, it was decided to concentrate on modes 2 and 4.

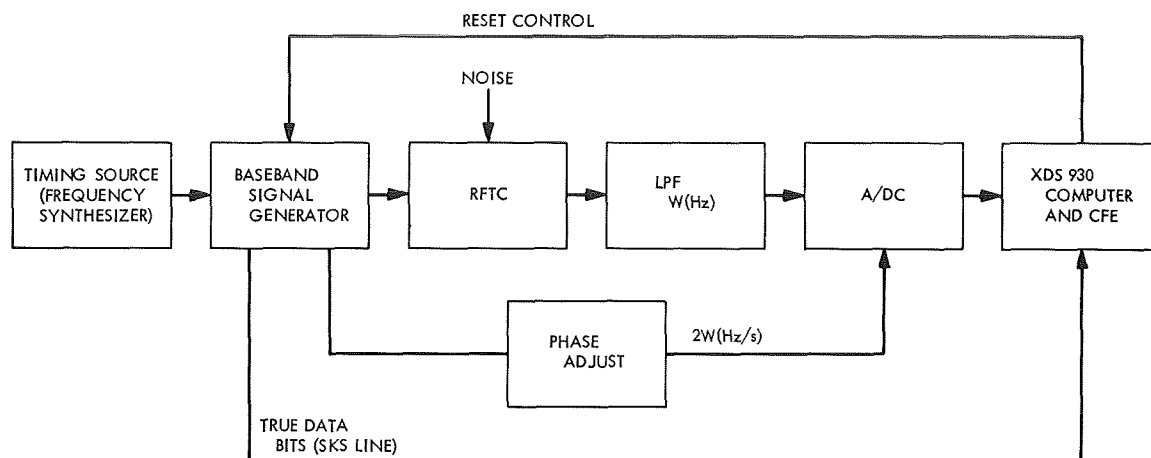


Fig. 1. Experiment setup

Since the subcarrier acquisition and tracking properties of the digital algorithm represented the least known about the approach, both from analytical and experimental viewpoints, the probability of best subcarrier phase became one subject of the experimental investigation. The second area of study became the probability of bit error measured in mode 4. It is felt that these statistical measures give a basis for "goodness of operation" and that the results can be considered as lower bounds on the system performance that would be expected in a full flight hardware configuration. Input parameters employed in the study included signal energy per bit vs noise spectral density ST_B/N_0 , sampled signal-to-noise ratio (SNR), sampling clock vs subcarrier phase, the number of bits in the A/DC, and, in the case of the RF tests, carrier power vs total power P_C/P_T and the SNR in the threshold two-sided tracking of the phase-locked loop bandwidth $2B_{LO}$.

3. Experimental Setup and Software

The experimental setup (Fig. 1) consists primarily of a baseband signal generator, the RF test console (RFTC), and the XDS 930 computer that performs all the processor operations.

The baseband signal generator is a digital logic bread-board driven by a Hewlett-Packard frequency synthesizer. The sequence of signals generated is outlined in Fig. 2. The intervals implemented for each component of the signal sequence were chosen for convenience and are, therefore, not the same as in the originally proposed algorithm. The processor decision times, however, are the same. The sequence is initiated by a pulse (end of message) issued by the computer.

The baseband signals are input to the RFTC where they are phase-modulated and noise is added to the resulting signal. This noisy signal is then demodulated and the resulting noisy baseband signal is fed into an A/DC.

The signal samples from the A/DC are entered into the computer at a fixed rate controlled by the computer interrupt system. The true values of the data bits are also entered into the computer via "skip-if-external-signal-not-set" (SKS) lines which are sensed by the computer at the

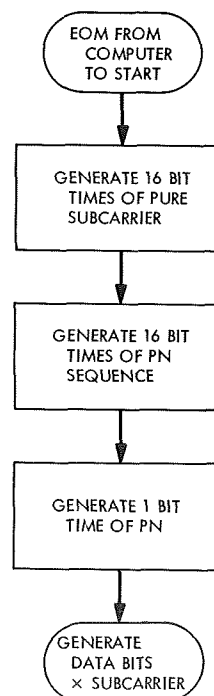


Fig. 2. Sequence of baseband signals

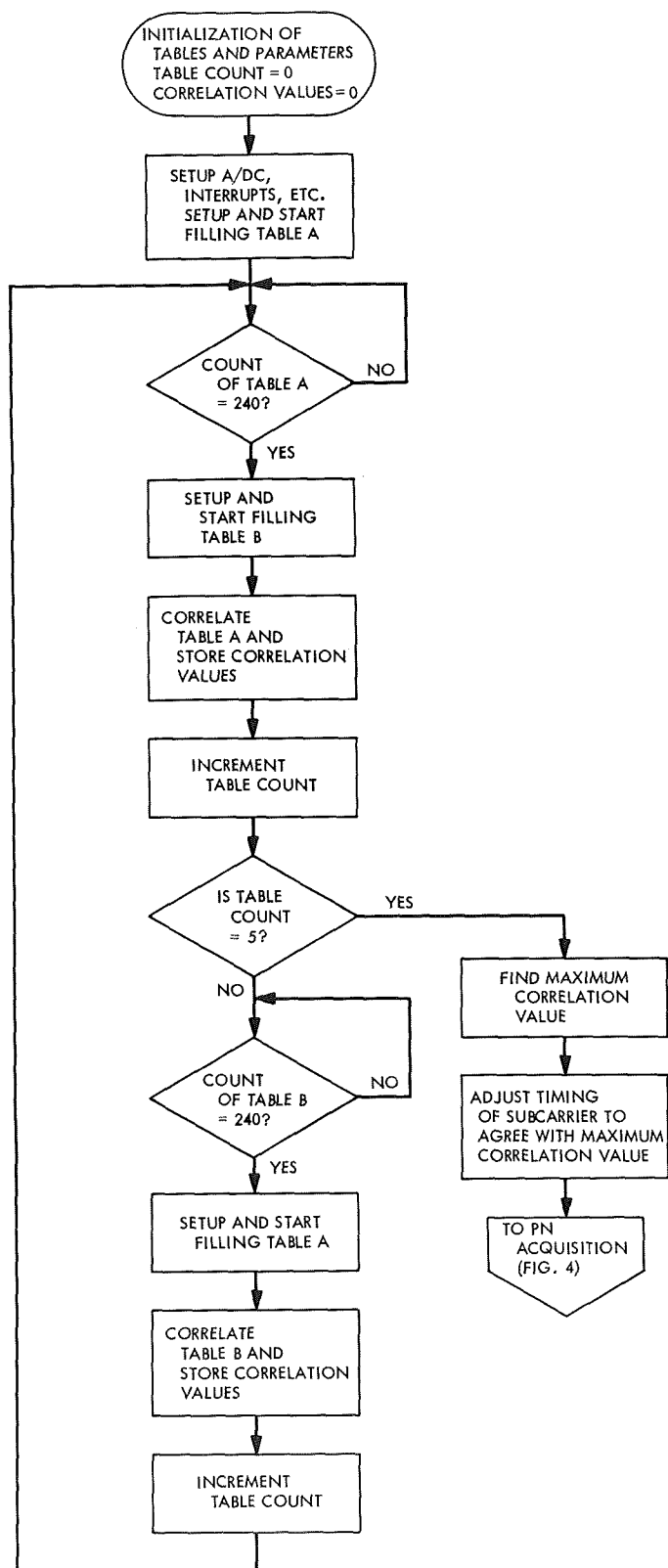


Fig. 3. Setup initialization and acquisition of subcarrier

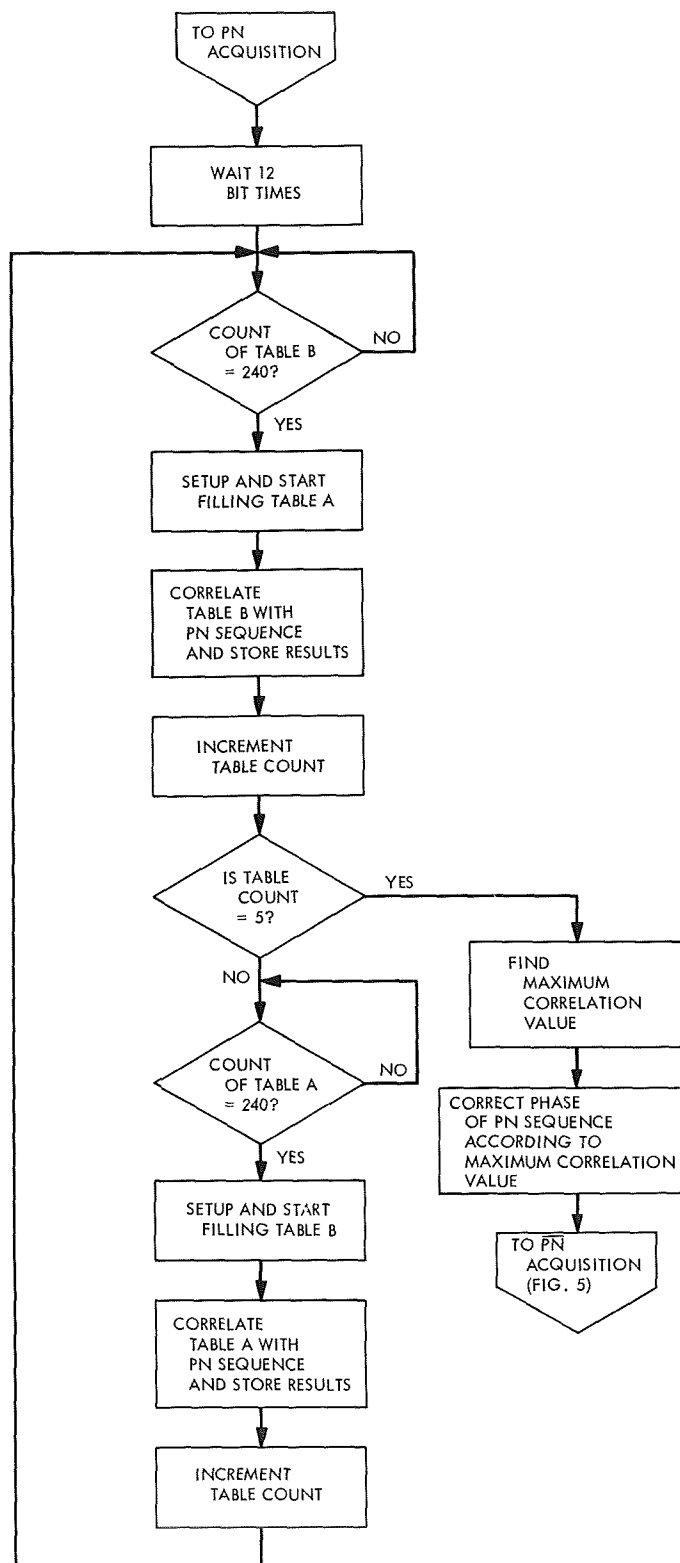


Fig. 4. Acquisition of PN

appropriate times to determine if a one or zero was actually sent.

The computer program, which is outlined in Figs. 3, 4, and 5, makes extensive use of the correlation and filtering unit (CFE) of the XDS 930 computer. The CFE is a special-purpose piece of hardware controlled by the com-

puter but which runs independently of the computer's central processing unit (CPU). The CFE is used to correlate the appropriate sampled data tables with fixed tables of subcarrier or a PN sequence table, depending on the stage of acquisition.

The basic technique used throughout the program is to continuously store data samples from the A/DC in two alternate data tables. While one table is being filled with new data, the other data table is being processed by the CFE and its output is then processed and stored. The process is then reversed as the second table is filled with new data while the first table is processed. One table is always being filled by means of the computer interrupt system.

Figure 3 shows the initialization and subcarrier acquisition mode of the program. In this mode, five tables of data are alternately filled each bit period, correlated with the subcarrier table for all phases, and accumulated. Since each table corresponds to one data bit time, this accumulating procedure corresponds to calculating the correlations over five bit times. The maximum correlation value is then picked and the phase reference in the computer is adjusted accordingly so as to agree with the predicted phase of the received subcarrier.

In the PN code acquisition phase (Fig. 4), a delay of 12 data bit times is first provided to wait for the generation of the PN sequences. The PN acquisition is essentially the same as the subcarrier acquisition; i.e., five data bit times are used and the correlations with the pure PN sequence are accumulated. The maximum correlation is then picked and the phase is changed so that data bit timing is correct.

As shown in Fig. 4, the next step in the acquisition is the data detection, which is preceded by the reception of one cycle (one data bit time) of a $\overline{\text{PN}}$ sequence. This is detected by correlating each data bit time with the pure PN sequence; when the $\overline{\text{PN}}$ is finally transmitted, the correlation value will become negative.

The final phase of the program performs bit detection. Each data table (or each data bit) is correlated with a pure subcarrier table and the sign of the correlation determines whether a one or zero was sent.

Not shown in the diagrams is the fact that the program also keeps a running accumulation of the signal mean and variance. Thus, a very accurate estimate of the sample's SNR is calculated for use in system performance evaluation.

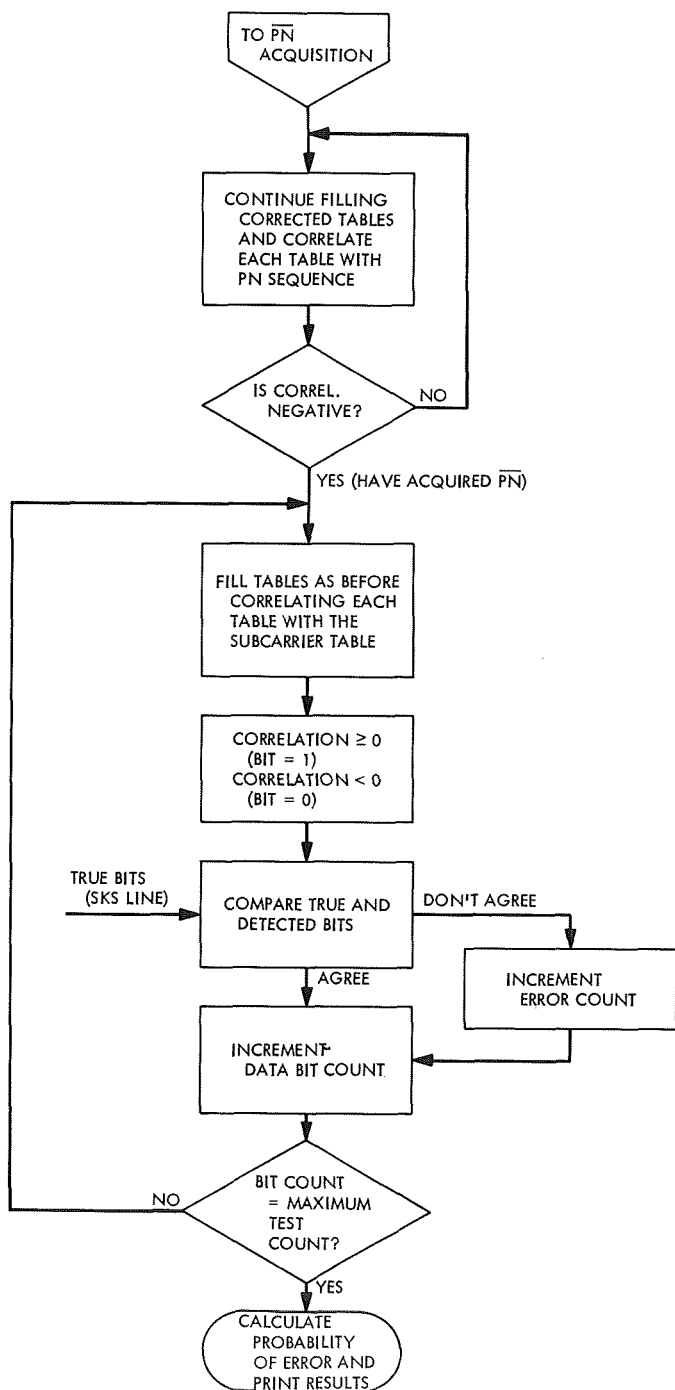


Fig. 5. Acquisition of PN and detection of data

The command detection algorithm under study was proposed to operate at 4 bits/s (the *Viking* command system data rate); however, in order to gain speed in data collection, the simulation runs were made at 25 bits/s. All parameters were scaled appropriately as shown in Table 1. Due to the ability of the CFE to perform two complete correlations per data bit time, it was possible to calculate data for two A/DC sizes (4 and 12 bits) simultaneously; therefore, the effective data rate per test was 50 bits/s. The LPF at the input to the A/DC had a 6-pole, linear phase characteristic.

Table 1. Simulation run parameter values

Parameter	Nominal value	Scaled value
Data rate (R)	4 bits/s	25 bits/s
Subcarrier frequency (f_{sc})	60 Hz	375 Hz
Input bandwidth (W)	480 Hz	3 kHz
Sampling rate (2W)	960 Hz	6 kHz
Threshold RF bandwidth ($2B_{Lo}$)	18 Hz	112.5 Hz

All additive noise was obtained with the RFTC using the wideband data channel with *no limiters*. The two modes employed were (1) with the tracking loop configured to provide the desired transfer function of the phase-locked loop $1 - H(s)$ characteristics without tracking a noisy carrier, and (2) with the tracking loop operated in a normal manner. This permitted both "baseband" and "noisy RF tracking" tests to be conducted using the RFTC.

4. Experimental Results

Since the computer interrupt clock was derived from the modulation sequence breadboard, the sampling clock vs filtered subcarrier phase was one parameter of interest. The "poor-sync" and "best-sync" conditions are shown in Fig. 6a and Fig. 6b, respectively. In Fig. 6a, one sample falls at the zero-crossing of the signal. In Fig. 6b, all samples are taken along the relatively flat portion of the signal. This difference amounts to a little more than 1 dB in average SNR. Most tests were run at the best-sync condition since it is assumed that the subcarrier tracking loop would be designed to seek this operating point.

Figure 7 shows the results of mode 4 testing for both baseband and RF situations. The data is summarized in Table 2. The nominal ST_B/N_0 is that theoretically required to give a particular bit error probability P_e . The carrier modulation index was chosen to give an $ST_B/N_0 = 9.6$ dB ($P_e = 1 \times 10^{-5}$) when SNR in $2B_{Lo}$ is 8.0 dB; thus, $P_C/P_T = -1.25$ dB. This is an arbitrary (but reasonable)

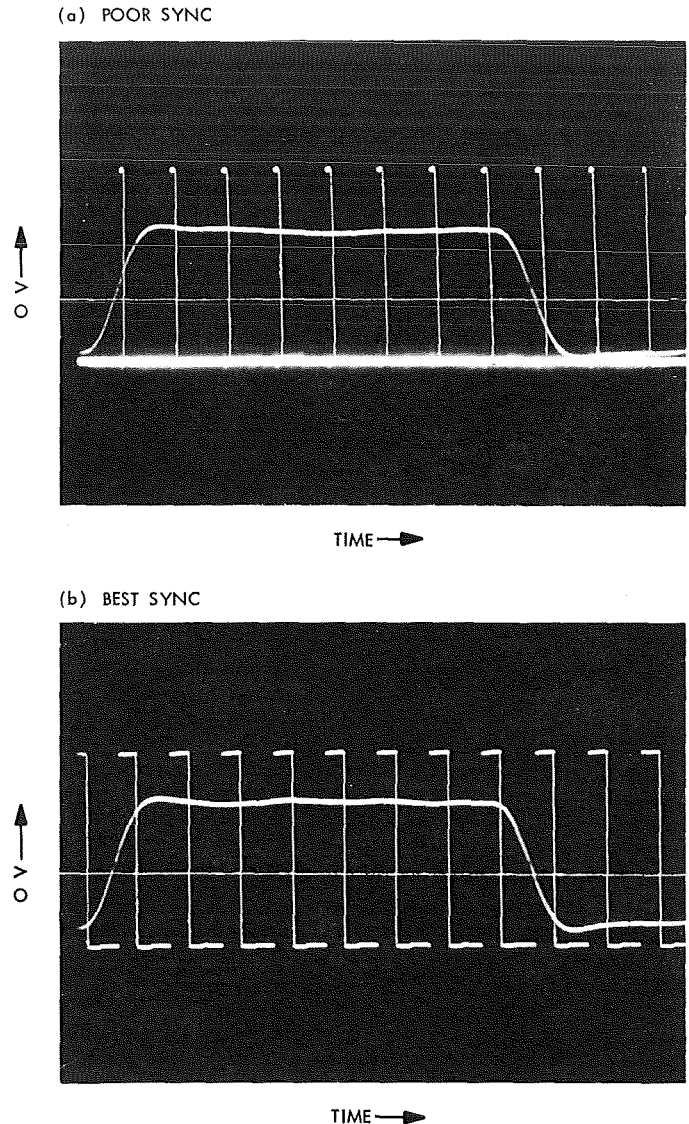


Fig. 6. Sync waveforms

choice of conditions and does not reflect any system loss estimates; it remains the function of the tests to determine the losses.

Tests were run for the two A/DC sizes (4 and 12 bits) using the best-sync sampling. The gross performance loss reflects the departure from ideal based upon the measured P_e and has been decomposed into three parts. The LPF and $1 - H(s)$ loss reflects the signal power filtering loss associated with the receiver tracking loop and the input filter to the A/DC. The detection efficiency includes all losses associated with the detection algorithm including the number of A/DC bits used, the sampling phase vs subcarrier waveform characteristics, the aliasing error, etc.

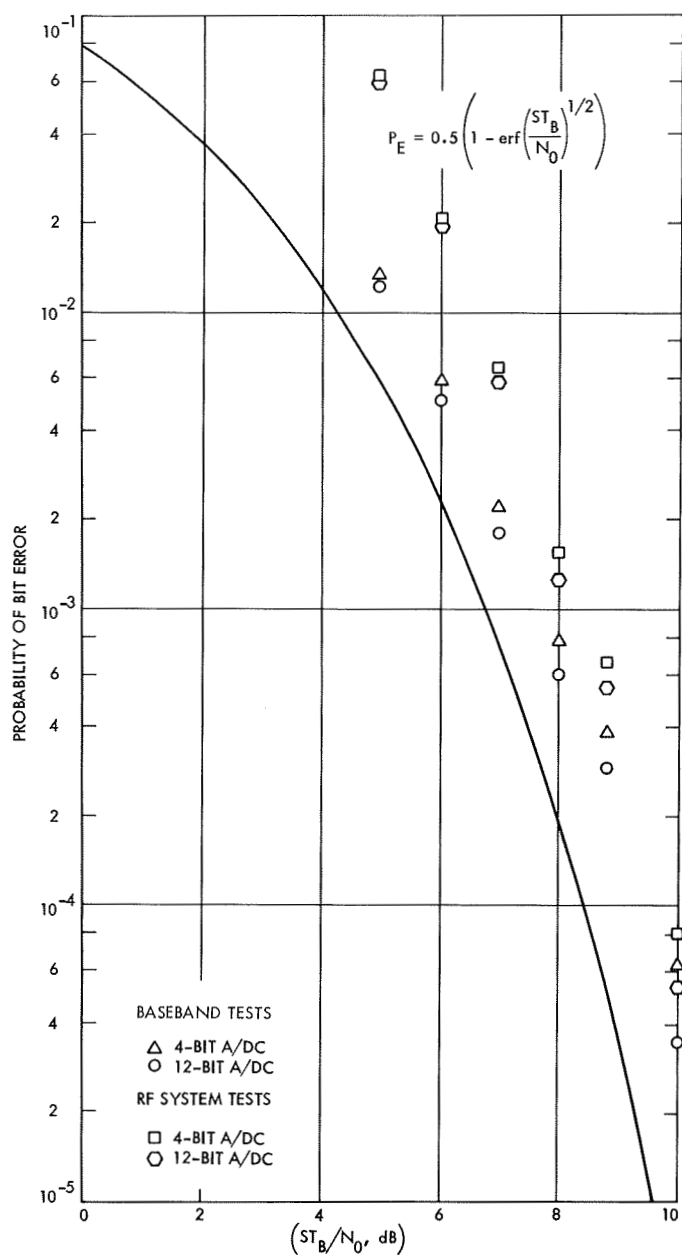


Fig. 7. Mode 4 test results (baseband and RF)

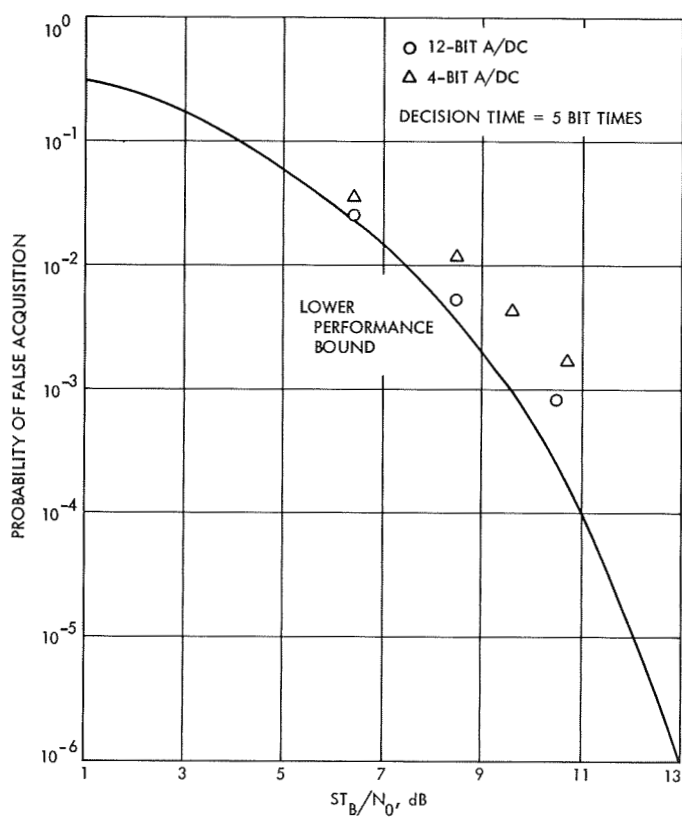


Fig. 8. Mode 2 test results (baseband only)

Table 2. Test summary

Nominal ST_B/N_0 , dB	A/DC, bits	Gross performance loss, dB (± 0.2)	Detection efficiency, dB (± 0.2)	LPF + 1 - H(s) loss, dB (± 0.1)	Noisy reference loss, dB	SNR ($2B_{LO}$), dB
5.0	12	4.3	0.4	0.5	3.4	3.7
	4	4.4	0.6	0.5	3.3	3.7
6.0	12	2.7	0.3	0.5	1.9	4.7
	4	2.3	0.5	0.5	1.8	4.7
7.0	12	2.0	0.2	0.5	1.3	5.6
	4	2.2	0.4	0.5	1.3	5.6
8.0	12	1.4	0.3	0.5	0.6	6.5
	4	1.6	0.5	0.5	0.6	6.5
9.0	12	1.5	0.5	0.5	0.5	7.4
	4	1.7	0.7	0.5	0.5	7.4
10.0	12	1.2	0.5	0.5	0.2	8.3
	4	1.5	0.9	0.5	0.1	8.3
9.6	∞					8.0

Finally, the noisy reference loss is that associated with the noisy RF carrier tracking and demodulation. Note that because no limiters were used in the data channel of the receiver, that this loss is negligible at the design point of $ST_B/N_0 = 10$ dB. Note also, that the effects of using the 4-bit A/DC are most pronounced at high ST_B/N_0 , and amount to about 0.4 dB at $ST_B/N_0 = 10$ dB.

Some results of the baseband mode 2 testing are shown in Fig. 8. Here, the probability of false acquisition was measured against choosing the "best-sync" point out of a total of eight possibilities. Because of the near-triangular correlation function, the most probable error was made to the adjacent neighbors. Note the greater degradation caused by the 4-bit A/DC relative to the mode 4 tests due to the fact that eight, rather than two, random variables or states are involved in the decision process.

B. Estimating the Phase of a Sampled Signal with Minimum Mean-Square Error, H. D. Chadwick

1. Introduction

In a digital implementation of a phase-coherent receiver, the subcarrier signal, together with additive noise, is low-pass filtered to a frequency band 0 to W Hz and is sampled at a rate $2W$ samples/s. The ratio of the sampling rate to the subcarrier frequency is the number of samples n per cycle of the subcarrier. In this estimation technique, the sampled subcarrier is correlated with each of the n possible discrete phase positions of a stored replica of the subcarrier waveform and the maximum correlation value is chosen as the estimate of the true subcarrier

phase position. The choice of the optimum sampling rate depends on both the quantization error, which decreases with increased sampling rate, and the variance of the estimate itself, which tends to increase with the sampling rate. In this article, the sampling rate that produces the minimum mean-square total error is derived as a function of the input signal-to-noise ratio for a square-wave signal. This problem differs from that treated by S. Butman (SPS 37-53, Vol. III, pp. 200-209) in that the phase estimation is performed discretely and not in the optimum continuous form.

2. Phase Estimation Technique

With a square-wave subcarrier, the signal is of the form

$$x_i = x\left(\frac{iT}{n}\right) = s_i + n_i \quad i = 0, 1, \dots, n-1 \quad (1)$$

where n_i is assumed to be a sample of a gaussian noise process, low-pass limited to the frequency $W = n/2T$ so that all sample values are independent and the values s_i are samples of a square wave with amplitude A . Thus,

$$s_i = \begin{cases} -A \frac{n\theta}{T} \leq i < \frac{n\theta}{T} + \frac{n}{2} \\ +A \frac{n\theta}{T} - \frac{n}{2} \leq i < \frac{n\theta}{T} \end{cases} \quad (2)$$

where θ is the true phase position, $0 \leq \theta < T$, marking the positive-to-negative transition of the square wave (Fig. 9). While θ is a continuous variable, the estimation of θ is

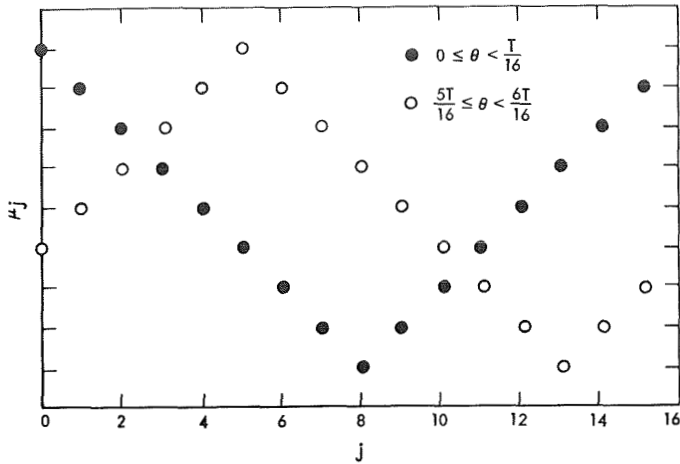


Fig. 10. Correlation value means, μ_j ($n = 16$)

The covariance matrix contains the terms

$$\begin{aligned}\sigma_{jl}^2 &= E[R_j R_l | \theta] - \mu_j \mu_l \\ &= E \left[\sum_{i_1=0}^{mn-1} x_{i_1} y_{i_1-j} \sum_{i_2=0}^{mn-1} x_{i_2} y_{i_2-l} | \theta \right] - \mu_j \mu_l \\ &= E \left[\sum_{i_1=0}^{mn-1} \sum_{i_2=0}^{mn-1} x_{i_1} x_{i_2} y_{i_1-j} y_{i_2-l} | \theta \right] - \mu_j \mu_l \\ &= \sum_{i_1=0}^{mn-1} \sum_{i_2=0}^{mn-1} E[(s_{i_1} + n_{i_1})(s_{i_2} + n_{i_2}) y_{i_1-j} y_{i_2-l} | \theta] - \mu_j \mu_l\end{aligned}\quad (13)$$

The term involving $s_{i_1} s_{i_2}$ in this expansion cancels the $\mu_j \mu_l$ term, and the $s_{i_1} n_{i_2}$ and $s_{i_2} n_{i_1}$ terms have zero expected value so that the only remaining term in the expansion gives

$$\sigma_{jl}^2 = \sum_{i_1=0}^{mn-1} \sum_{i_2=0}^{mn-1} E(n_{i_1} n_{i_2} | \theta) y_{i_1-j} y_{i_2-l} \quad (14)$$

which becomes, because of the independence of the noise samples,

$$\sigma_{jl}^2 = \sigma^2 \sum_{i=0}^{mn-1} y_{i-j} y_{i-l} \quad (15)$$

or

$$\sigma_{jl}^2 = \begin{cases} \sigma^2 m [n - 4|j-l|] & |j-l| \leq \frac{n}{2} \\ \sigma^2 m [4|j-l| - 3n] & \frac{n}{2} < |j-l| \leq \frac{3n}{2} \end{cases} \quad (16)$$

where $\sigma^2 = E n_i^2 = N_0 W$ is the variance of the samples of the noise with two-sided spectral density $N_0/2$.

The $n \times n$ covariance matrix of the σ_{jl}^2 is singular because the rows (or columns) numbered $j + (n/2)$ are the exact negatives of those numbered j . This is true because the correlation of the x_2 values with one square wave produces the exact negative as the correlation with the square wave shifted by $T/2$. Thus, the use of n correlation values is redundant (half of them are the exact negatives of the other half) and the estimation procedure could be replaced by one in which only $n/2$ values are calculated and the one with the largest magnitude is chosen. The sign of this correlation value is then used to resolve the $T/2$ ambiguity in the phase. The performance of such a technique is identical to that of the previously described technique. Using this fact, the joint density function of the R_j 's may be written in terms of the $n/2 \times n/2$ covariance matrix $M = [\sigma_{jl}^2]$

$$\begin{aligned}p(\mathbf{R} | \theta) &= \\ &[(2\pi)^{1/2}]^{-n/2} |\mathbf{M}|^{-1/2} \exp \left[-\frac{1}{2} (\mathbf{R} - \boldsymbol{\mu})^T \mathbf{M}^{-1} (\mathbf{R} - \boldsymbol{\mu}) \right]\end{aligned}\quad (17)$$

The probability distribution of $\hat{\theta}$ can then be written

$$\begin{aligned}\text{Prob} \left[\theta = \frac{\hat{j}T}{n} | \theta \right] &= \text{Prob} [|R_j| > |R_i|; \text{all } i \neq j | \theta] \\ &= \int_S p(\mathbf{R} | \theta) d\mathbf{R}\end{aligned}\quad (18)$$

where S is the $n/2$ dimensional region in which $|R_j| > |R_i|$ for all $i = 0 \leq i \leq n/2, i \neq j$. As this equation implies an $n/2$ dimensional integration, some simplifying assumptions are in order.

4. Approximation to the Distribution of $\hat{\theta}$

The probability

$$\text{Prob} [|R_j| > |R_i|; \text{all } i \neq j | \theta]$$

may be simplified by the use of some inequality relationships. Specifically,

$$\text{Prob} [|R_j| > |R_i|; \text{all } i \neq j | \theta]$$

$$\begin{aligned}&= \text{Prob} [|R_j| > |R_k| | \theta] \text{Prob} [|R_j| > |R_i|; \text{all } i \neq k, j | \theta, |R_j| > |R_k|] \\ &\leq \text{Prob} [|R_j| > |R_k| | \theta] \leq \text{Prob} [R_j > R_k | \theta]\end{aligned}\quad (19)$$

By judicious choice of the index k , specifically that k that corresponds to correlation value with the largest mean

$$\frac{kT}{n} \leq \theta \leq \frac{kT}{n} + \frac{1}{n}$$

the approximation is reasonably close when the signal-to-noise ratio is high. The approximation implies that if $R_j > R_k$, then R is also greater than all the other R_i . This two-dimensional integral can be shown to have the solution

$$\text{Prob}[R_j > R_k | \theta] = \frac{1}{2} \text{erfc} \left[\frac{\mu_i}{2\sigma_{kk}} (1 - \rho_{jk})^{1/2} \right] \quad (20)$$

where σ_{kk}^2 is the variance of the k th correlation value and

$$\rho_{jk} = \frac{\sigma_{jk}^2}{\sigma_{kk}^2} = 1 - 4 \frac{|k-j|}{n}$$

In terms of the input signal-to-noise ratio, $\gamma^2 = A^2 T / N_0$, Eq. (20) can be written

$$\text{Prob}[R_j > R_k | \theta] = \frac{1}{2} \text{erfc} \left[2 \left(\frac{m}{2} \right)^{1/2} \gamma \left(\frac{|k-j|}{n} \right)^{1/2} \right] \quad (21)$$

Equation (21) was used to calculate a lower bound to the probability of false acquisition. Correct acquisition is defined as $\hat{\theta} \leq \theta \leq \hat{\theta} + (1/n)$ so that false acquisition would occur if any R_j is greater than R_k . The lower bound is illustrated by J. C. Springett and W. J. Weber (Section A) in the description of their experimental results.

5. Minimization of the Mean-Squared Error

Two terms are included in the expression for the mean-squared error of the phase estimation. The first is due to the probability distribution of the estimator, which depends only on the quantized position of the true phase. The second term is the quantization error, which arises because the true phase is a continuous variable and is assumed to have a uniform distribution within the quantization interval. The complete expression is

$$\bar{\epsilon}^2 = \left(\frac{T}{n} \right)^2 \sum_{j=0}^{(n/2)-1} (k-j)^2 \text{Prob} \left[\hat{\theta} = \frac{jT}{n} \right] + \frac{T^2}{12n^2} \quad (22)$$

where again $\hat{\theta} = kT/n$ is the "correct" phase position estimate.

Substituting the approximation derived in Subsection 4 (Eq. 21) gives

$$\bar{\epsilon}^2 = \frac{1}{2} \left(\frac{T}{n} \right)^2 \sum_{j=0}^{(n/2)-1} (k-j)^2 \text{erfc} \left[2\gamma \left(\frac{m|k-j|}{2n} \right)^{1/2} \right] + \frac{T^2}{12n^2} \quad (23)$$

The solutions to this equation are plotted in Fig. 11 for various values of ST_B/N_0 . (ST_B/N_0 is defined as $15 ST/N_0$.) It can be seen that each curve has a definite minimum for some value of n . These minimum values are plotted versus ST_B/N_0 in Fig. 12.

6. Summary

It has been shown that the mean-squared error of a discrete phase estimate performed by the digital correlation technique can be minimized by the proper choice of the sampling rate. Other factors, such as computational

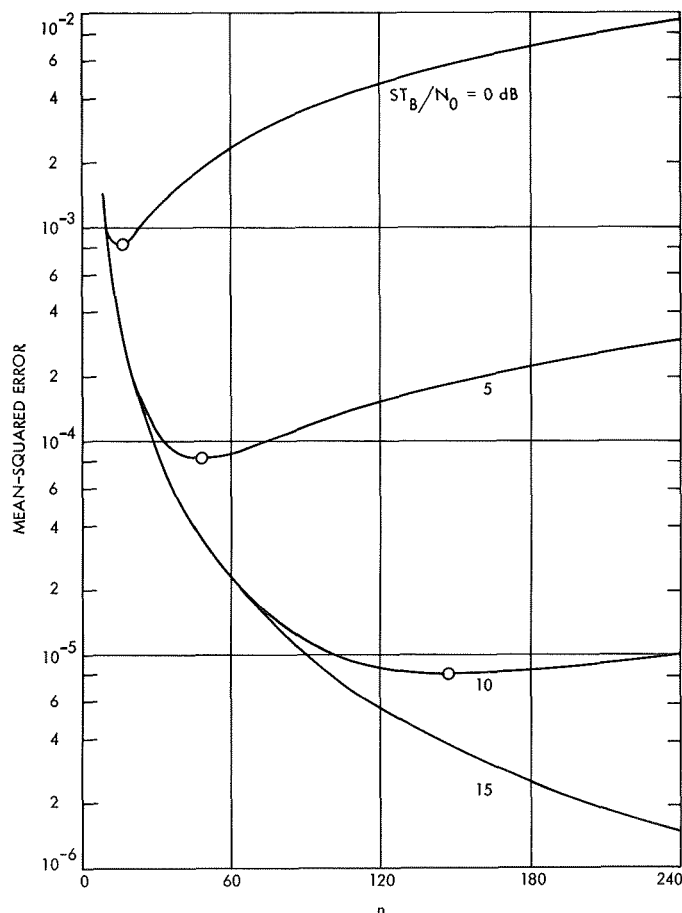


Fig. 11. Total mean-squared error vs n

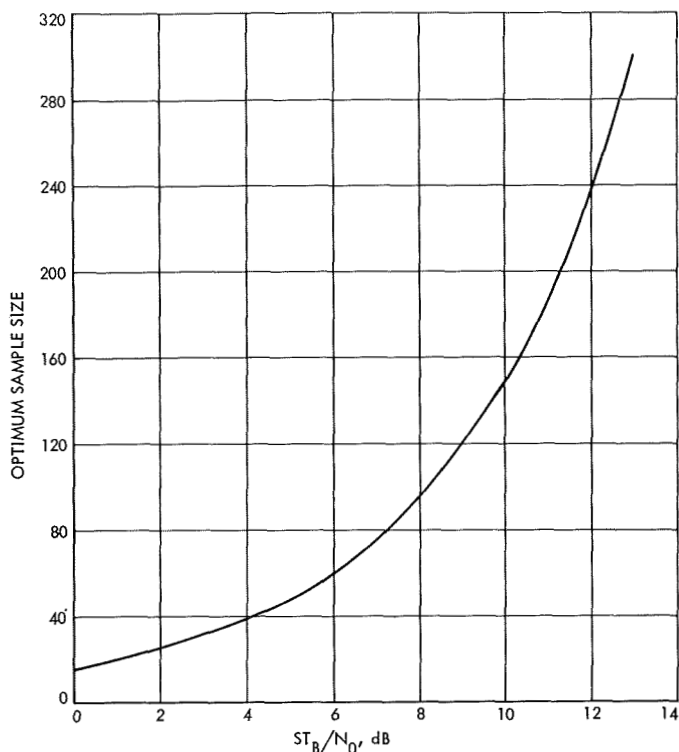


Fig. 12. Optimum sample size vs signal-to-noise ratio ST_B/N_0

complexity and maximum speed, will also dictate the choice of the sampling rate; however, these results show that the choice of sampling rate can effect the phase estimator.

C. On the Receiver Structure for a Single-Channel, Phase-Coherent Communication System,

M. K. Simon and S. Butman

1. Introduction

The problem of estimating or tracking the random phase and frequency in a phase-coherent communication system where a single data channel phase modulates a transmitted RF carrier has received widespread attention (Refs. 1, 2 and 3). Typically, a standard phase-locked loop is employed for tracking the RF carrier phase and frequency and no attempt is made to use the power in the data channel for improving carrier tracking performance. Recently, a data-aided tracking loop was proposed (SPS 37-60, Vol. III, pp. 46-56) that makes use of the decision-directed feedback idea for reducing the degradation due to RF phase noise.

The purpose of this article is to describe a receiver structure that forms the maximum *a posteriori* estimate of

the RF phase (frequency assumed known) given a finite past record of the received signal plus noise. The method used follows the well-known theory of maximum likelihood estimation of an unknown parameter in gaussian noise (Ref. 4). The application of this theory to problems of symbol synchronization and suppressed carrier tracking loops has been studied in Refs. 5 and 6.¹

Two special cases of the general continuous analog of the result described here are of interest. For large signal-to-noise ratios, the closed-loop suboptimum configuration resembles the data-aided loop proposed in SPS 37-60, Vol. III; whereas for small signal-to-noise ratios, the configuration becomes the parallel combination of a standard phase-locked loop and a Costas loop (Ref. 7). A similar analog loop has been suggested by Lindsey (Section D) who refers to his configuration as a "hybrid modulation tracking loop."

2. Statement of the Problem

An RF transmitter is phase-modulated by a single data channel and provides an output signal of the form

$$x(t) = (2P)^{1/2} \sin [\omega_0 t + (\cos^{-1} m_1) m(t)] \quad (1)$$

where ω_0 is the nominal carrier frequency and m_1 is the modulation factor. The modulation $m(t)$ is assumed digital and may be placed directly on the RF carrier or, alternately, on a separate subcarrier. Thus, two cases exist: Either $m(t) = m_0(t)$ or $m(t) = m_0(t) x(t)$, where $m_0(t)$ is a random square-wave pulse train with period T and $x(t)$ is a square-wave subcarrier of period T/N . For simplicity, we will pursue the first case, although the second case is probably the more practical. However, we will indicate the modification of the receiver structure that is necessary if one assumes that the modulation is placed on a square-wave subcarrier. Additional information concerning the restrictions placed on the subcarrier demodulator is given in Subsection 4.

The transmitted signal $x(t)$ is sent over an additive white gaussian noise channel, which, in addition, introduces an arbitrary phase shift θ that is assumed uniformly distributed. Thus, $y(t)$ is of the form

$$y(t) = (2P)^{1/2} \sin [\omega_0 t + (\cos^{-1} m_1) m(t) + \theta] + n(t) \quad (2)$$

¹See also SPS 37-35, Vol. IV, pp. 349-357 and SPS 37-37, Vol. IV, pp. 268-275.

where $n(t)$ is a white gaussian noise process of single-sided spectral density N_0 in W/Hz. Since the modulation $m(t)$ (in either of the two previous cases) is a ± 1 digital sequence, the signal component of Eq. (2) can be decomposed into two components resulting in

$$y(t) = (2P_c)^{1/2} \sin(\omega_0 t + \theta) + (2S)^{1/2} m(t) \cos(\omega_0 t + \theta) + n(t) \\ = s(t) + n(t) \quad (3)$$

where

$$\left. \begin{aligned} P_c &= m_i^2 P \\ S &= (1 - m_i^2) P \end{aligned} \right\} \quad (4)$$

The problem then is to form the *a posteriori* density function $p(\theta|y(t))$ over an interval of time, say KT , during which it can be assumed that θ is constant, and then choose that value of $\hat{\theta}$, i.e., θ , that maximizes this functional. The procedure will be carried out in vector space by expanding the signal and noise in a weighted series of orthogonal basis functions during each symbol interval T .

3. Derivation of the Optimum Receiver Structure

We begin by specifying the vector space that appropriately characterizes $s(t)$ and $n(t)$ in each symbol interval. Expanding the signal and noise processes in Eq. (3) in terms of the basis functions $\sin m\omega_0 t$ and $\cos m\omega_0 t$, we get, for the k th symbol interval ($k = 1, 2, \dots, M$)

$$y(t) = [(2P_c)^{1/2} \cos \theta - (2S)^{1/2} d_k \sin \theta] \sin \omega_0 t \\ + [(2P_c)^{1/2} \sin \theta + (2S)^{1/2} d_k \cos \theta] \cos \omega_0 t \\ + \sum_{m=1}^{\infty} \eta_{mk} \cos m\omega_0 t + \sum_{m=1}^{\infty} \xi_{mk} \sin m\omega_0 t \\ = \sum_{m=1}^{\infty} a_{mk}(\theta, d_k) \cos m\omega_0 t \\ + \sum_{m=1}^{\infty} b_{mk}(\theta, d_k) \sin m\omega_0 t \quad (k-1)T \leq t \leq kT \quad (5)$$

where $d_k = m(t)$ is the polarity of the k th transmitted symbol; η_{mk} and ξ_{mk} are zero mean, uncorrelated gaussian variates all with variance $N_0/2$; and $a_{mk}(\theta, d_k)$, $b_{mk}(\theta, d_k)$ are conditionally gaussian on θ and d_k . Letting η_k , ξ_k , $\mathbf{a}_k$, and $\mathbf{b}_k$ denote vectors with components η_{mk} , ξ_{mk} , $a_{mk}(\theta, d_k)$, and $b_{mk}(\theta, d_k)$, respectively ($m = 1, 2, \dots$), then the following properties are noted:

- (1) η_k and ξ_k are independent for all $k, \ell = 1, 2, \dots, K$.

- (2) $E\{\eta_k, \eta_\ell\} = E\{\xi_k, \xi_\ell\} = \sigma^2 I$, where I is the identity matrix.

- (3) The vectors $\mathbf{a}_k$ and $\mathbf{b}_k$ are gaussian conditioned on fixed θ and d_k and are independent from symbol interval to symbol interval. (Presently, for simplicity, this dependence will be omitted in the notation.) Furthermore, all components of these vectors are zero mean except for the first components in each, which represent the signal in each symbol interval and hence entirely contain the dependence on θ and d_k .

It is convenient to assume finite dimensionality for the vectors (e.g., M) and later let M approach infinity. In this case, $\sigma^2 = (N_0/2)^M$.

From the above, we see that the vectors $\mathbf{a}_k$ and $\mathbf{b}_k$ are sufficient to represent the received signal $y(t)$ in the k th symbol interval. Furthermore, the received signal may be specified over the entire observation interval $0 \leq t \leq KT$ by the partitioned vector

$$\mathbf{c} = (\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_K, \mathbf{b}_1, \mathbf{b}_2, \dots, \mathbf{b}_K)$$

and thus the *a posteriori* density function $p(\theta|y(t))$, $0 \leq t \leq KT$, is stochastically equivalent to the probability density function $p(\theta|\mathbf{c})$. Letting $\mathbf{d}$ be the data vector with components $d_1, d_2, \dots, d_K$ and noting that the random phase and symbol sequence are independent, one obtains from Bayes' rule:

$$p(\theta|\mathbf{c}) = \frac{p(\theta)}{p(\mathbf{c})} \int_{\mathbf{d}} p(\mathbf{c}|\theta, \mathbf{d}) p(\mathbf{d}) d\mathbf{d} \quad (6)$$

Since the transmitted symbols are assumed to be independent, their joint density function $p(\mathbf{d})$ can be expressed as:

$$p(\mathbf{d}) = \prod_{k=1}^K p(d_k) \quad (7)$$

where $p(d_k)$ for equiprobable symbols is specified by

$$p(d_k) = \frac{1}{2}; \quad d_k = \pm 1 \quad k = 1, 2, \dots, K \quad (8)$$

Furthermore, from properties (1), (2), and (3) following Eq. (5),

$$p(\mathbf{c}|\theta, \mathbf{d}) = \prod_{k=1}^K p(\mathbf{a}_k, \mathbf{b}_k|\theta, d_k) \quad (9)$$

Hence, combining Eqs. (6), (7), and (8),

$$p(\theta|c) = \frac{p(\theta)}{p(c)} \int_{\mathbf{d}} \prod_{k=1}^K p(\mathbf{a}_k, \mathbf{b}_k | \theta, d_k) p(d_k) d\mathbf{d} \quad (10)$$

Since $p(\theta)$ is assumed uniformly distributed between $-\pi$ and π and $p(c)$ is independent of θ , maximizing $p(\theta|c)$ with respect to θ is equivalent to maximizing

$$\int_{\mathbf{d}} \prod_{k=1}^K p(\mathbf{a}_k, \mathbf{b}_k | \theta, d_k) p(d_k) d\mathbf{d} = \prod_{k=1}^K \int_{d_k} p(\mathbf{a}_k, \mathbf{b}_k | \theta, d_k) p(d_k) d d_k \quad (11)$$

The conditional density function $p(\mathbf{a}_k, \mathbf{b}_k | \theta, d_k)$ is 2M-dimensional gaussian with a diagonal covariance matrix $(N_0/2)I$, i.e.,

$$p(\mathbf{a}_k, \mathbf{b}_k | \theta, d_k) = \frac{1}{[2\pi(N_0/2)]^M} \exp \left\{ - \frac{\sum_{m=1}^M \{a_{mk}(\theta, d_k) - E[a_{mk}(\theta, d_k)]\}^2}{N_0} - \frac{\sum_{m=1}^M \{b_{mk}(\theta, d_k) - E[b_{mk}(\theta, d_k)]\}^2}{N_0} \right\} \quad (12)$$

Recalling that only $a_{1k}(\theta, d_k)$ and $b_{1k}(\theta, d_k)$ have non-zero mean, we can rewrite Eq. (12) as

$$p(\mathbf{a}_k, \mathbf{b}_k | \theta, d_k) = \frac{1}{[\pi N_0]^M} \exp \left(- \sum_{m=2}^M \frac{a_{mk}^2 + b_{mk}^2}{N_0} \right) \times \exp \left\{ - \frac{[a_{1k} - (2P_c)^{1/2} \sin \theta - (2Sd_k)^{1/2} \cos \theta]^2}{N_0} - \frac{[b_{1k} - (2P_c)^{1/2} \cos \theta + (2Sd_k)^{1/2} \sin \theta]^2}{N_0} \right\} \quad (13)$$

Averaging over the distribution of d_k as given in Eq. (8), one obtains

$$\begin{aligned} \int_{d_k} p(\mathbf{a}_k, \mathbf{b}_k | \theta, d_k) p(d_k) d d_k &= \frac{1}{2(\pi N_0)^M} \exp \left[- \frac{\sum_{m=1}^M a_{mk}^2 + b_{mk}^2}{N_0} - \frac{(2S + 2P_c)}{N_0} \right] \\ &\times \left\{ \exp \left[\frac{2}{N_0} a_{1k} ((2P_c)^{1/2} \sin \theta + (2S)^{1/2} \cos \theta) \right. \right. \\ &\quad \left. \left. + \frac{2}{N_0} b_{1k} ((2P_c)^{1/2} \cos \theta - (2S)^{1/2} \sin \theta) \right] \right. \\ &\quad \left. + \exp \left[\frac{2}{N_0} a_{1k} ((2P_c)^{1/2} \sin \theta - (2S)^{1/2} \cos \theta) \right. \right. \\ &\quad \left. \left. + \frac{2}{N_0} b_{1k} ((2P_c)^{1/2} \cos \theta + (2S)^{1/2} \sin \theta) \right] \right\} \quad (14) \end{aligned}$$

From the orthogonality property of the basis functions, the weighting coefficients can be expressed as

$$\begin{aligned} a_{1k} &= \frac{2}{T} \int_{(k-1)T}^{kT} y(t) \cos \omega_0 t dt \\ b_{1k} &= \frac{2}{T} \int_{(k-1)T}^{kT} y(t) \sin \omega_0 t dt \end{aligned} \quad (15)$$

Substituting Eq. (15) into Eq. (14), combining terms, and taking the limit as $M \rightarrow \infty$, we get

$$\int_{d_k} p(\mathbf{a}_k, \mathbf{b}_k | \theta, d_k) p(d_k) dd_k = C_k \exp \left[\frac{4}{N_0 T} \int_{(k-1)T}^{kT} (2P_c)^{1/2} y(t) \sin(\omega_0 t + \theta) dt \right] \cosh \left[\frac{4}{N_0 T} \int_{(k-1)T}^{kT} (2S)^{1/2} y(t) \cos(\omega_0 t + \theta) dt \right] \quad (16)$$

where

$$C_k = \lim_{M \rightarrow \infty} \frac{1}{(\pi N_0)^M} \exp \left[\frac{\sum_{n=1}^M a_{nk}^2 + b_{nk}^2}{N_0} - \frac{(2S + 2P_c)}{N_0} \right] \quad (17)$$

The best estimate of θ in the maximum *a posteriori* sense, based on a KT observation of $y(t)$ is then that value of θ that maximizes the function

$$f(\theta) = \prod_{k=1}^K \exp \left[\frac{4}{N_0 T} \int_{(k-1)T}^{kT} (2P_c)^{1/2} y(t) \sin(\omega_0 t + \theta) dt \right] \cosh \left[\frac{4}{N_0 T} \int_{(k-1)T}^{kT} (2S)^{1/2} y(t) \cos(\omega_0 t + \theta) dt \right] \quad (18)$$

or, alternately,

$$\ln f(\theta) = \sum_{k=1}^K \left\{ \frac{4}{N_0 T} \int_{(k-1)T}^{kT} (2P_c)^{1/2} y(t) \sin(\omega_0 t + \theta) dt + \ln \cosh \left[\frac{4}{N_0 T} \int_{(k-1)T}^{kT} (2S)^{1/2} y(t) \cos(\omega_0 t + \theta) dt \right] \right\} \quad (19)$$

The interpretation of Eq. (19) is as follows: For each value of θ , one correlates $y(t)$ with the stored replicas

$$\frac{4}{N_0 T} [(2P_c)^{1/2} \sin(\omega_0 t + \theta)]$$

and

$$\frac{4}{N_0 T} [(2S)^{1/2} \cos(\omega_0 t + \theta)]$$

over each symbol interval, adding the result of the first correlation to the log hyperbolic cosine of the second. This statistic is then accumulated over K successive symbol intervals and stored. The value of θ that yields the largest accumulated value is then declared as the best estimate; i.e., $\hat{\theta}$. Actually, θ takes on a continuum of values between $-\pi$ and π and hence, to practically apply the above technique in each KT observation interval, one must quantize θ in a number of levels compatible with the size of the storage one is willing to provide.

An alternate procedure is to incorporate the above ideas in a tracking loop that would continuously provide an estimate of the input θ . To see this, we differentiate $f(\theta)$ (or equivalently $\ln f(\theta)$) with respect to θ and equate to zero. The solution of this equation is then $\hat{\theta}$, the maximum *a posteriori* estimate. Hence

$$g(\hat{\theta}) = \frac{d}{d\theta} \ln f(\theta) |_{\theta=\hat{\theta}} = \sum_{k=1}^K \left[\frac{4}{N_0 T} \int_{(k-1)T}^{kT} (2P_c)^{1/2} y(t) \cos(\omega_0 t + \hat{\theta}) dt - \tanh \left\{ \frac{4}{N_0 T} \int_{(k-1)T}^{kT} (2S)^{1/2} y(t) \cos(\omega_0 t + \hat{\theta}) dt \right\} \times \frac{4}{N_0 T} \int_{(k-1)T}^{kT} (2S)^{1/2} y(t) \sin(\omega_0 t + \hat{\theta}) dt \right] = 0 \quad (20)$$

By using $g(\hat{\theta})$ to control the voltage-controlled oscillator in a tracking loop, one obtains the suboptimum configuration of Fig. 13 or its continuous equivalent, shown in Fig. 14. In practice, the receiver structure of Fig. 14 is difficult to construct so it is interesting to examine its limiting forms for small and large signal-to-noise ratios. Specifically, we make the following approximations to the hyperbolic tangent function:

$$\left. \begin{aligned} \tanh x &\approx x && \text{for small } x \\ \tanh x &\approx \operatorname{sgn} x && \text{for large } x \end{aligned} \right\} \quad (21)$$

Then, for small signal-to-noise ratio, the configuration of Fig. 14 reduces to the parallel combination of a standard phase-locked loop and a Costas loop (Ref. 7 and Footnote 2); whereas for large signal-to-noise ratio, we observe a resemblance to the data-aided loop suggested in SPS 37-60, Vol. III. Perhaps the most interesting point is the way in which the power splits between the upper and lower branches of each configuration as a function of

m_1 . Specifically, from Eq. (4) one sees that the signal power in the upper branch is proportional to m_1^2 , whereas that in the lower branch varies as $1 - m_1^2$. In SPS 37-60, Vol. III, it was shown that for the data-aided loop specifically, splitting the power between upper and lower branches as m_1^2 and $1 - m_1^2$, respectively, gives the maximum improvement in signal-to-noise ratio in the loop bandwidth relative to a standard phase-locked loop when both are operated in the linear region.

4. Concluding Remarks

In this article, no attempt has been made to analyze the performance of the general configuration or either of its two special cases for low and high signal-to-noise ratio. For such a discussion, the reader is referred to SPS 37-60, Vol. III and Footnote 2.

Finally, for the case where $m_0(t)$ is placed on $x(t)$, the configuration of Fig. 13 is modified as in Fig. 15. The

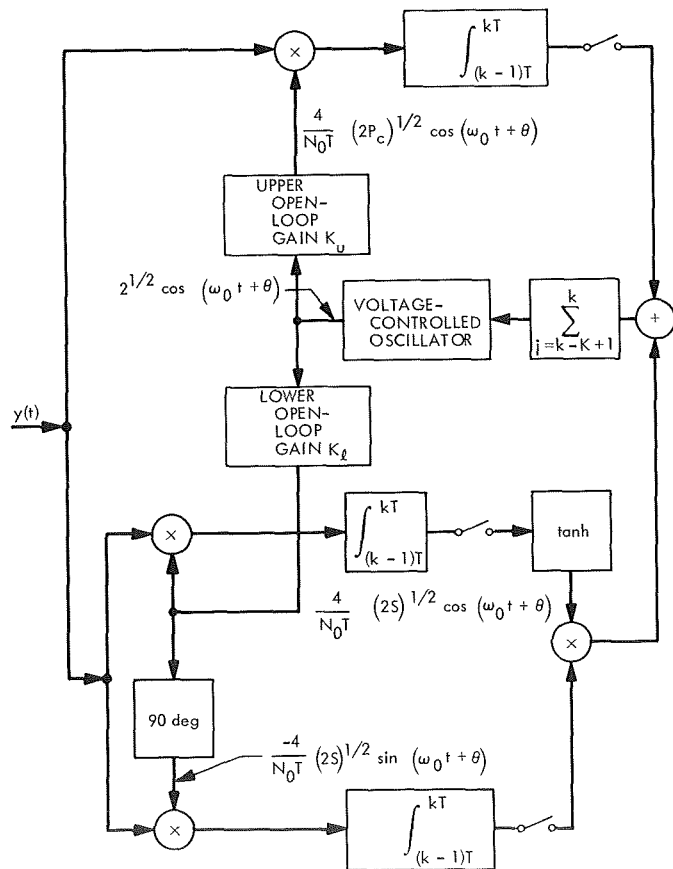


Fig. 13. Discrete model of suboptimum receiver configuration

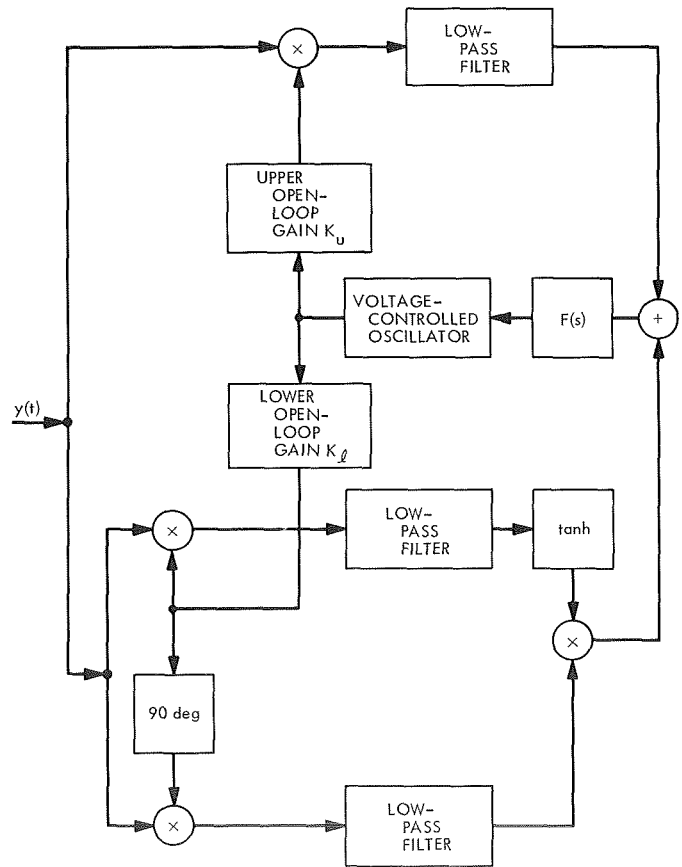


Fig. 14. Continuous model of suboptimum receiver configuration

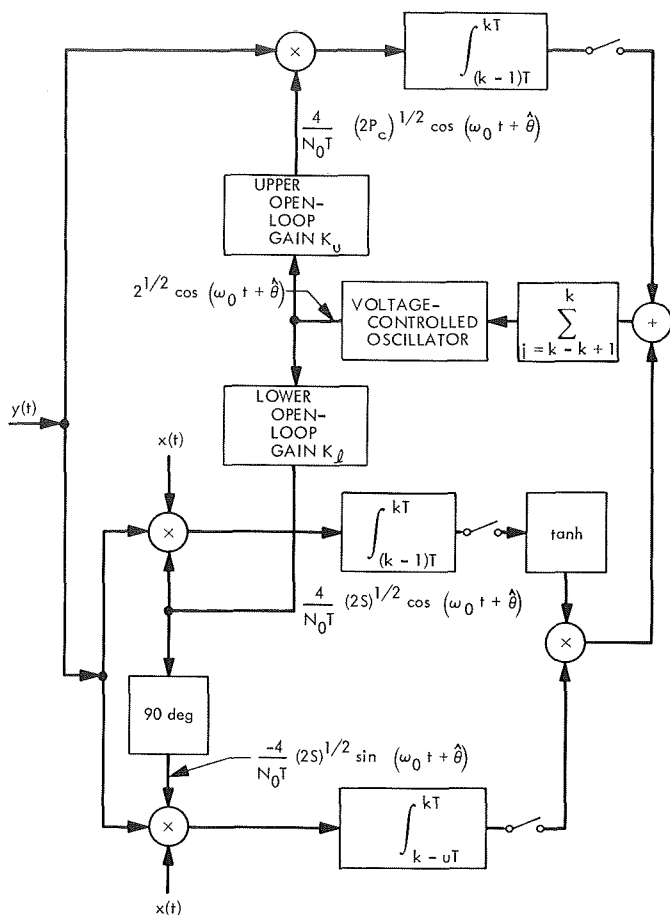


Fig. 15. Discrete model of suboptimum receiver configuration (modulation on subcarrier)

assumption regarding the subcarrier is that it be a perfect reference (i.e., no phase jitter) or that its random phase is independent of the RF phase being tracked or estimated. In practice, the subcarrier reference signal is derived in a loop (typically a squaring loop) whose input amplitude is degraded by the cosine of the RF phase error and, hence, is dependent on the input RF phase. However, the subcarrier phase jitter is usually negligibly small relative to the RF phase noise component, and hence, to a first-order approximation, it can be ignored. If such an approximation is not valid, then one must consider the problem of joint estimation of the RF and subcarrier phases.

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D. Hybrid Carrier and Modulation Tracking Loops, W. C. Lindsey

1. Introduction

The loop which is discussed in what follows is best motivated by one aspect of generalized harmonic analysis normally referred to as the cross-spectrum. In the usual form of generalized harmonic analysis, the physical significance of the cross-spectrum consists of the spectrum of mutual power. Consider, for instance, two signals immersed in noise coming from two different channels. The cross-spectrum represents the spectral distribution of power that is mutually shared between the two signals in a phase-coherent manner. In order to have mutual power, the signals must be phase-coherent.

In the field of optics, the cross-spectrum is related to the coherency of the fields. In nonlinear systems, however, mutual power may often be shared at other than the same frequency. In a nonlinear system there may be coherence between one frequency in one channel and some multiple of that frequency in a separate channel. This power may, in fact, occur in any combination of multiples, sums, and differences of frequencies. Such interactions frequently remain undetected by the usual linear schemes; however, by utilizing nonlinear methods, the nonlinear interactions can be detected and used to improve system efficiency.

For example, in phase-coherent communications the radiated signal frequently assumes the form (Refs. 1 and 2)

$$u(t) = (2P)^{1/2} \sin [\omega_0 t + \theta(t)] \quad (1)$$

where $\theta(t) = (\cos^{-1} m) x_k(t) + \theta_0 + \Omega_0 t$ represents the applied modulation, θ_0 is a slowly varying quantity, and $\Omega_0 = \omega - \omega_0$ is the loop detuning. The modulation index

$(\cos^{-1} m)$ serves to apportion the total transmitter power between the carrier and sidebands and the signal $x_k(t)$ represents a bi-phase modulated data subcarrier (Ref. 1). In Refs. 1 and 2, it is shown that $u(t)$ can be broken into two components, viz.,

$$u(t) = \underbrace{(2P_c)^{1/2} \sin \Phi(t)}_{\text{carrier component}} + \underbrace{(2S)^{1/2} x_k(t) \cos \Phi(t)}_{\text{sideband component}} \quad (2)$$

where $\Phi(t) = \omega_0 t + \theta(t)$, $P_c = m^2 P$ represents the power retained in the carrier component and $S = (1 - m^2) P$ represents sideband power.

Excluding for the moment any additive noise, it is intuitively clear that any efficient coherent receiver, which is to track the carrier coherently and extract the modulation $x_k(t)$ from $u(t)$, should make effective use of the power in both components; i.e., coherency of the sideband component at the carrier frequency should be exploited in the carrier tracking loop. In Ref. 3, it has been shown how one can track the frequency ω and phase θ_0 of the sideband components by means of a squaring or Costas loop. For quite some time, the sinusoidal phase-locked loop (PLL) has been successfully used to track the frequency and phase of the carrier component. It seems apparent that the hybrid tracking (HT) system illustrated in Fig. 16 can be used to estimate ω and θ_0 for the case where $0 \leq m \leq 1$. By appropriately choosing the gains in the upper loop filter $F_u(p)$ and the lower loop filter $F_l(p)$, the hybrid-tracker in Fig. 16 operates as a sinusoidal PLL when $m = 1$ and its operation is somewhat analogous to the Costas loop when $m = 0$. This difference is discussed in Subsection 3. The qualitative operation that leads to this conclusion is discussed in Subsection 2. We first note that $x(t) = u(t) + n_i(t)$ where $\{n_i(t)\}$ is the channel noise characterized in (Ref. 4).

2. System Model

Coherency of sideband component at the carrier frequency is exploited at the carrier frequency as follows: The harmonic oscillations x are applied to two phase detectors (PDs) (assumed to be multipliers with gain K_{1m} and K_{2m}) driven by signals r_u and r_l respectively. The output ϵ_1 of the upper loop PD is applied to the bandpass filter $H(p)$ and to the loop filter $F_u(p)$ which removes the modulation sidebands. The signal z_u is formed by filtering ϵ_1 in the loop filter $F_u(p)$. Thus, in operator form,

$$z_u = K_{1m} F_u(p) \epsilon_1 = K_{1m} F_u(p) x r_u \quad (3)$$

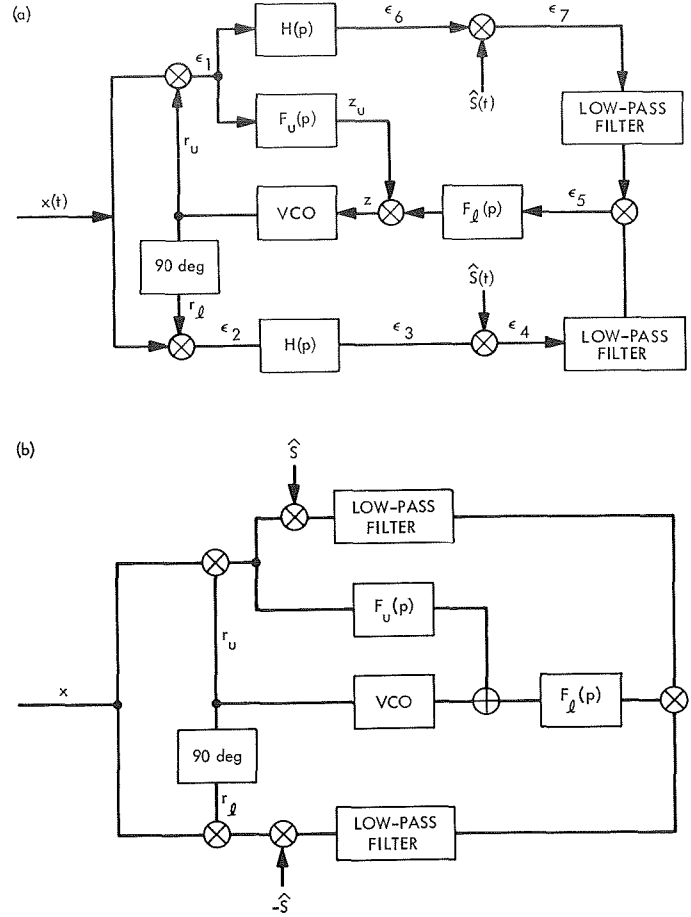


Fig. 16. Hybrid loop mechanization: (a) proposed, (b) alternate

where K_{1m} is the multiplier gain in the upper loop. The method used by the lower loop to make use of the power in the sidebands to estimate $\Phi(t)$ is as follows: The upper PD output ϵ_1 and the lower PD output $\epsilon_2 = K_{2m} r_l x$ are applied to bandpass filters $H(p)$ to remove the low-frequency noise and dc signal components. This produces signals ϵ_3 and ϵ_6 , which are then shifted to baseband by insertion of the subcarrier $S(t)$. This filtering, frequency-translation process yields the signals ϵ_4 and ϵ_7 . The signal z_l is obtained by filtering $\epsilon_5 = \epsilon_4 \epsilon_7$ in the lower loop filter $F_l(p)$. Thus, in operator form,

$$z_l = F_l(p) \epsilon_5 = K_{1m} K_{2m} F_l(p) \epsilon_4 \epsilon_7 \quad (4)$$

where K_{2m} is the total gain of the multipliers effecting the signal in the lower loop.

Assuming that the voltage-controlled oscillator (VCO) acts as an ideal integrator with gain K_V , the phase estimate

$\hat{\theta}$ of the VCO output is related to z_u and z_i through

$$\begin{aligned}\hat{\theta} &= \frac{K_V}{p} (z_u + z_i) \\ &= \frac{K_V}{p} [K_{1m}F_u(p)r_{ux} + K_{1m}K_{2m}F_i(p)\hat{S}(H(p)r_{ux})\hat{S}(H(p)r_{ix})]\end{aligned}\quad (5)$$

The first term in Eq. (5) is due to the PLL nature of the HT system while the second term is attributable to the squaring operation familiar in Costas loops. Both terms combined yield the hybrid system. Defining the loop phase-error as $\phi = \Phi - \hat{\Phi}$, then the stochastic differential Eq. (5) of loop operation becomes

$$\dot{\phi} = \Omega_0 - K_V [K_{1m}F_u(p)r_{ux} + K_{1m}K_{2m}F_i(p)\hat{S}(H(p)r_{ux})\hat{S}(H(p)r_{ix})]\quad (6)$$

Denoting the reference signals by

$$\left. \begin{aligned}r_u(t) &= 2^{1/2} K_1 \cos \hat{\Phi} \\ r_i(t) &= 2^{1/2} K_2 \sin \hat{\Phi}\end{aligned} \right\} \quad (7)$$

and, assuming a perfect subcarrier reference $\hat{S}(t) = 2^{1/2} \sin \omega_s t$ (we shall discuss the implication of this assumption later), the loop equation reduces to the form

$$\begin{aligned}\dot{\phi} &= \Omega_0 - KF_u(p) [(m^2 P)^{1/2} \sin \phi + N_u(t, \phi)] \\ &\quad - K_i F_i(p) \left[\frac{S}{2} \sin 2\phi + N_i(t, \phi) \right]\end{aligned}\quad (8)$$

$K_i = K_2 K_{2m} K$ and $K = K_1 K_{1m} K_V$ where $N_u(t, \phi)$ is the usual "white" noise (single-sided spectral density of N_0 W/Hz) process (Ref. 4) which affects the performance of the upper loop and

$$N(t, \phi) = n_1 n_2 + d S^{1/2} [n_1 \sin \phi + n_2 \cos \phi] \quad (9)$$

where n_1 and n_2 are orthogonal, bandlimited white gaussian noise process. Their bandwidths are assumed to be wide in comparison with the bandwidth of the phase-error process $\phi(t)$ and have a constant spectral density of N_0 W/Hz single-sided over this band. We also note that $N_i(t, \phi)$, although spectrally equivalent, is not the same as that arising in the Costas or squaring loop. The filters $F_u(p)$ and $F_i(p)$ are lowpass since $F_u(p)H(p) \approx 0$; therefore the processes $N_u(t, \phi)$ and $N_i(t, \phi)$ are statistically independent.

If we now assume that $F_u(p) = F_i(p)$, then Eq. (8) reduces to

$$\begin{aligned}\dot{\phi} &= \Omega_0 - KF(p) \\ &\quad \times \left\{ \left[(m^2 P)^{1/2} \sin \phi + \frac{GS}{2} \sin 2\phi \right] \right. \\ &\quad \left. + N_u(t, \phi) + GN_i(t, \phi) \right\}\end{aligned}\quad (10)$$

where $G \triangleq K_i/K$ is the ratio of the gain in the lower loop to the open-loop gain of a standard PLL. One principal point to note is that the shape of the equivalent system S-curve (the term in brackets) is modified by the addition of a double harmonic term that serves to make the S-curve more "rectangular." It has been pointed out (Refs. 4, 5) that the optimum loop S-curve, for zero loop detuning, is one that is "rectangular" when the additive noise is white and gaussian. In fact, when $F_u(s) = F_i(s)$, the loop S-curve is given by

$$S(\phi) = A \sin \phi + A_{sb} \sin 2\phi$$

where $A = (m^2 P)^{1/2}$ is proportional to the power in the carrier and $A_{sb} = GS/2$ is proportional to the power in the signal's sidebands. It is for this reason that the loop has been referred to as a hybrid tracker; viz., when $A_{sb} = 0$, the S-curve reduces to that of a standard PLL, and, when $A = 0$, the S-curve is analogous to that of a squaring loop or a Costas loop (Ref. 3). We also note the differences between the hybrid tracker and a Costas loop when $m = 0$. Ignore for the moment that branch in Fig. 16 that contains $F_u(p)$. The remaining block diagram is somewhat

reminiscent of the Costas loop (Ref. 3). However, there are significant differences. First, a Costas loop does not require the subcarrier references $\hat{S}(t)$. Ignoring this point, however, a Costas loop operates on the baseband signals appearing at the outputs of the upper and lower PDs, whereas the outer branches of the HT operates on the bandpass spectra appearing at the phase detector outputs. The loop operation when the $\hat{S}$'s are not presented as inputs to the loop is discussed in *Subsection 4*.

3. Probability Density Function of the Phase-Error and the n th Moment of the First-Slip Time

For narrowband loops, the "fluctuation equation" approach (Ref. 6) applies to a good approximation and the results given in Ref. 4 can be used to solve for the statistical dynamics of interest. If we write the loop filter in the form

$$F(s) = F_0 + \sum_{k=1}^N \frac{1 - F_k}{1 + \tau_k p} \quad (11)$$

and let

$$g(\phi) \triangleq \frac{S(\phi)}{A} = \sin \phi + \frac{GS}{2A} \sin 2\phi \quad (12)$$

where $A = (m^2 P)^{1/2}$, then using the method and results presented in Ref. 4, the steady-state probability density function of the phase-error process is approximated by

$$p(\phi) \approx C_0 \exp[-U_0(\phi)] \times \int_{\phi}^{\phi+2\pi} \exp[U_0(x)] dx; \quad |\phi| \leq \pi \quad (13)$$

where

$$\begin{aligned} U_0(\phi) &= \beta(N)\phi + \alpha(N) \left\{ \cos \phi + \frac{GS}{A} \cos 2\phi \right\} \\ \beta(N) &= \frac{2}{K_{00}} \left[\Omega_0 - AK\bar{g} \sum_{k=1}^N (1 - F_k) \left(1 + \frac{S_g(0)}{2\tau_k \sigma_g^2} \right) \right] \\ \alpha(N) &= \frac{2}{K_{00}} \left[AKF_0 - \frac{AKS_g(0)}{2\sigma_g^2} \sum_{k=1}^N \left(\frac{1 - F_k}{\tau_k} \right) \right] \end{aligned} \quad (14)$$

$$G = g - \bar{g}$$

and

$$K_{00} = \frac{K^2 N_0}{2} [1 + SG^2 \mathcal{S}_L] \quad (15)$$

In Eq. (15), $\mathcal{S}_L$ is the circuit squaring loss² defined by

$$\mathcal{S}_L \triangleq 1 + \frac{2}{N_0 S} \int_{-\infty}^{\infty} R_{n_1}^2(\tau) d\tau \quad (16)$$

In writing Eq. (13), it is assumed that $p(\phi)$ exists and C_0 is the normalization constant

$$C_0 = \left[\exp[-U_0(\phi)] \int_{\phi}^{\phi+2\pi} \exp[U_0(x)] dx \right]^{-1} \quad (17)$$

The probability current defined in Ref. 4 is easily found using Eq. (13).

The first-order loop is of particular interest since it serves to illustrate the hybrid nature of the loop. Letting $N = 0$, we find that

$$\begin{aligned} p(\phi) &= C_0 \exp \left\{ \frac{2\Omega_0 \phi}{K_{00}} + \frac{\rho \left[\cos \phi + \frac{mGM}{4} \cos 2\phi \right]}{1 + m^2 \mathcal{G}^2 M \mathcal{S}_L} \right\} \\ &\times \int_{\phi}^{\phi+2\pi} \exp \left\{ -\frac{2\Omega_0 x}{K_{00}} - \frac{\rho \left[\cos x + \frac{mGM}{4} \cos 2x \right]}{1 + m^2 \mathcal{G}^2 M \mathcal{S}_L} \right\} dx \end{aligned} \quad (18)$$

²Lindsey, W. C., and Simon, M. K., "Nonlinear Performance of Suppressed Carrier Tracking Loops in the Presence of Frequency Detuning" (to be published in the *Proc. IEEE*).

where the loop signal-to-noise ratio is given by

$$\rho = \frac{2A^2}{N_0 W_L} = 2m^2 P / N_0 W_L \quad (19)$$

and

$$M \stackrel{\Delta}{=} \frac{1-m^2}{m^2} \quad \mathcal{G} \stackrel{\Delta}{=} G^{P\frac{1}{2}} = K_2 K_{2m}^{P\frac{1}{2}} \quad (20)$$

Note also that Brennan's rule (Ref. 7) can be used to maximize the signal-to-noise ratio in the combined signal z that controls the VCO. Thus,

$$\frac{G = m^2}{1 - m^2} = M^{-1}$$

and

$$\mathcal{G} = \frac{P^{1/2}}{M}$$

The corresponding n th moment of the mean-time-to-first-slip is found by applying the theory given in Ref. 4. For the first-order loop, $N = 0$ and we have

$$\begin{aligned} \tau^n(2\pi) = & \frac{2}{K_{00}} \int_{-2\pi}^{2\pi} \int_{-2\pi}^{\phi} [C'_0(n-1) - \tau^{n-1}(x)] \\ & \times \exp[U_0(x) - U_0(\phi)] dx d\phi \end{aligned} \quad (21)$$

where $\tau^0(x) = u(x - \phi_0)$ is the unit step function and $\phi = \phi_0$ at $t = t_0$.

4. Elimination of the Subcarrier Reference

Development of the loop equation (Eq. 8) has assumed a perfect subcarrier reference in the receiver. Inclusion of the subcarrier reference jitter can easily be carried out in order to see how this jitter would ultimately effect loop performance. More important is the possibility of eliminating the provision for a subcarrier reference at the RF level. This simplifies the loop mechanization as well as the signal acquisition procedure, i.e., the signal can be acquired without opening the lower loop until the RF carrier is acquired; acquiring the subcarrier and then closing the lower loop.

When the subcarrier references are omitted, the corresponding simplified loop is shown in Fig. 17. The loop equation is now given by Eq. (8); however, $N_i(t, \phi)$ must be redefined as

$$N_t(t, \phi) \stackrel{\Delta}{=} 2^{1/2} n_1 n_2 + d S^{1/2} [n_1 \sin \phi + n_2 \cos \phi] \quad (22)$$

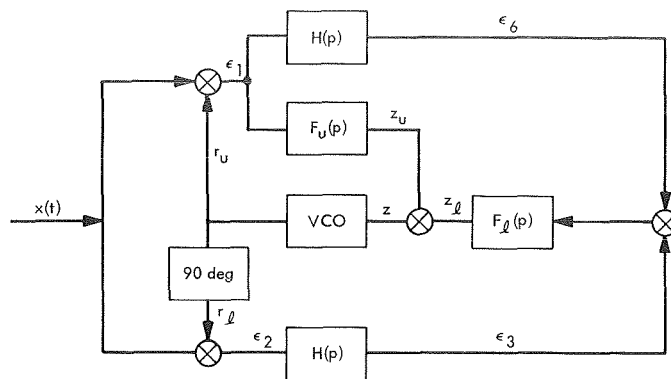


Fig. 17. Loop mechanization eliminating the subcarrier reference

and loop performance is obviously inferior since twice as much noise variance appears in the noise $\times$ noise term. For this circuit, loop performance is found from Eqs. (12) through (15) with $\mathcal{O}_L$ defined by

$$\mathcal{S}_L = 1 + \frac{4}{N_0 S} \int_{-\infty}^{\infty} R_{n_1}^2(\tau) d\tau \quad (23)$$

5. Conclusions

This article has proposed a coherent receiver that is essentially a linear combination of a standard PLL and a modified Costas loop. It should be pointed out that this receiver is easier to implement than the data-aided loop proposed in Ref. 7, which employs decision directed feedback. The reason for this is that the hybrid tracker proposed here does not require the implementation of a delay in the loop nor does it require the use of the data-estimate-subcarrier estimate product.³ It is well known that extraneous delays in a PLL reduces the loop acquisition range, etc. Moreover, loop performance is not dependent upon system data rate.

When $\theta(t)$ consists of several data modulated subcarriers, coherency of the multiple sideband components (Ref. 9) can be exploited at the carrier frequency by means of multiple loops. It is intuitively clear how this can be accomplished.

We also point out that the input signal component $u(t)$ could be remodeled to consider amplitude variations, say $A(t)$. Such variations could well account for any linear modulation produced at the transmitter or variations due to a randomly time-varying propagation medium. For this case, $(m^2P)^{1/2}$ and S in Eq. (8) would be replaced by $mA(t)$

³Lindsey, W. C., and Simon, M. K., "Data-Aided Carrier Tracking Loops" (to be published in the *Proc. IEEE*).

and $(1 - m^2) A^2(t)$, respectively. Therefore, it appears that the loop could be used in a multipath environment either to produce a coherent reference or for amplitude modulation extraction if the phase variations of the channel are sufficiently stable.

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E. The Error Probability of a Wide-Band FSK Receiver in the Presence of Multipath Fading, H. D. Chadwick

1. Introduction

Some relay links use a binary frequency-shift-keyed (FSK) modulation technique. The receiver for this link is illustrated in Fig. 18. The two band-pass filters, F_0 and F_1 ,

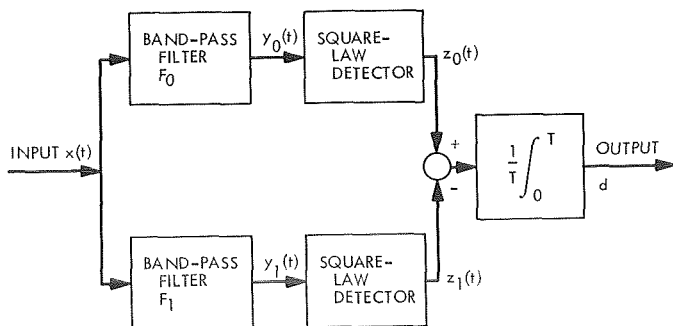


Fig. 18. FSK receiver block diagram

are centered at the nominal transmitted frequencies, ω_0 and ω_1 , but, because of severe doppler shift conditions and frequency instability in the transmitter, these filters must have a considerably wider bandwidth than for the case of no doppler shift. For this reason, the error probability performance of the receiver is worse than that of the ideal FSK receiver. The error probability of a wide-band FSK receiver, similar to the one proposed for *Viking*, has been approximated by A. B. Glenn (Ref. 1) and by D. W. Boyd (SPS 37-41, Vol. IV, pp. 233-239), who used the approach followed by J. I. Marcum (Ref. 2). These calculations do not take into account the effects of multipath distortion on the received signal due to reflection of the signal off the surface of the planet. In this article, the error performance of the FSK receiver is calculated including the effects of multipath distortion using both a low and a high fading bandwidth. The analysis follows that of Boyd, which yielded results close to those obtained experimentally by C. Carl (SPS 37-54, Vol. III, pp. 211-220).

2. Multipath Fading Model

The signal at the receiver consists of a direct component

$$x_d(t) = A \cos \omega t \quad (1)$$

a reflected component

$$x_r(t) = B(t) \cos [\omega t + \phi(t)] \quad (2)$$

and white gaussian noise $n(t)$ with two-sided spectral density $N_0/2$. Since the receiver is incoherent, the phase of the direct signal is unimportant and only the relative phase between the direct and reflected signals need be considered in the term $\phi(t)$. In the usual model for multipath distortion (Ref. 3), $B(t)$ is assumed to have a Rayleigh distribution

$$p(B) = \frac{B}{\psi^2} e^{-B^2/2\psi^2} \quad (3)$$

The term ψ^2 is the average power in the reflected signal, which is difficult to compute, but may be approximated by a method such as that of P. Beckmann and A. Spizzichino (Ref. 4). The term $\phi(t)$ is assumed to have a uniform distribution. If the bandwidth of the reflected component process is considerably higher than the signal bandwidth, then "fast fading" exists and the signal amplitude varies rapidly during one bit interval T . In this case, the reflected signal may be written

$$x_r(t) = x_{rc}(t) \cos \omega t + x_{rs}(t) \sin \omega t \quad (4)$$

where $x_{rc}(t)$ and $x_{rs}(t)$ are independent gaussian processes of mean zero and variance ψ^2 . A similar expansion can be performed on the noise process at the bandpass filter output

$$n(t) = n_c(t) \cos \omega t + n_s(t) \sin \omega t \quad (5)$$

where $n_c(t)$ and $n_s(t)$ are independent gaussian processes with mean zero and variance $\sigma^2 = N_0 W$, where W is the filter bandwidth. The sum of reflected signal and noise can thus be expressed

$$x_r(t) + n(t) = [x_{rc}(t) + n_c(t)] \cos \omega t + [x_{rs}(t) + n_s(t)] \sin \omega t \quad (6)$$

and the combined process considered to have gaussian distributed coefficients with mean zero and variance $\sigma^2 + \psi^2$.

In the "slow fading" model, where the fading bandwidth is considerably smaller than the data bandwidth, the reflected signal amplitude is assumed to be constant during T and to vary randomly according to the Rayleigh distribution from T to T .

The slow fading model is probably a reasonable representation of the specular reflection term and the fast fading model of the diffuse term that exists in the generally accepted multipath reflection approximation. While both specular and diffuse reflection can occur simultaneously, their relative magnitudes depend on the geometry and relative motion of the receiver, transmitter, and surface; in most cases, one term or the other will predominate.

If we now examine the outputs of the bandpass filters and assume that during the present T the direct signal is at frequency ω_0 , two possible situations may occur. These are:

- (1) **Case 1.** Direct and reflected signals at the same frequency ω_0 .
- (2) **Case 2.** Direct signal at ω_0 and reflected signal at ω_1 .

Case 2 is possible because of the relative delay between direct and reflected signals due to different path lengths. Obviously, over the entire bit time, both conditions can occur. However, these two cases give rise to the best and worst performance, respectively, as will be shown, and any other condition can be expected to produce performance that falls between these limits.

In order to simplify, two properties of multipath reflected signals have been ignored in this development. These are the doppler offset, which may exist between

the direct and reflected signals in frequency, and the time spreading that may occur in the reflected signal. These properties tend to complicate the model and further work is required to determine their effects.

3. Output Probability Distribution

The outputs of the band-pass filters are processed by the square-law detectors to produce signals that are proportional to the square of the envelope of the input. These outputs, z_0 and z_1 , can be shown to have the non-central chi-squared probability distributions⁴

$$p(z_0) = \frac{1}{2\sigma_0^2} \exp \left[-\frac{1}{2\sigma_0^2} (z_0 + B_0^2) \right] I_0 \left[\frac{B_0(z_0)^{1/2}}{\sigma_0^2} \right] \quad (7)$$

and

$$p(z_1) = \frac{1}{2\sigma_1^2} \exp \left[-\frac{1}{2\sigma_1^2} (z_1 + B_1^2) \right] I_0 \left[\frac{B_1(z_1)^{1/2}}{\sigma_1^2} \right] \quad (8)$$

where σ_0^2 and σ_1^2 are the "noise" variances at the output of filters F_0 and F_1 , respectively, and B_0 and B_1 are the "signal" amplitudes. σ_0^2 and σ_1^2 include the noise and fast fading components, while B_0 and B_1 include the direct signal and slow fading components. These envelope signals are then differenced and integrated in the integrate and dump circuit to produce the decision variable d where

$$d = \frac{1}{T} \int_0^T [z_0(t) - z_1(t)] dt \quad (9)$$

which may be expressed

$$d = \frac{1}{T} \int_0^T z_0(t) dt - \frac{1}{T} \int_0^T z_1(t) dt = d_0 - d_1 \quad (10)$$

The decision is made to accept signal 0 if $d > 0$, or equivalently, if $d_0 > d_1$. To determine the probability distribution of the decision variables d_0 and d_1 , it is necessary to use an approximation to the integral. One technique, used by Boyd, which may be justified by the sampling theorem, is to represent the integral as a sum.

$$d_0 \approx \sum_{k=0}^{N-1} z_0 \left(\frac{kT}{N} \right) \quad (11)$$

⁴Henceforth, all density functions apply only for non-negative values of the variable and are zero for negative values.

and

$$d_1 \approx \sum_{k=0}^{N-1} z_1 \left(\frac{kT}{N} \right) \quad (12)$$

where $N = WT$.

The sampled variables $z(kT/N)$ then have the probability density functions given by Eqs. (7) and (8) and are independent. Thus, the probability densities of d_0 and d_1 can be found approximately by the N -fold convolution of these densities with themselves. Equivalently, they can be found by transforming to the characteristic function, raising this characteristic function to the N th power, and inverse transforming back to the density function. The probability of error is then given by

$$P_E = \text{prob} [d_0 < d_1 | \text{signal } 0] \\ = \int_0^\infty \int_0^{d_1} p(d_0) p(d_1) dd_0 dd_1 \quad (13)$$

where the densities $p(d_0)$ and $p(d_1)$ are evaluated for signal 0 received. (Because of the symmetry of the receiver, it is only necessary to consider the signal 0 case. The signal 1 case is exactly equivalent if the receiver channels are balanced.) Following the characteristic function approach with the densities $p(z_0)$ and $p(z_1)$ yields the following density for d_0 and d_1 :

$$p(d_j) = \frac{1}{2\sigma_j^2} \left(\frac{d_j}{NB_j^2} \right)^{(N-1)/2} \exp \left[-\frac{1}{2\sigma_j^2} (d_j + NB_j^2) \right] \\ \times I_{N-1} \left[\frac{B_j (Nd_j)^{1/2}}{\sigma_j^2} \right] \quad (14)$$

When B_j goes to zero, this expression reduces to

$$p(d_j) = \frac{1}{2\sigma_j^2} \left(\frac{d_j}{2\sigma_j^2} \right)^{N-1} \frac{\exp \left(-\frac{d_j}{2\sigma_j^2} \right)}{(N-1)!} \quad (15)$$

4. Probability of Error

The probability of error for a specific, fixed set of the parameters σ_j^2 and B_j^2 can be found by performing the integration of Eq. (13) on the density functions of Eq. (14). For the slow fading situation, in which B_j^2 is a random variable, it is then necessary to average this probability of error over the distribution of B_j^2 .

The result obtained by Boyd is reproduced here for comparison with the fading results. His result is

$$P_E = e^{-\rho^2/2} \sum_{k=0}^{N-1} \frac{L_k^{(N-1)}(-\rho^2/2)}{2^{N+k}} \quad (16)$$

where

$$\rho^2 = \frac{NA^2}{2\sigma^2} = \frac{STb}{N_0}$$

is the ratio of signal energy per bit to the noise spectral density and $L_k^{(N-1)}(\cdot)$ is a generalized Laguerre polynomial.

The fast and slow fading cases, combined with delays producing Case 1 and Case 2 as defined in Subsection 2, are now considered separately. For convenience, these cases are designated F1, F2, S1, and S2. The parameter γ^2 is defined as the ratio of direct power to reflected power or

$$\gamma^2 = \frac{A^2}{2\psi^2}$$

Thus, $\gamma^2 = \infty$ corresponds to the no-fading case analyzed by Boyd.

a. Case F1. In this case, $B_0 = A$, the direct signal amplitude, $B_1 = 0$, $\sigma_0^2 = \sigma^2 + \psi^2$, because the reflected signal is at the same frequency as the direct signal, and $\sigma_1^2 = \sigma^2$, due to noise only. After performing the integration following Marcum and Boyd, the result is obtained.

$$P_E^{F1} = \exp \left(-\frac{\gamma^2 N \rho^2}{2N\gamma^2 + \rho^2} \right) \left(\frac{N\gamma^2}{2N\gamma^2 + \rho^2} \right)^N \sum_{k=0}^{N-1} \left(\frac{\gamma^2 N + \rho^2}{2N\gamma^2 + \rho^2} \right)^k L_k^{(N-1)} \left[\frac{-N^2 \gamma^4 \rho^2}{(N\gamma^2 + \rho^2)(2N\gamma^2 + \rho^2)} \right] \quad (17)$$

b. Case F2. Here, $B_0 = A$ and $B_1 = 0$, as before, but $\sigma_0^2 = \sigma^2$ and $\sigma_1^2 = \sigma^2 + \psi^2$ because the reflected signal is now contributing "noise" in the non-signal channel. The result in this case becomes

$$P_E^{F2} = \exp \left(-\frac{\gamma^2 N \rho^2}{2N\gamma^2 + \rho^2} \right) \left(\frac{N\gamma^2 + \rho^2}{2N\gamma^2 + \rho^2} \right)^N \sum_{k=0}^{N-1} \left(\frac{N\gamma^2}{2N\gamma^2 + \rho^2} \right)^k L_k^{(N-1)} \left[-\frac{\rho^2 (N\gamma^2 + \rho^2)}{2N\gamma^2 + \rho^2} \right] \quad (18)$$

c. *Case S1.* Here, B_0 is a random variable (constant over a bit period) having the probability distribution

$$p(B_0^2) = \frac{1}{2\psi^2} \exp\left[-\frac{1}{2\psi^2}(B_0^2 + A^2)\right] I_0\left(\frac{AB_0}{\psi^2}\right) \quad (19)$$

while $B_1 = 0$, and $\sigma_0^2 = \sigma_1^2 + \sigma^2$. Integrating Eq. (13) over the densities of d_0 and d_1 , using these parameters with fixed B_0 , an expression is obtained for the probability of error as a function of B_0 . This expression is identical to Eq. (16) with A^2 replaced by B_0^2 .

$$P_E^{S1}(B_0^2) = \exp\left(-\frac{B_0^2}{4\sigma^2}\right) \sum_{k=0}^{N-1} \frac{1}{2^{N+k}} L_k^{(N-1)}\left(-\frac{B_0^2}{4\sigma^2}\right) \quad (20)$$

The overall average probability of error for Case S1 can then be found from the relation

$$P_E^{S1} = \int_0^\infty P_E^{S1}(B_0^2) p(B_0^2) dB_0^2 \quad (21)$$

by substituting Eqs. (19) and (20). The integration can be performed by expanding the generalized Laguerre polynomial as an infinite series, and integrating term-by-term using the method of I. S. Gradshteyn and I. M. Ryzhik (Ref. 5, Eq. 6.631.10). The result is

$$P_E^{S1} = \frac{\gamma^2}{\rho^2 + \gamma^2} \exp\left(-\frac{\gamma^2 \rho^2}{\rho^2 + \gamma^2}\right) \times \sum_{k=0}^{\infty} \frac{L_k\left(-\frac{\gamma^4}{\rho^2 + \gamma^2}\right)}{2^{N+k} \Gamma(N+k)} \left(\frac{\rho^2}{\rho^2 + \gamma^2}\right)^k D_k \quad (22)$$

where

$$D_k = \sum_{i=0}^{N-1} \frac{\Gamma(N+k+i)}{i! 2^i} \quad (23)$$

$L_k(\cdot) = L_k^{(0)}(\cdot)$ is the Laguerre polynomial.

d. *Case S2.* In this case, $B_0^2 = A^2$, $\sigma_0^2 = \sigma_1^2 = \sigma^2$, and B_1^2 is a random variable with the density function

$$p(B_1^2) = \frac{1}{2\psi^2} \exp\left(-\frac{B_1^2}{2\psi^2}\right) \quad (24)$$

The solution to Eq. (13) for this case can be shown, expanding $I_{N-1}(\cdot)$ as a series and again using the relation in

Gradshteyn and Ryzhik, to be

$$P_E^{S2}(B_1^2) = \exp\left(-\frac{\rho^2}{2}\right) \sum_{k=0}^{\infty} \frac{L_k^{(N-1)}(-\rho^2/2)}{2^{N+k}} \mu_k\left(\frac{NB_1^2}{2\sigma^2}\right) \quad (25)$$

where

$$\mu_k(\beta^2) = \begin{cases} 1 & \text{for } k < N \\ 1 - e^{-\beta^2} \sum_{i=0}^{k-N} \frac{\beta^{2i}}{i!} & \text{for } k \geq N \end{cases} \quad (26)$$

This expression is for a fixed value of B_1^2 and would be valid, for example, if a constant interfering signal of amplitude B_1 were present in the output of the non-signal channel detector.

To obtain the average probability of error for a randomly varying B_1 , this expression must be averaged over the distribution of B_1 , using the relation

$$P_E^{S2} = \int_0^\infty P_E^{S2}(B_1^2) p(B_1^2) dB_1^2 \quad (27)$$

Performing this integration leads to the result

$$P_E^{S2} = \exp\left(-\frac{\rho^2}{2}\right) \sum_{k=0}^{\infty} \frac{L_k^{(N-1)}(-\rho^2/2)}{2^{N+k}} Q_k(\gamma^2, \rho^2) \quad (28)$$

where

$$Q_k(\gamma^2, \rho^2) = \begin{cases} 1 & \text{for } k < N \\ 1 - \frac{\gamma^2}{\rho^2 + \gamma^2} \sum_{i=0}^{k-N} \left(\frac{\rho^2}{\rho^2 + \gamma^2}\right)^i & \text{for } k \geq N \end{cases} \quad (29)$$

All four results are plotted in Figs. 19 and 20 for various values of N . In addition, a set of curves are plotted for

$$P_E^F = \frac{1}{2} (P_E^{F1} + P_E^{F2}) \quad (30)$$

and

$$P_E^S = \frac{1}{2} (P_E^{S1} + P_E^{S2}) \quad (31)$$

These curves provide a reasonable approximation to the average performance with a random 0,1 pattern and a differential delay at least as great as one T . For smaller

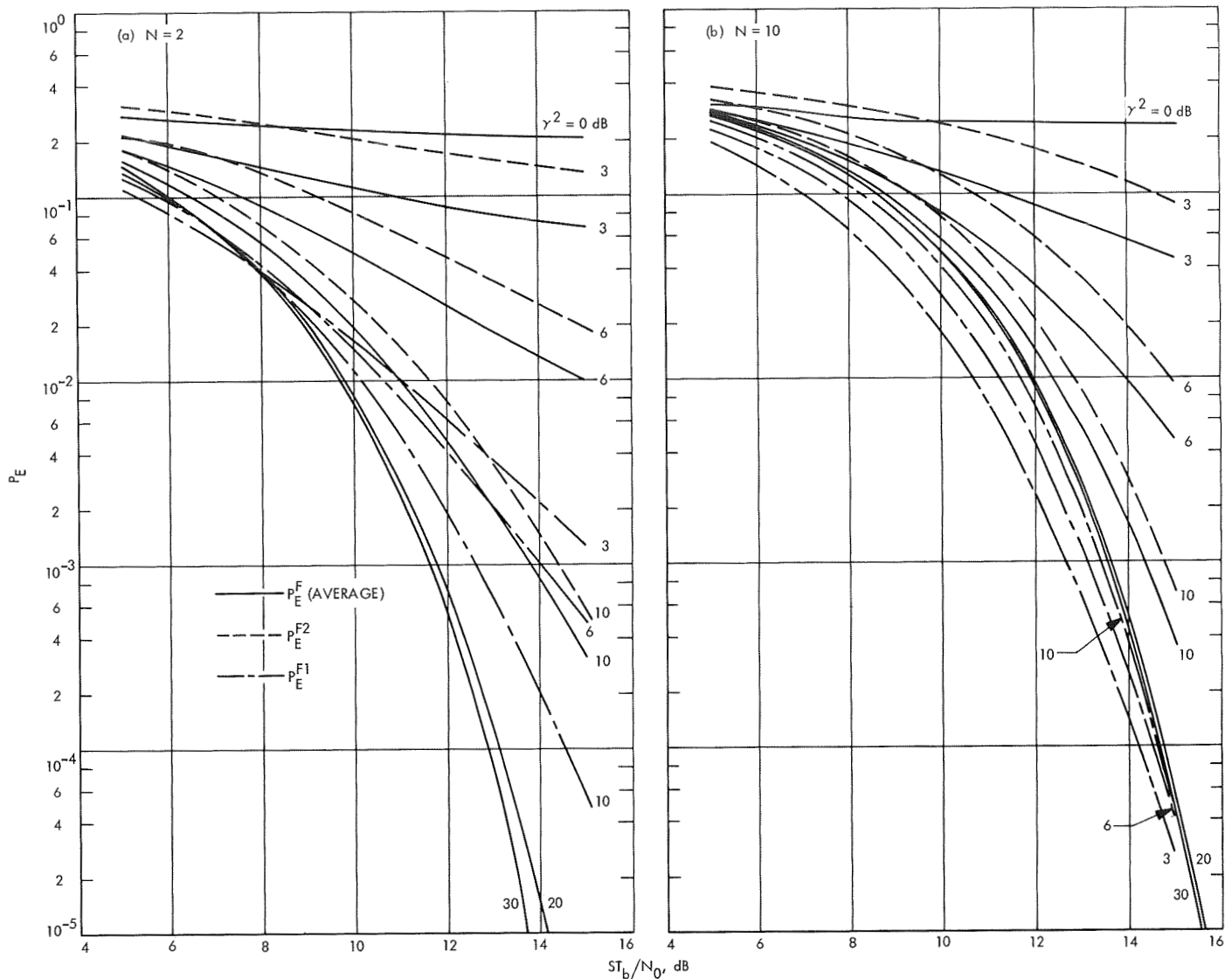


Fig. 19. Fast fading P_E vs signal-to-noise ratio ST_b/N_0

delays, the curves for Case 1 should provide a closer approximation.

5. Discussion of Results

The following points may be inferred from the plotted results:

- (1) For large N and small ST/N_0 , it is possible for multipath distortion to improve the error performance of the system for Case 1. This is probably due to the fact that the FSK receiver is an incoherent energy detector and the reflected signal tends to increase the total signal energy.
- (2) In general, slow fading limits performance more than does fast fading.

- (3) Case 1 produces uniformly better performance than does Case 2. For this reason, it is apparent that any type of alternating 0, 1 sequence, used for synchronization or any other purpose, can produce, with the proper differential time delay, the Case 2 condition with subsequent poor performance.

6. Conclusions

Results have been derived for the probability of error of a relay type wide band FSK receiver undergoing multipath distortion. While the results are an approximation, neglecting some properties of the reflected signal, they provide an indication of the performance that can be expected, particularly if the signal bandwidth is considerably below the coherence bandwidth. It is hoped that

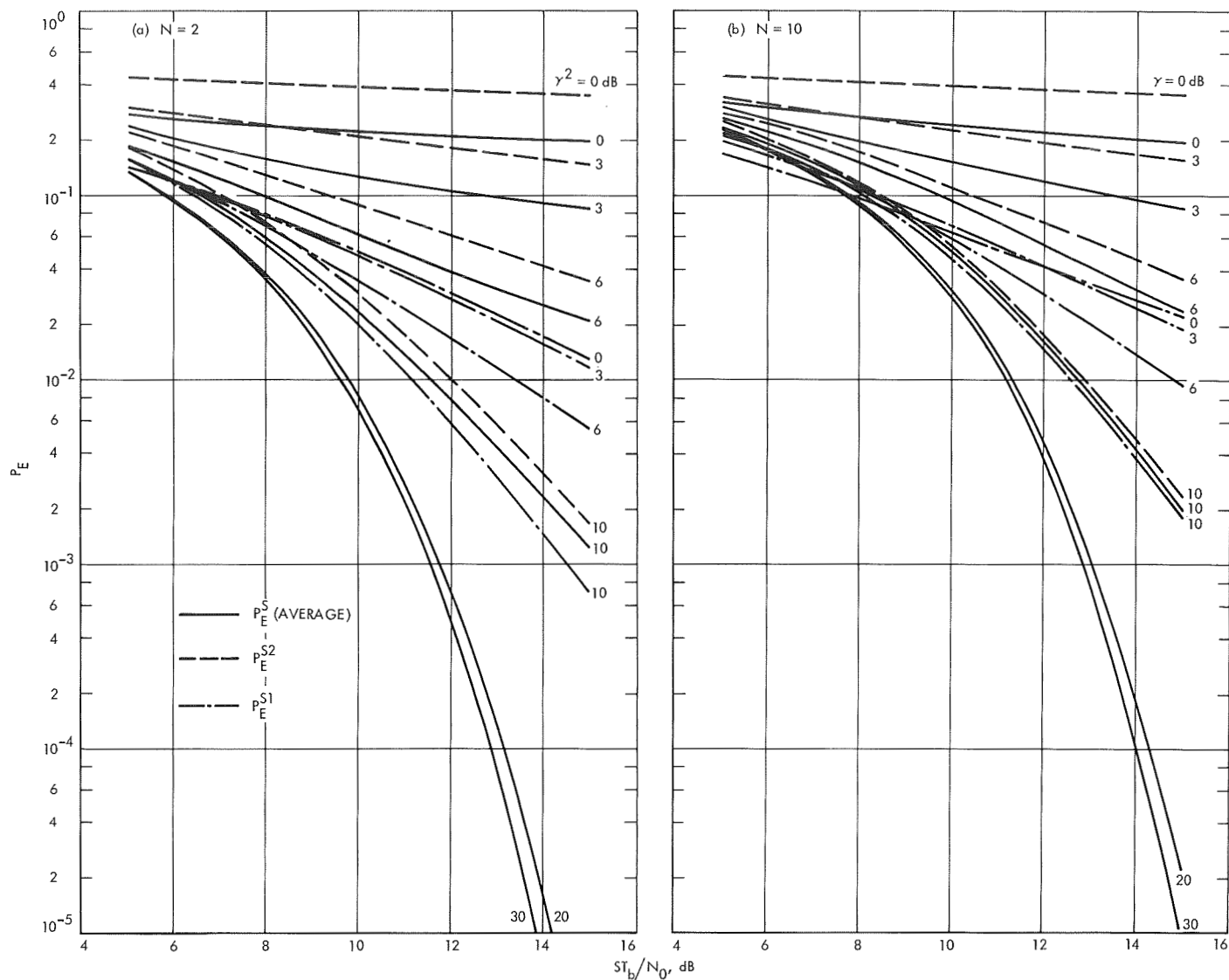


Fig. 20. Slow fading P_E vs signal-to-noise ratio ST_b/N_0

experimental work performed in the future will verify these results. The problem of determining the proper value of the parameter γ^2 remains.

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F. Effects of Synchronization Jitter on Relay Performance, J. G. Smith

1. Introduction

It might be expected that detection of "jitter" data with a "clean" bit synchronizer would provide the same steady-state performance as detection of "clean" data with a "jittery" bit synchronizer. This is true when successive jitters are completely dependent, but not in general. In

this article, these results are applied to a lander-orbiter-earth relay communication link, and steady-state performance is presented under a gaussian jitter density function assumption.

2. Relay System Example

In any deep-space mission involving orbiters and landers, a one-way lander-orbiter-earth relay communication link is of possible interest. In this article, attention is restricted to communication links utilizing square-wave binary data trains and bit synchronizing loops (as opposed to separate synchronization channels). Attention is further, and most importantly, restricted to a relay system in which the orbiter system introduces the only significant data jitter, and this due to imperfect bit synchronizer loop tracking.

If, during the lander descent phase, a real-time, direct-coupled, uncoded-data relay mode is required to avoid total loss in the event of hard impact, the lander-orbiter link must have a relatively short acquisition time to permit rapid recovery of synchronization after entry blackout (assuming planetary atmosphere). This precludes a non-trivial tracking bandwidth in the orbiter bit synchronizer loop and, hence, because of noise in the lander-orbiter link (up link), imperfect loop tracking. Thus, the orbiter bit synchronizer makes "noisy" estimates of the detection

intervals required by the detector, and the detector output is thus "jittery" data. This data is direct-coupled to the orbiter transmitter, sent over the orbiter-earth link (down link) and received by ground receiver. If the lander utilizes a fairly stable clock, and the ground station tracks the jittery data closely (since it has no comparable limit in its acquisition time), the orbiter will in fact introduce the only significant data jitter.

Figure 21 is a block diagram of the essential elements of the motivating relay link. Since only the effects of jitter are to be considered, the low-pass equivalent system (Fig. 22) can be considered where $n_1(t)$ and $n_2(t)$ are the low-pass equivalents of $N_1(t)$ and $N_2(t)$. It is assumed that the original data stream is

$$d_1(t) = S^{1/2} \sum_i b_i g[t, iT, (i+1)T]$$

where the b_i 's, the original set of uncoded data, are independent random variables equally likely to be +1 or -1, and $g(t, T_1, T_2)$ is a gate pulse that is unity in $[T_1, T_2]$ and zero elsewhere. It is assumed that $n_1(t)$ and $n_2(t)$ are independent zero-mean gaussian white noise processes with two-sided spectral density levels of $N_{01}/2$ and $N_{02}/2$, respectively, and

$$y_1(t) = d_1(t) + n_1(t)$$

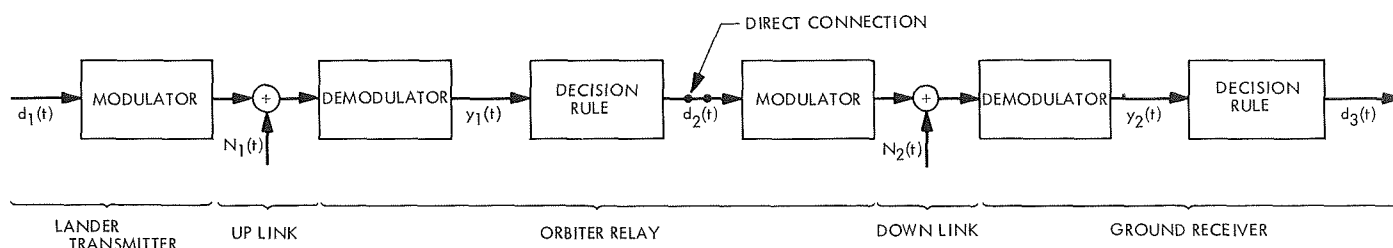


Fig. 21. Essential elements of the relay system

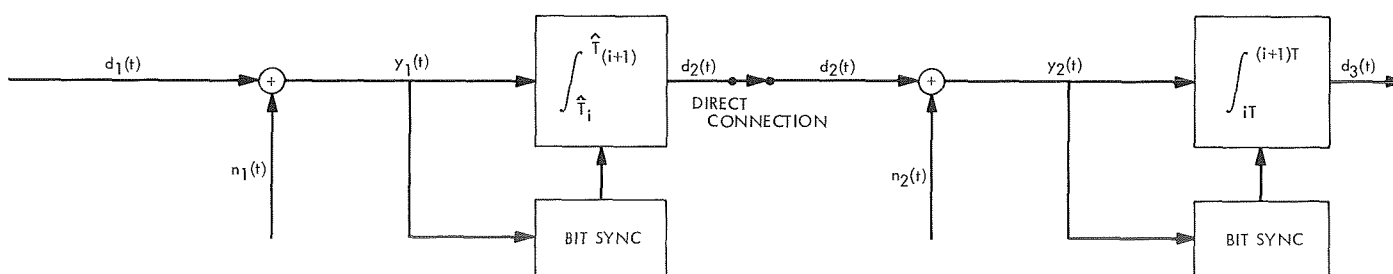


Fig. 22. Essential elements of the low-pass equivalent system

The orbiter bit synchronizer makes estimates $\hat{T}_i$ of the integration limits, and the receiver performs the test

$$\int_{\hat{T}_i}^{\hat{T}_{(i+1)}} y_1(t) dt \geq 0 \quad : \quad b_i = \pm 1$$

The resulting (jittery) data stream $d_2(t)$ is of the form

$$d_2(t) = S^{1/2} \sum_i a_i g[t, \hat{T}_i, \hat{T}_{(i+1)}]$$

where $a_i = b_i$ if no error occurs in the i th test and $a_i = -b_i$ if an error does occur. The signal $y_2(t)$ received at the ground receiver is of the form

$$y_2(t) = d_2(t) + n_2(t)$$

The ground receiver bit-synchronizer makes estimates⁵ T of the integration limits and performs the test

$$\int_{iT}^{(i+1)T} y_2(t) dt \geq 0 \quad : \quad a_i = \pm 1$$

An error occurs in the i th interval if the test produces the wrong value of a_i . The final data stream $d_3(t)$ is

$$d_3(t) = S^{1/2} \sum_i c_i g[t, iT, (i+1)T]$$

where $c_i = a_i$ if no error occurs in the ground receiver in the i th interval and $c_i = -a_i$ if an error does occur.

Under the assumptions made, it is evident that the process at the ground receiver involves detection of jittery data with a clean bit synchronizer (i.e., the zero crossings of the binary data are randomly perturbed by jitter, as well as noise, around the noise-free crossings, while the integrate-and-dump limits are exact because the bit synchronizer estimates them correctly) while at the orbiter, detection is of clean (noisy butunjittered) data with a jittery bit synchronizer (data zero crossings are not randomly perturbed by jitter, but the integrate-and-dump limits are because of imperfect bit synchronizer performance).

Intuitively, it might be argued that the individual degradations in the relay performance due to these "dual" effects

⁵Since steady-state, no-cycle-slip performance is of interest, the random variables $\hat{T}_i$ may be assumed identically distributed on $[iT - T/2, iT + T/2]$. Hence, the ground receiver will track the average of $\hat{T}_i$. Further, if it is assumed that the bit synchronizer reacts in a sufficiently balanced fashion to the input stream such that the jitter density function P is symmetric, the ground receiver will integrate over the correct interval $[iT, (i+1)T]$.

should be the same (assuming identical transmitter/receiver configurations and independent, identically distributed noise in each link). Although this is the case if complete dependence exists, it is not true in general.

3. Probability of Down-Link Error

In the presence of jitter, three cases are possible in the ground receiver in the interval $[iT, (i+1)T]$:

- (1) No data transition occurs in $[iT, (i+1)T]$. That is, either no data transitions occur at all, or the data transition is jittered out of the interval.
- (2) A single data transition occurs in $[iT, (i+1)T]$. That is, either a data transition occurs at iT and is jittered forward into the first half-interval $[iT, iT + T/2]$, or a data transition occurs at $(i+1)T$ and is jittered backward into the second half-interval $[iT + T/2, (i+1)T]$.
- (3) Two data transitions occur in $[iT, (i+1)T]$, one in each half-interval. That is, two data transitions occur: The first is jittered forward into the first half-interval and the second is jittered backward into the second half-interval.

Let S_j , $j = 0, 1, 2$, denote the events that occur in these three cases. Then the probability of error in $[iT, (i+1)T]$, denoted $P(e_i)$, is

$$P(e_i) = \sum_{j=0}^2 P(e_i | S_j) P(S_j)$$

Define

$$\mu = \frac{\hat{T}_i - iT}{T}$$

$$\nu = \frac{(i+1)T - \hat{T}_i}{T}$$

These are the (normalized) jitter distances into the interval $[iT, (i+1)T]$ from the edges. That is, $\mu > 0$ implies a forward jitter of the i th transition time and $\nu > 0$ implies a backward jitter of the $(i+1)$ th transition time. Note that μ and ν are zero-mean on $[-1/2, 1/2]$ with joint density function $p(\mu, \nu)$.

Evaluation of the three *a priori* state probabilities $P(S_j)$ is straightforward. There are 16 essential cases of interest

($a_{i+1}a_i \geq 0$, $a_{i-1}a_i \geq 0$, $\mu \geq 0$, $\nu \geq 0$, and simple arithmetic reveals that nine cases cause state S_0 , six cases yield S_1 and only one contributes to S_2 . Thus, $P(S_0) = 9/16$, $P(S_1) = 3/8$, and $P(S_2) = 1/16$. Note that these probabilities are independent of whether a_i is $+1$ or -1 , as are the probabilities $P(e_i|S_j)$ and $P(e_i)$. Thus, it suffices to assume $a_i = -1$.

Denote the down-link signal-noise ratio ST/N_{02} by R_2 , define N_i by

$$N_i \triangleq \int_{iT}^{(i+1)T} n_2(t) dt$$

and note that the N_i 's are independent, identically distributed zero-mean gaussian random variables with variance $N_{02}T/2$.

If S_0 occurs, the test becomes

$$N_i + S^{1/2}Ta_i \geq 0 \quad : \quad a_i = \pm 1$$

That is, either no data change occurs ($a_{i-1} = a_i = a_{i+1}$) or the transition occurs outside the interval. Thus, the probability of error (assuming $a_i = -1$) is

$$\begin{aligned} P(e_i|S_0) &= \text{prob}[N_i > S^{1/2}T] \\ &= \frac{1}{2} \text{erfc}[R_2^{1/2}] \end{aligned}$$

where $\text{erfc}[X]$ is, as usual,

$$\text{erfc}[X] \triangleq \frac{2}{\pi^{1/2}} \int_X^\infty e^{-t^2} dt$$

If S_1 occurs for some $\mu > 0$ (it suffices to assume the transition occurs in the first half-interval and to double the result), then the test becomes

$$\begin{aligned} &\int_{iT}^{(i+1)T} n_2(t) dt \\ &+ S^{1/2} \left[\int_{iT}^{\hat{T}_i} a_{i-1} dt + \int_{\hat{T}_i}^{(i+1)T} a_i dt \right] \geq 0 \quad : \quad a_i = \pm 1 \end{aligned}$$

or

$$N_i + S^{1/2}Ta_i(1 - 2\mu) \geq 0 \quad : \quad a_i = \pm 1$$

Thus, assuming again $a_i = -1$, the conditional error density is

$$\begin{aligned} p(e_i|\mu) &= \text{prob}[N_i > S^{1/2}T(1 - 2\mu)] \\ &= \frac{1}{2} \text{erfc}[R_2^{1/2}(1 - 2\mu)] \end{aligned}$$

and

$$p(e_i|S_1) = \int_0^{1/2} \text{erfc}[R_2^{1/2}(1 - 2\mu)] p(\mu) d\mu$$

If S_2 occurs for some μ and ν (both necessarily positive), then the test becomes

$$\begin{aligned} &\int_{iT}^{(i+1)T} n_2(t) dt + S^{1/2} \left[\int_{iT}^{\hat{T}_i} a_{i-1} dt + \int_{\hat{T}_i}^{\hat{T}_{(i+1)}} a_i dt \right. \\ &\quad \left. + \int_{T_{(i+1)}}^{(i+1)T} a_{i+1} dt \right] \geq 0, \quad a_i = \pm 1 \end{aligned}$$

and since, for S_2 , $-a_{i-1} = +a_i = -a_{i+1}$, the test becomes

$$N_i + S^{1/2}Ta_i(1 - 2\mu - 2\nu) \geq 0 \quad : \quad a_i = \pm 1$$

Thus, again assuming $a_i = -1$, the conditional error density is

$$\begin{aligned} p(e_i|\mu, \nu) &= \text{prob}[N_i > S^{1/2}T(1 - 2\mu - 2\nu)] \\ &= \frac{1}{2} \text{erfc}[R_2^{1/2}(1 - 2\mu - 2\nu)] \end{aligned}$$

and conditional probability $p(e_i|S_2)$ becomes

$$\begin{aligned} p(e_i|S_2) &= 4 \int_0^{1/2} \int_0^{1/2} p(e_i|\mu, \nu) p(\mu, \nu) d\mu d\nu \\ &= 2 \int_0^{1/2} \int_0^{1/2} \text{erfc}[R_2^{1/2}(1 - 2\mu - 2\nu)] p(\mu, \nu) d\mu d\nu \end{aligned}$$

Thus, the probability of down-link error in the interval $[iT, (i+1)T]$ is

$$\begin{aligned} p(e_i) &= \frac{9}{32} \text{erfc}[R_2^{1/2}] \\ &+ \frac{3}{8} \int_0^{1/2} \text{erfc}[R_2^{1/2}(1 - 2\mu)] p(\mu) d\mu \\ &+ \frac{1}{8} \int_0^{1/2} \int_0^{1/2} \text{erfc}[R_2^{1/2}(1 - 2\mu - 2\nu)] p(\mu, \nu) d\mu d\nu \end{aligned}$$

Note that if p is assumed independently uniform on an infinitesimally small centered region of $[-1/2, 1/2]$, the probability of error reduces to $1/2 \operatorname{erfc} [R_1^{1/2}]$, as expected.

4. Probability of Up-Link Error

Consider the 16 possible sets of circumstances that may occur in the orbiter receiver in the interval $[iT - T/2, (i+1)T + T/2]$ that affect the performance:

$$\hat{T}_i \geq iT, \hat{T}_{(i+1)} \geq (i+1)T, b_{i-1}b_i \geq 0$$

and

$$b_{i+1}b_i \geq 0$$

Figure 23 sketches examples of each of these 16 cases C_j , $j = 1, \dots, 16$. For a fixed $\hat{T}_i$ and $\hat{T}_{(i+1)}$, let μ , ν , N_i , and D be defined by

$$\mu \triangleq \frac{\hat{T}_i - iT}{T}$$

$$\nu \triangleq \frac{(i+1)T - \hat{T}_{(i+1)}}{T}$$

$$N_i \triangleq \int_{\hat{T}_i}^{\hat{T}_{(i+1)}} n_1(t) dt$$

$$D \triangleq \int_{\hat{T}_i}^{\hat{T}_{(i+1)}} d_1(t) dt$$

Let $\sigma^2 \triangleq N_0 T/2$ and $R_1 \triangleq ST/N_{01}$, note that the test becomes

$$N_i \geq -D \quad : \quad b_i = \pm 1$$

and that the N_i are independent, identically distributed zero-mean gaussian random variables of variance $\sigma^2(1 - \mu - \nu)$.

Table 3 lists C_j , and the value of D for each case. Assuming $b_i = -1$, letting

$$u \triangleq |\mu|, v \triangleq |\nu|, \beta \triangleq \frac{-D}{S^{1/2} T}$$

and

$$\alpha \triangleq \frac{EN_1^2}{\sigma^2}$$

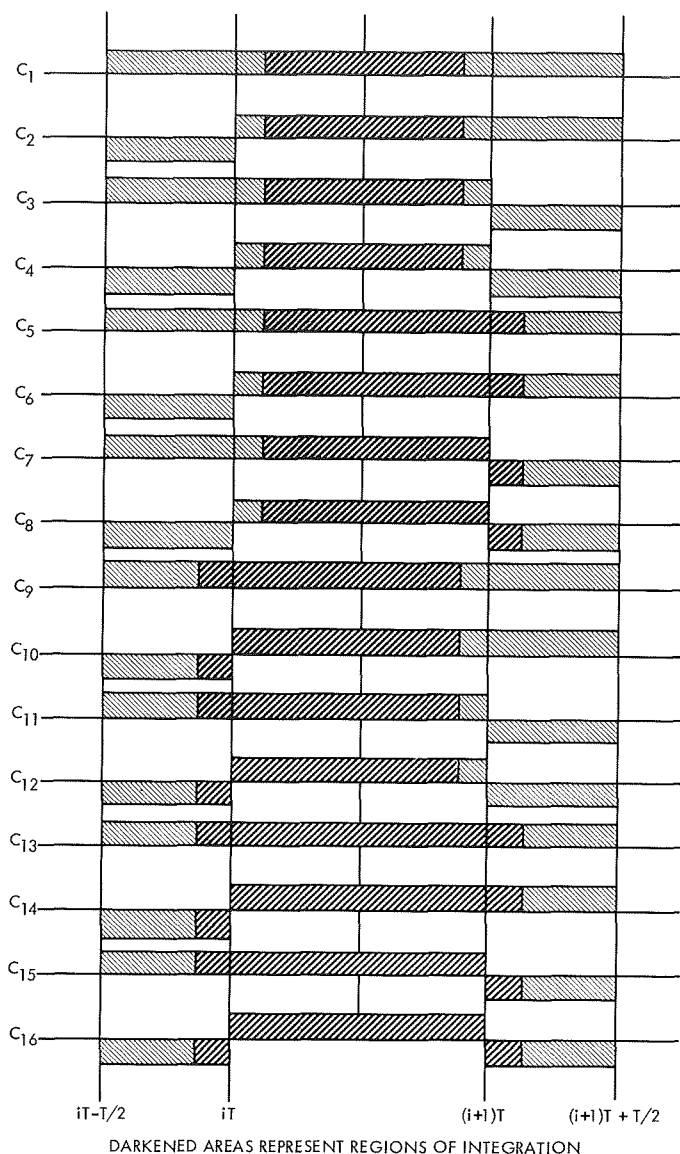


Fig. 23. Examples of the 16 cases of interest

(the 16 possible values of α and β are also tabulated in Table 3), yields for any C_j

$$p(e_i | C_j, u, v) = P[N_i > -D] \\ = \frac{1}{2} \operatorname{erfc} \left(\frac{R_1 \beta^2}{\alpha} \right)^{1/2}$$

Thus,

$$P(e_i | C_j) = 2 \int_0^{1/2} \int_0^{1/2} p(u, v) \operatorname{erfc} \left(\frac{R_1 \beta^2}{\alpha} \right)^{1/2} du dv$$

Table 3. Equation parameters for the 16 cases

Case	μ	ν	$b_i + 1 \times b_i$	$b_i - 1 \times b_i$	$D/(b_i S^{1/2} T)$	α	β
1	+	+	+	+	$1 - \mu - \nu$	$1 - u - v$	$1 - u - v$
2	+	+	+	-	$1 - \mu - \nu$	$1 - u - v$	$1 - u - v$
3	+	+	-	+	$1 - \mu - \nu$	$1 - u - v$	$1 - u - v$
4	+	+	-	-	$1 - \mu - \nu$	$1 - u - v$	$1 - u - v$
5	+	-	+	+	$1 - \mu - \nu$	$1 - u + v$	$1 - u + v$
6	+	-	+	-	$1 - \mu - \nu$	$1 - u + v$	$1 - u + v$
7	+	-	-	+	$1 - \mu + \nu$	$1 - u + v$	$1 - u - v$
8	+	-	-	-	$1 - \mu + \nu$	$1 - u + v$	$1 - u - v$
9	-	+	+	+	$1 - \mu - \nu$	$1 + u - v$	$1 + u - v$
10	-	+	+	-	$1 + \mu - \nu$	$1 + u - v$	$1 - u - v$
11	-	+	-	+	$1 - \mu - \nu$	$1 + u - v$	$1 + u - v$
12	-	+	-	-	$1 + \mu - \nu$	$1 + u - v$	$1 - u - v$
13	-	-	+	+	$1 - \mu - \nu$	$1 + u + v$	$1 + u + v$
14	-	-	+	-	$1 + \mu - \nu$	$1 + u + v$	$1 - u + v$
15	-	-	-	+	$1 - \mu + \nu$	$1 + u + v$	$1 + u - v$
16	-	-	-	-	$1 - \mu - \nu$	$1 + u + v$	$1 - u - v$

and, since all 16 cases are equally likely,

$$\begin{aligned}
 P(e_i) = & \frac{1}{2} \int_0^{1/2} \int_0^{1/2} p(u, v) \operatorname{erfc} [R_1 (1 - u - v)]^{1/2} du dv \\
 & + \frac{1}{2} \int_0^{1/2} \int_0^{1/2} p(u, v) \operatorname{erfc} [R_1 (1 - u + v)]^{1/2} du dv \\
 & + \frac{1}{2} \int_0^{1/2} \int_0^{1/2} p(u, v) \operatorname{erfc} \left[\frac{R_1 (1 - u - v)^2}{(1 - u + v)} \right]^{1/2} du dv \\
 & + \frac{1}{4} \int_0^{1/2} \int_0^{1/2} p(u, v) \operatorname{erfc} \left[\frac{R_1 (1 - u + v)^2}{(1 + u + v)} \right]^{1/2} du dv \\
 & + \frac{1}{8} \int_0^{1/2} \int_0^{1/2} p(u, v) \operatorname{erfc} [R_1 (1 + u + v)]^{1/2} du dv \\
 & + \frac{1}{8} \int_0^{1/2} \int_0^{1/2} p(u, v) \operatorname{erfc} \left[\frac{R_1 (1 - u - v)^2}{(1 + u + v)} \right]^{1/2} du dv
 \end{aligned}$$

5. Special Case

As a special case, if the successive jitters are completely dependent, the probabilities of error for the two links are *identical* in form.

For the jittery *data* situation (the down-link), if successive jitters are completely dependent, two data transitions within the interval $[iT, (i+1)T]$ are not possible (S_2 cannot occur) and S_1 and S_0 are equally likely. Thus, the probability of error becomes

$$P(e_i) = \frac{1}{4} \operatorname{erfc} [R_2^{1/2}] + \frac{1}{2} \int_0^{1/2} p(u) \operatorname{erfc} [R_2^{1/2} (1 - 2u)] du$$

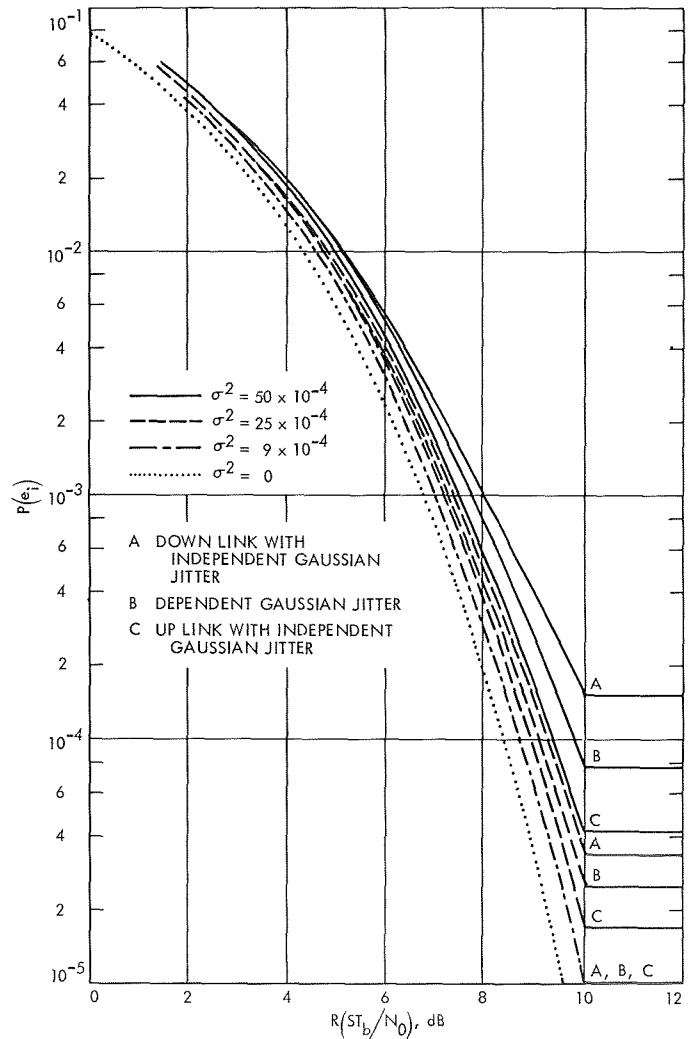


Fig. 24. Error probabilities

For the jittery *integration* situation (the up link), if successive jitters are completely dependent, the noise N_i always has variance σ^2 , and only C_5 through C_{12} are possible. Four of the cases (those involving no data transitions, i.e., C_5 , C_6 , C_9 , and C_{11}) are not affected by jitter, while the remaining four, all equally affected, have an α of $1 - 2u$, u in $[0, 1/2]$. Thus, the probability of error is

$$P(e_i) = \frac{1}{4} \operatorname{erfc} [R_1^{1/2}] + \frac{1}{2} \int_0^{1/2} p(u) \operatorname{erfc} [R_1^{1/2}(1 - 2u)] du$$

and if R_1 and R_2 are equal, the up-link and down-link probabilities of error are equal.

6. Comparison

Figure 24 shows plots of $P(e_i)$ for the up and down links assuming equal signal-to-noise ratios for the cases of complete dependence and complete independence assuming the jitter is zero-mean gaussian truncated $[-1/2, 1/2]$, with $\sigma^2 = 9 \times 10^{-4}$, 25×10^{-4} , and 50×10^{-4} . The down-link curves are meaningful measures of degradation in steady-state, no-cycle-slip performance. The up-link curves are included for comparison, but it must be remembered that the jitter variance is, in general, a function of the up-link signal-to-noise ratio as well as the orbiter bit synchronizer loop bandwidth. More meaningful up-link curves would require a detailed study of the actual bit synchronizer loop and an examination of the above functional relationship.

X. Spacecraft Power

GUIDANCE AND CONTROL DIVISION

A. Thermoelectric Properties of 80-at. % Silicon-20-at. % Germanium Alloy as a Function of Time and Temperature, V. Raag and F. de Winter

1. Introduction

Study of outer planet missions requires extrapolation of the properties, or of the operation, of many materials and parts to lifetimes far beyond those for which actual test data are available. One of the more critical materials is the silicon-germanium thermoelectric material used in the radioisotope thermoelectric generator (RTG) which supplies electrical power to the spacecraft. If this material degrades more than is estimated at the time the spacecraft design and objectives are determined, the mission will have to be curtailed accordingly. A detailed study of this problem has been performed for JPL.¹

Prediction of the time dependence of the properties of the silicon-germanium thermoelectric material requires an understanding of the mechanism of change of

these properties. The degradation mechanism in the silicon-germanium alloy is discussed below, and results are presented for the properties over a 12-yr period for the 80-at. % silicon-20-at. % germanium alloy. This material seems to be the most likely candidate for outer planet missions because of its thermoelectric "figure of merit," its high melting temperature, its strength, and the fact that the properties of the *p* and *n* legs are close to being equal. The earliest and still most complete study of the relative merits of the silicon-germanium alloys is that of Dismukes, et al (Ref. 1).

2. Silicon-Germanium Degradation Mechanism and Corresponding Effects for a 12-yr Period

At present it appears that the most important degradation mechanism by far in the silicon-germanium alloy couples involves a precipitation of the phosphorus dopant used to produce the carriers in the *n* leg. However, there may be other mechanisms involved; for example, possible long-term bond degradation, sublimation, or compatibility effects at high temperatures may exist that have remained undiscovered because all the test data are for relatively short periods. Presently, only the precipitation mechanism seems amenable to analysis; however, other effects should be investigated as soon as possible.

¹Raag, V., *Thermoelectric Properties of 80 a/o Si-20 a/o Ge Alloy as a Function of Time and Temperature*, Resalab Scientific Memorandum 6. Resalab Inc., Menlo Park, Calif., Mar. 3, 1970 (prepared for JPL).

In silicon-germanium alloy boron, a group III element, is generally used as dopant in the *p* leg of the thermoelectric couple. Each atom, with one valence electron less than the four required to form complete bonds with the surrounding silicon and germanium atoms, yields one carrier. The solid solubility of boron in the silicon-germanium alloys is more than high enough to dope the alloy to approximately 2×10^{20} carriers/cm³—the required level for optimum thermoelectric performance. The resulting alloy is stable with no precipitation problems.

The most soluble of the group V elements that can be used to produce the excess electrons needed as carriers in the *n* leg is phosphorus. However, the solid solubility of phosphorus is sufficient only at temperatures of around 1000°C to dope the silicon-germanium alloy to the required level of about 2×10^{20} carriers (or atoms)/cm³. Phosphorus has the property of retrograde solubility, with lower solubility at both higher and at lower temperatures.

At temperatures higher than that corresponding to maximum solubility, the excess phosphorus precipitates very quickly because of rapid high temperature diffusion. This is not too important, however, because only a very small part of the material operates in this region. The force for precipitation increases as solubility decreases at lower temperatures. At low temperatures, the diffusion coefficient is so low that there are no discernible effects. The worst precipitation (i.e., degradation) effects occur in the range between 300 and 700°C.

Prediction of long-term properties involves description of this precipitation mechanism. A study of this subject is given by Slyozov and Lifshitz (Ref. 2). Precipitation is postulated to occur by diffusion to nucleation centers of precipitate particles, which occur in a virtually continuous spectrum of sizes. If the radius of a precipitate particle is below a critical radius, then the surface energy is so large that it will redissolve even though it is in a supersaturated medium. This surface energy effect causes a constant adjustment of the size spectrum of the particles. An understanding of the precipitation mechanism and its long-term effects requires the description of this process.² The spectral shift and re-absorption process of the precipitate particles tends to yield similar long-term behavior for materials even when preparation techniques have been somewhat different.

The thermoelectric properties of the 80-at.% silicon-20-at.% germanium alloy were predicted³ based on the Slyozov-Lifshitz precipitation model (Ref. 2). The results are shown in Tables 1 and 2. Ekstrom and Dismukes (Ref. 3) had previously obtained good agreement for times up to 1000 h. Approximate verification of the theory has been obtained from longer time-device results. The

²Raag, V., *The Performance Characteristics of Silicon-Germanium Alloys in Thermoelectric Applications*, Resalab Scientific Report, Resalab Inc., Menlo Park, Calif. (prepared for JPL).

³Raag, V., *Thermoelectric Properties of 80 a/o Si-20 a/o Ge Alloy as a Function of Time and Temperature*, Resalab Scientific Memorandum 6, Resalab Inc., Menlo Park, Calif., Mar. 3, 1970 (prepared for JPL).

Table 1. Thermoelectric properties of phosphorus-doped 80-at.% Si-20-at.% Ge alloy

Temperature, °C	Resistivity, mΩ-cm					Seebeck coefficient, ^a μV/°C					Thermal conductivity W/°C-cm
	Initial	1500 h	1 yr	5 yr	12 yr	Initial	1500 h	1 yr	5 yr	12 yr	
0	0.90	0.90	0.90	0.90	0.90	113	113	113	113	113	0.0516
100	1.08	1.08	1.08	1.08	1.08	145	145	145	145	145	0.0495
200	1.29	1.35	1.45	1.59	1.74	176	181	187	196	206	0.0475
300	1.51	1.98	2.30	2.80	3.30	204	229	245	265	282	0.0456
400	1.75	2.99	3.74	4.50	4.90	228	279	302	322	330	0.0440
500	1.97	3.25	3.63	4.00	4.21	246	294	305	315	320	0.0427
600	2.04	2.79	3.10	3.35	3.54	254	284	294	302	307	0.0416
700	1.88	2.28	2.54	2.75	2.91	252	269	279	287	292	0.0412
800	1.61	1.84	2.03	2.21	2.32	246	256	264	271	275	0.0426
900	1.40	1.49	1.61	1.73	1.81	242	246	251	256	258	0.0459
1000	1.22	1.26	1.30	1.34	1.37	239	240	242	244	245	0.0533

^aReferenced to absolute.

Table 2. Thermoelectric properties of boron-doped 80-at. % Si-20-at. % Ge

Temperature, °C	Electrical resistivity, mΩ-cm	Seebeck coefficient, ^a μV/°C	Thermal conductivity, W/°C-cm
0	1.19	121	0.0638
100	1.32	156	0.0610
200	1.48	179	0.0585
300	1.66	200	0.0561
400	1.86	219	0.0540
500	2.06	238	0.0521
600	2.28	255	0.0506
700	2.50	271	0.0497
800	2.72	288	0.0495
900	2.95	304	0.0502
1000	3.19	320	0.0545

^aReferenced to absolute.

results are based on measurements of the electrical resistivity between 400 and 800°C for periods of up to 1200 h,⁴ during which the thermal conductivity was found to stay relatively constant. The values for thermal conductivity were based on thermal diffusivity measurements⁴ and specific heat values by Steigmeier and Kudman (Ref. 4). These values seem to be confirmed by measurements (Ref. 5), although the experimental technique has been questioned (Ref. 6).

In the near future, predicted values will be presented for the 63-at.% silicon-37-at.% germanium material that has often been proposed for RTGs. Experimental studies are in progress to investigate the degradation mechanism in depth to confirm the predictions.

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⁴Raag, V., *Research and Development of a Silicon-Germanium Alloy*, Final Report, Contract NAS-5-3410. Dec. 1963 (prepared for JPL).

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B. Spacecraft-RTG Thermal Interface Considerations, W. D. Leonard⁵

1. Introduction

The thermoelectric outer planet spacecraft (TOPS) system configuration 12J (Fig. 1) uses three radioisotope thermoelectric generators (RTGs) to supply the system electrical power. After additional information became available concerning the design and performance of the RTG, it was apparent that four RTGs would be required. The choice of mounting arrangement of four RTGs depended upon various thermal and other interface considerations. The advantages of having two RTGs mounted in tandem on each of two near-parallel axes, from a structural weight point of view, had to be balanced against the effect of the interaction of the RTGs with each other and the possible effect of interfacing with the thermal control system (TCS). These two aspects were examined in light of the RTG information currently available.

2. RTG Interaction

If two or more RTGs are mounted on axes that are near parallel, the sides of the RTGs that face each other have reduced heat rejection capability because the waste heat is rejected by radiation. The magnitude of this reduction depends upon the distance between the RTGs and the geometry of the facing sides. The curve in Fig. 2 for no reflector shows how the heat rejection capability of the facing sides is reduced as a function of the distance between the closest points of the two RTGs. At zero spacing, the adjacent sides have only about 33% of the heat rejection capability of the case where no member obstructs the view of the RTG of deep space. As the distance between the RTGs is increased to 20 in. the efficiency reaches 86%.

One method to reduce the effect of the second RTG upon the first is to interpose a member that blocks their view of each other. To be effective in this manner, the

⁵Mr. Leonard is employed at the Jet Propulsion Laboratory by Resalab Scientific (a division of Resalab Inc., Menlo Park, Calif.).

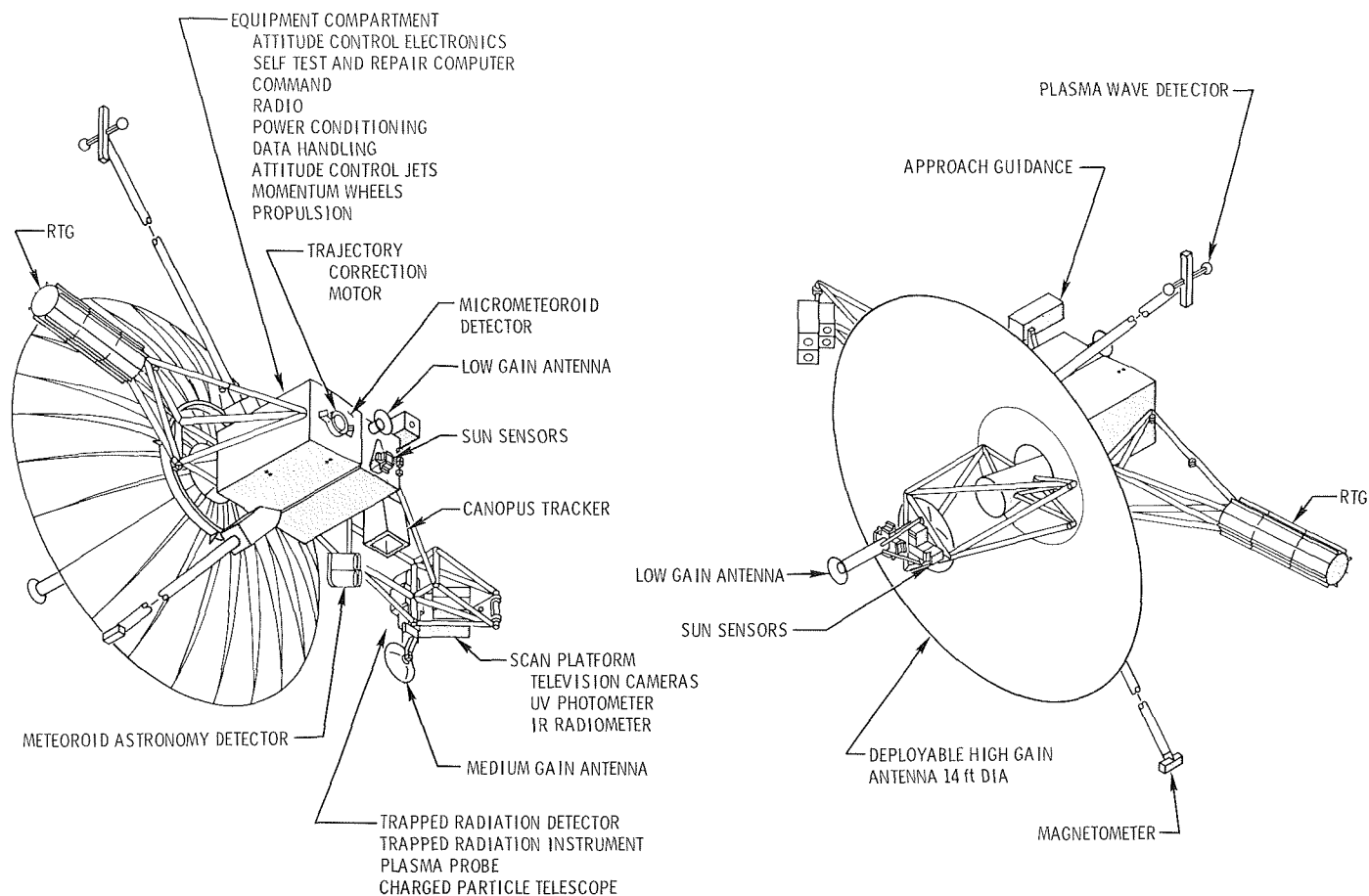


Fig. 1. Thermoelectric outer planet spacecraft (configuration 12J)

interposed member must be either a specular reflector or a radiator.

If the member has a high reflectivity, is specular, and is shaped so that a minimum of energy is reflected back to the emitting RTG, the member would appear as deep space to the RTG. Specular surfaces with reflectivity as high as 95% have been achieved. However, the ability to maintain the specularity and reflectivity for mission time in excess of 10 yr has not been established. The surface would be subjected to both solar wind and dust erosion, as well as micrometeorite damage.

From a design standpoint, it is perhaps easier to predict the long-term behavior of a reradiating surface than a reflecting surface. For this reason, several reradiating members were examined for possible use in reducing the effect of RTG interaction. The reradiator must be able to absorb and reradiate enough thermal energy to offset the shielding effect of its insertion. To do this, the reradiator must have a high absorptance and a high emis-

sivity. Its shape should be such that it has an adequate view of space without reducing the view that the RTG itself has of space. The reradiator should have high thermal conductivity so that the absorbing surfaces can easily transmit the thermal energy to the radiating surfaces.

For purposes of analysis, a hexagon-shaped reradiator was used that had the same cross section as the RTG. The effectiveness depends strongly upon the ratio of the area projected toward the RTG to the area projected directly to space. In Fig. 2, these areas are shown as A_1 and A_2 , respectively.

Area A_1 should be fixed at a value equal to or slightly less than the projected area of the RTG facing the reradiator. Larger values of A_1 reduce the direct view of the RTG to space; smaller values increase the view of one RTG of the other. As A_2 is increased, the efficiency of the reradiator increases. This effect is shown in Fig. 2 for values of the area ratio of 0.655, 1.0, and 2.0.

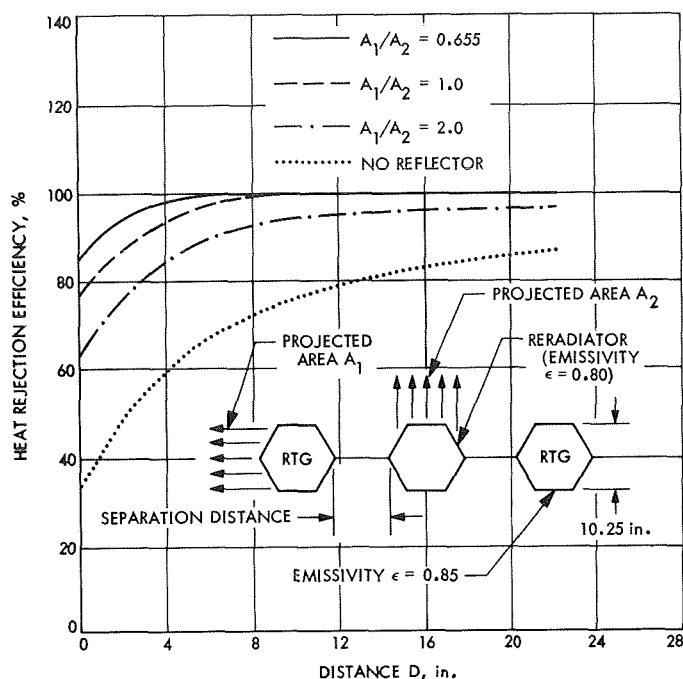


Fig. 2. Effect of RTG spacing and reradiator size on heat rejection

The primary disadvantage of a reradiator is the additional space required for its insertion. For instance, assume that the RTGs are 20-in. apart. Without a reradiator the heat rejection efficiency is 86%. If the RTG spacing remains unchanged, but a reflector is inserted that has an area ratio of 1.0 (the width of the reradiator is equal to the width of the RTG; i.e., about 10 in.) the space between the RTG and the reradiator drops to 5 in. At a distance D of 5 in. and an area ratio A_1/A_2 of 1.0, the heat rejection efficiency is 96%.

The effect of reduced RTG heat rejection efficiency on RTG performance is shown in Fig. 3. The data shown are for beginning-of-life conditions with an initial fuel loading of 2000 W. The load voltage is held at 30 V.

The most significant effect of reduced heat rejection efficiency is an increase in the cold junction temperature at the point closest to the reradiator or second RTG, as the case may be. As the heat rejection efficiency of the near side of the RTG decreases, the amount of heat transmitted through the near side decreases; this in turn increases the amount which must be transmitted through the remainder of the RTG. The net effect is an increase in all of the cold junction temperatures, in the hot junction and fuel capsule temperatures, and a slight decrease in the electrical power of the RTG.

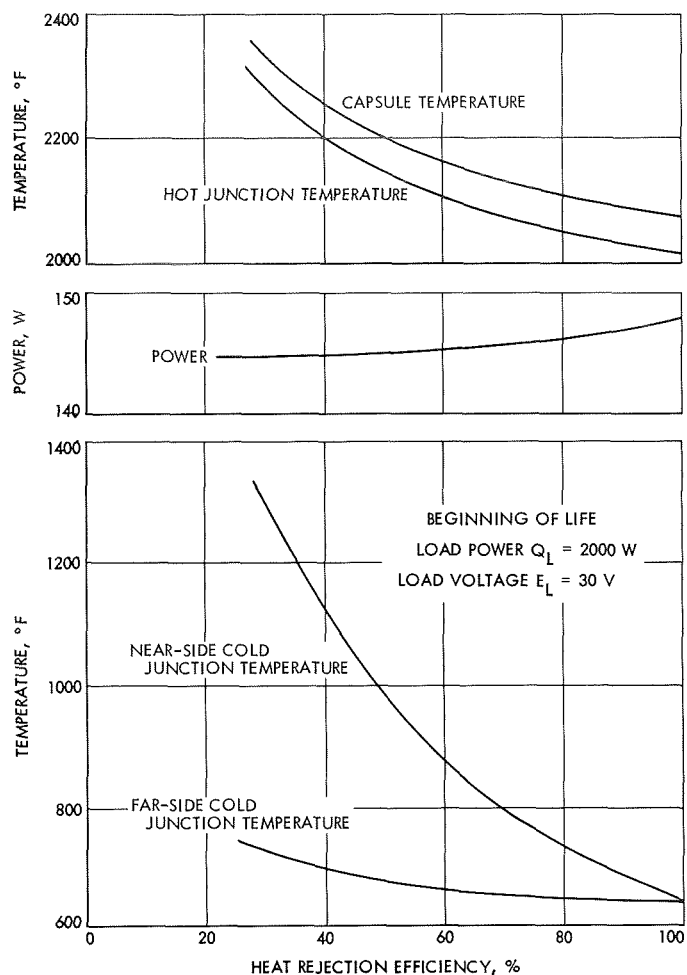


Fig. 3. Effect of heat rejection on performance

For the example cited where no reradiator is used, the near-side cold junction temperature increases by 65°F and the hot junction and capsule temperatures increase by 25°F; power would decrease by less than 1%. If a reradiator having an area ratio of 1.0 is inserted, the near-side cold junction temperature would increase by 25°F and the hot junction and capsule temperature would increase by 10°F.

The temperature at which the reradiator would operate is shown in Fig. 4. These temperatures are based upon a reradiator that has an emissivity of 0.8. In general, the temperatures vary from 200 to 600°F depending upon the spacing of the RTGs and the reradiator area ratio.

3. Thermal Control System Interface

In the TOPS system, several science packages and electronics modules require heat to maintain them at

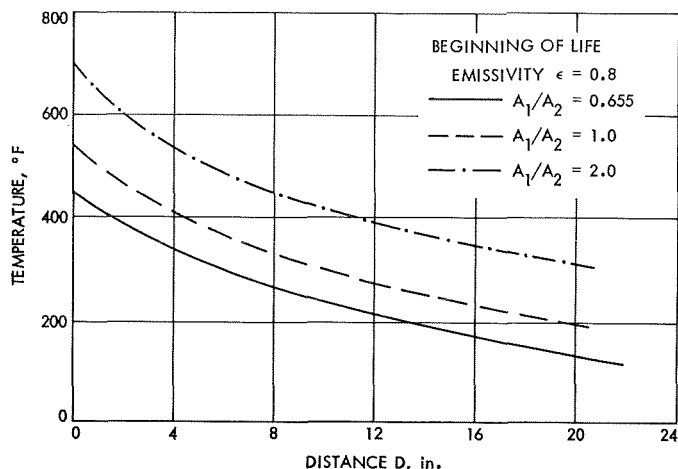


Fig. 4. Reradiator temperature vs distance

satisfactory temperatures during the life of the mission. It is the purpose of the TCS to provide this heat. It has been proposed that the TCS absorb thermal energy from the RTGs and deliver it to the various spacecraft locations by a pumped liquid loop.

To determine the impact upon the RTG of various methods of delivering the required thermal energy to the TCS, the life behavior of the RTG was determined. The temperatures of the RTG are shown in Fig. 5 as a function of mission time. The decrease in temperature with time is due to two effects: (1) the radioisotope power source decays about 10% during the 12-yr life of the system, and (2) the properties of the silicon-germanium material degrade with time. The combined effect of these two mechanisms results in about 100°F decrease in hot junction and fuel capsule temperatures and a 45°F decrease in cold junction temperature. With these temperatures as a reference point, the effect of the TCS on the RTG was evaluated.

Two primary methods were considered for acquiring heat from the RTG: radiation and conduction. Also, two sources at the RTG were considered: the waste heat processed by the thermoelements and the RTG end losses. The amount of heat needed by the TCS is 125–150 W at end of life.

a. End losses. The estimated end loss from each end of each RTG is about 50 W. About 25 W of this are conductive losses through the heat source support structure and the remainder are radiative losses through the end insulation. In the two-axis tandem arrangement where one end of each RTG will be supported by a

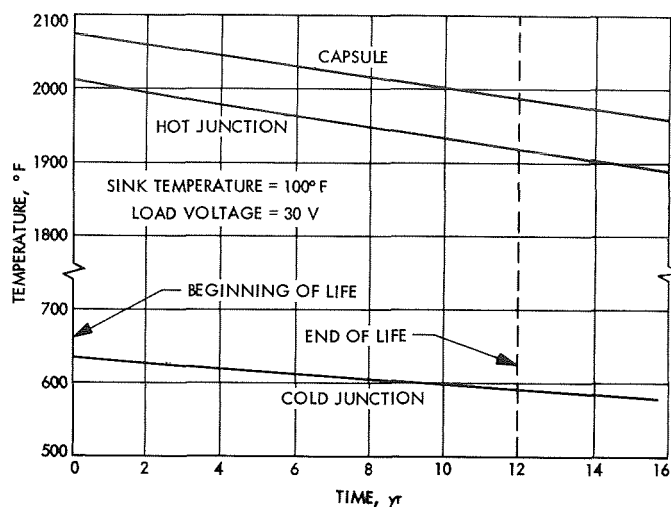


Fig. 5. Temperature of RTG as a function of time

common structure, a heat exchanger could be designed around the structure to acquire heat for the TCS. If this method were used, little perturbation on the RTG would result. Furthermore, any postulated failure in the TCS would not result in a temperature or power effect upon the RTG.

If a receptor plate were used to acquire the waste heat from the ends of the RTGs by radiation, failure of the TCS could result in increased RTG temperatures. This occurs because of the need to remove the energy previously acquired by the TCS by transmitting it through other points of the RTG. In the extreme case where all of the heat for the TCS is picked up at one end of one RTG and the TCS system fails early in life (due to a leak, for example), the hot junction temperature would increase about 100°F in the affected RTG.

b. Waste heat. If heat that has been processed by the thermoelements is used as a source by the TCS, the overall power loading of the RTG can be less (assuming that the end losses are less than the TCS requirements) than for the case where end losses are used. If the acquisition scheme uses conduction as the mode of heat transfer from the RTG to the TCS, the perturbation to RTG design would be intolerable. Thus, if the waste heat is used, the only practical method of transferring it to the TCS is by radiation.

From an RTG design standpoint, the best arrangement would be a TCS receptor that operates at the RTG design sink temperature of about 100°F. If the TCS receptor temperature is at this value, a receptor area of

about 7% of the radiating area of one RTG would be adequate. In the event of failure of the TCS, the receptors would act as a partial insulator to the RTG and, in the case where all of the TCS thermal requirement is being met by a single RTG, hot junction temperatures would increase by about 40°F. Higher receptor temperatures would result in larger receptor areas and a more significant effect on RTG temperatures in the event of TCS failure.

Because of the power decay of the isotopic fuel, the end-of-life power available for the TCS is less than at the beginning of life. Because the heat required by the TCS is greatest at end of life (in fact, the beginning-of-life thermal requirement of the TCS is probably zero), a method has to be available to regulate the temperature of the TCS if changes in input power level are not to be reflected back into the RTG. Thus, for any TCS arrangement, some kind of heat dump with an active temperature control is required. A shuttered auxiliary radiator could serve this purpose.

4. Conclusions

The analysis of the effect of RTG interaction and RTG-TCS interface upon RTG performance has indicated the following:

- (1) From a thermal standpoint, mounting the RTG on two axes results in little or no effect upon the RTGs if they are ≥ 20 -in. apart.
- (2) The use of a reflector or reradiator between the RTGs to reduce the effect of one RTG viewing a second is of minimal benefit for comparable RTG spacing. It would appear preferable from a weight standpoint to add a short fin to the affected side of the RTG rather than to interpose a reflector or reradiator.
- (3) The non-parallel, two-axis arrangement provides a convenient acquisition point for thermal energy for the TCS without a direct attachment to the RTG.
- (4) A heat exchanger for the TCS that absorbs heat by conduction from the mounting structure, rather than directly from the RTG by conduction or radiation, results in the minimal perturbation on RTG temperatures and performance.
- (5) An active heat dump is required in the TCS to control temperature and account for decay of the isotopic heat source.

C. Solar Cell Standardization, R. F. Greenwood

1. Introduction

The solar cell standardization program is a continuing effort to provide calibrated solar cells at near-zero air mass conditions. These standard solar cells are used to aid in the evaluation of solar arrays, to procure additional solar cells to specific power levels, and to assist in the evaluation of newly developed solar cells. The standard solar cells are mounted in modular form and loaded with precision resistors at the short-circuit current point. The cells are calibrated on high-altitude balloons that ascend to 120,000 ft. Formerly, balloon flights were made to an altitude of only 80,000 ft but the higher altitude was chosen to eliminate as much error as possible resulting from the residual air mass.

Through a cooperative effort between JPL and the NASA and government agencies, the balloon-calibrated standard solar cell has become widely used and recognized as a useful and accurate tool to aid in solar array and solar cell evaluation for a variety of space projects and advanced development work.

2. Cooperative Calibration Effort

Invitations to participate in the cooperative-effort solar cell standardization program have been sent to interested NASA and government agencies. Several responses to the invitations have been received. The cooperative effort permits the government agencies to obtain primary solar cell standards and use them in their respective solar cell programs. Since the inception of the cooperative effort, eight different NASA and government organizations have participated in this program and the same organizations usually continue to participate year after year.

3. Plans for the 1970 Balloon Flight Series

Negotiation of a contract providing for three 120,000-ft altitude balloon flights is in progress. The flights will be conducted during June and July in the vicinity of Minneapolis, Minnesota. Payload scheduling is now taking place. Due to the fact that the solar trackers are now six years old and have experienced severe use, extensive preventative maintenance will be performed on the two trackers prior to the scheduled flights.

Temperature control of the solar cell mounting platform at the float altitude has been a problem during past

flights and some effort was expended to design a system to cool the solar cell mounting platform. The method proposed was a system utilizing a refrigerant vented to the atmosphere. The system will not be used since there is the possibility of the vented refrigerant forming a cloud around the solar tracker and attenuating the sunlight being measured by the solar cells. Alternate methods of temperature control will be investigated.

Some abnormal performance was experienced with the balloons during the 1969 balloon flight series. The difficulty was with the balloon design in that improper descent valve size and placement did not allow sufficient helium to escape from the balloons to cause them to descend to the earth. This problem is expected to be corrected by installation of the proper descent valve size and placement in the balloons used in the 1970 series. Until the new balloon design is proven, a backup radio-controlled electric valve will be installed in the balloon apex.

D. Lightweight Integrated Silicon Solar Cell Array, R. K. Yasui

1. Introduction

The work presented here represents an extension of the theoretical studies reported in SPS 37-60, Vol. III, pp. 63-65. This program consists of the study, analysis, and development of an advanced integrated lightweight flexible silicon solar cell array. The scope of the program includes the assembly and testing of solar cells, glass coverslides, interconnectors, and substrates, individually and as a complete system. Tests have been conducted on sample assemblies to gain information on the effects of thermal stresses and flexural and tensile fatigue on soldered solar cell interconnectors.

A sample solar cell module that uses a new wrap-around interconnector design and consists of 19 cells in parallel by 20 cells in series was fabricated. This module is to be bonded to a lightweight solar panel substrate that will be environmentally and functionally tested as a part of the roll out array development program. This array program is being conducted in conjunction with General Electric and is entitled, "30 Watts Per Pound Solar Cell Array." The solar cell module shown in Figs. 6 and 7 incorporates a new wraparound interconnector concept developed at Heliotek, a division of Textron, under contract to JPL. The solar cell module specifications are listed in Table 3.

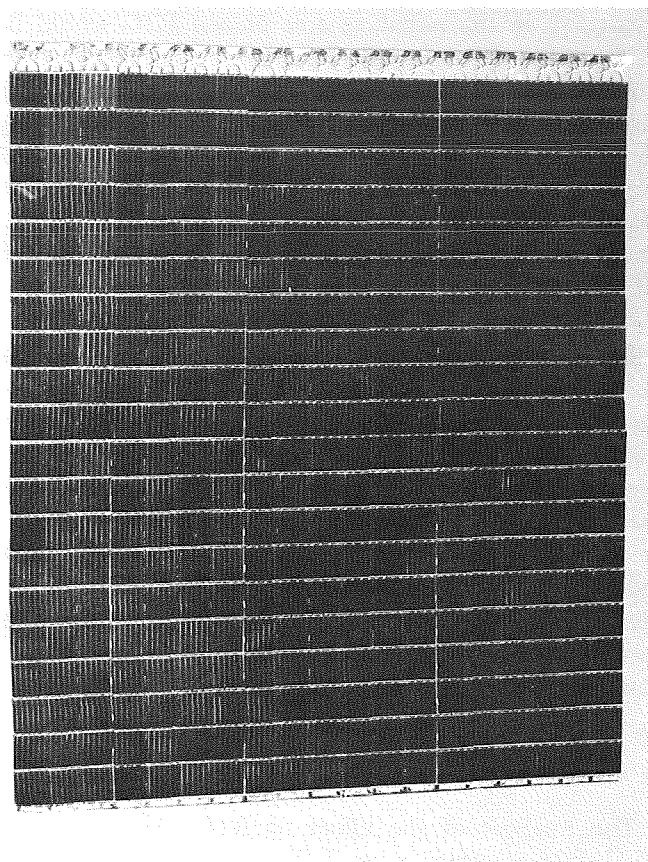


Fig. 6. Bottom view of solar cell module with wraparound interconnectors

2. Interconnector Design

The interconnector design partially shown in Figs. 6 and 7 incorporates stress-relief features that were designed to alleviate the environmentally induced thermal stresses at the interconnector-solar cell interface. The major difference between this and conventional inter-

Table 3. Solar cell module specifications

Item	Description
Solar cells	2- $\times$ 2-cm $\times$ 8-mil-thick, 2 Ω -cm, Ti-Ag solderless bar contact
Glass coverslides	3-mil-thick microsheet
Interconnect	2-mil silver plated copper
Soldering technique	Front contacts, pulse soldering, and rear contacts, resistance soldering
Module size	19 cells in parallel by 20 cells in series
Insulation (wrap-around)	1-mil glass cloth impregnated with RTV 3145 silicone adhesive

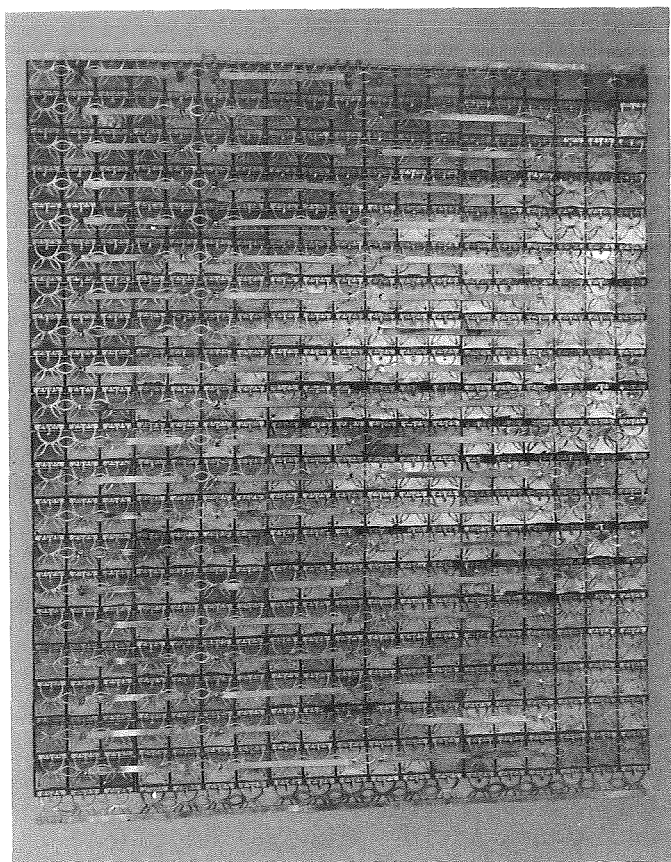


Fig. 7. Top view of solar cell module with wraparound interconnectors

connect designs is that the interconnection is primarily accomplished on the back side of the solar cell and does not require a direct electrical connection to the relatively weak top surface ohmic contact. Further, this new design interconnect allows transverse movement due to thermal expansion, without exerting large stresses at the cell and tab solder joint interface. To ensure against potential cell shorts across the semiconductor junction, a small amount of adhesive was applied to the back side of the cell opposite the ohmic strip. A strip of 1-mil woven glass cloth was placed in the adhesive and additional adhesive was applied to the glass cloth and the interconnector was then rolled around the edge of the cell. The assembly was then placed between sheets of Teflon-coated Kapton and a pressure of approximately 0.5 lb/in.² was applied and the assembly then cured under room ambient conditions for 48 h.

3. Thermal Cycling

Thermal cycling tests were begun on interconnector solar cell samples to provide failure rate data to aug-

ment the theoretical interconnector failure analysis previously reported to JPL. At present, the test samples consist of copper tabs, 0.050-in. wide, soldered to 14-mil-thick silicon solar cells.

Interconnector thicknesses under examination are 8, 6, 4, and 2 mils. Two interconnectors of each thickness were soldered to each of four solar cells, one on the *n* contact and the other on the *p* contact. The resulting 16 solar cells (32 interconnectors) have begun the thermal cycling phase. The first phase consists of cycling between ambient and -196°C (liquid nitrogen) with a dwell period at -100°C for temperature stabilizing to reduce any thermal shock. A similar sequence of tests will consist of cycling from ambient to -100°C . The result of these tests will be correlated to stress vs interconnector curves to provide information on safe and non-safe thermally induced stress regions.

4. Flexure/Tensile Test

The effect of thermal motions on interconnector fatigue was examined. It was noted that vibrational motions could be treated once the vibrationally induced motions and frequencies were defined. However, such knowledge is dependent on a complete definition of not only the vibrational environment, but also of the array configuration. Some empirical data that had been determined by earlier tests at Ryan Aeronautical Company in San Diego on a roll-out type array showed failure-inducing vibrations that produced a change in the cell spacing distance of approximately 4 mils. With this number, Heliotek has assembled a test apparatus that will bend and stretch a solar cell module polyamide substrate system around a drum to provide a similar cell spacing change. Calculations indicated that a 1-mil-thick polyamide substrate is needed to survive a force of the order of 12 lb/lineal in.

A test apparatus was designed and built that consisted of an eccentric cam on a motor that produces a cyclic vertical motion to a 6-in.-diam drum. This drum in turn pushes against the fixed module/substrate to induce the required loading. The actual loading is measured by examination of the deflection of the springs that fix one end of the polyamide substrate to the test fixture. The spring calibration has indicated that a total spring deflection of 0.050-in. will be produced by a total force of 36 lb. This will equal the required 12 lb/lineal in. across the 3-in.-wide substrate. Measurements of the cell spacing have further indicated an actual spacing change of approximately 3 to 5 mils.

E. Radiation Damage in Lithium-Doped

Silicon, P. A. Berman

1. Introduction

One of the major concerns in the use of silicon solar cells for deep space missions is the fact that significant decreases in power can occur when the cells are subjected to irradiation by electrons and protons. Within the past few years, work on solar cells with lithium dopant has indicated that cells can be fabricated that exhibit considerable annealing of radiation damage at temperatures of 28°C or greater. The objective of this program is to study the properties of lithium-doped silicon (without the additional complicating effects of a *p-n* junction) as a function of electron and neutron irradiation, and to investigate the applicability of lithium-doped silicon for the radiation hardening of silicon solar cells. The investigations are being carried out by Gulf General Atomic, under contract to JPL.

2. Irradiation Studies

The investigations undertaken in this program are being performed on lithium-doped bulk silicon, rather than on solar cells, to obtain information on the basic interaction between radiation-induced defects, lithium, and silicon. The use of bulk silicon circumvents the more complex problems associated with the analysis of solar cell radiation degradation because it eliminates such factors as high field effects in the depletion region, large lithium concentration gradients near the junction, and possible degradation of the cell electrical contacts.

Two samples of lithium-diffused, float-zone silicon were irradiated by 30-MeV electrons and examined for changes in minority carrier lifetime τ . The first sample had a lithium concentration of 4.5×10^{14} Li/cm³. The initial τ of 55 μ s was reduced to 1 μ s by a fluence of 2.2×10^{13} e/cm². The degradation constant decreased from 7×10^{-8} to 3×10^{-8} cm²/e-s at fluences greater than 1×10^{13} e/cm², possibly because of lithium depletion or room temperature annealing during the irradiation. These constants are lower than those previously measured for more highly lithium-doped (10^{16} – 10^{17} Li/cm³) silicon and are similar to those obtained for silicon without lithium doping. Annealing at 355°C and 380°C resulted in an increase in τ to 40 μ s, which indicates annealing of 90% of the irradiation-induced recombination centers. The sample was re-irradiated to a total fluence of 6.9×10^{13} e/cm², which resulted in the creation of a severe trapping center that could not be removed by annealing at 430°C for 66 h. The second sample had a higher lithium concen-

tration of 1.5×10^{16} Li/cm³ and was irradiated and annealed three times. After a total fluence of 4×10^{14} e/cm² and a 30-min anneal at 370°C and 420°C, 80% of the radiation-induced recombination centers were annealed. The degradation constant for heavily lithium-diffused silicon seems to be greater than for lightly, or non-lithium-diffused silicon. Annealing is inhibited as fluence is increased; the higher the lithium concentration, the greater the fluence required to inhibit recovery. These observations strongly indicate that lithium is depleted during production and annealing of radiation produced recombination centers.

Two 3.7- Ω -cm, lithium-doped, float-zone silicon samples were irradiated by neutrons to fluences up to 6.75×10^{10} n/cm² with energy greater than 10 keV. Minority carrier lifetime was monitored for both isothermal and isochronal anneals. More than 90% of the recombination centers were annealed at temperatures of about 400°C. The degradation constant was 6.4×10^{-6} cm²/e-s, independent of the total fluence, and is quite similar to that observed in non-lithium-diffused silicon. The annealing kinetics appear to be first order. The activation energy was of the order of 0.67 eV, which suggests that the anneal is due to lithium diffusion because it is quite near the activation energy of 0.66 for diffusion of lithium in silicon. The effective frequency factor of 10^7 s⁻¹ indicates long-range migration.

The temperature dependence of minority carrier lifetime τ was investigated to establish the density and energy levels of recombination centers. Two techniques were used to measure τ , the photoconductivity decay method, and the steady-state photoconductivity method. All samples were fabricated from initially high purity (10^4 Ω -cm), float-zone, *n*-type silicon having a τ of 10^3 μ s. Lithium was introduced to obtain samples ranging from 0.4 to 11 Ω -cm resistivity. The samples were irradiated by 30-MeV electrons. The temperature dependence of the pre-irradiation τ indicated a recombination center in the highly lithium-doped silicon (0.4 Ω -cm) deeper than 0.17 eV from either band edge. The degradation constant of this material after irradiation was found to be twice that of non-lithium-containing silicon of equivalent resistivity, indicating that the lithium is participating in the production of radiation-induced recombination centers. After a high fluence of 4×10^{14} e/cm², annealing at 370°C did not result in complete recovery, while at lower fluences complete recovery was obtained.

The analysis of the more lightly lithium-doped (11 Ω -cm) silicon indicated that the pre-irradiation τ was

determined by at least two centers, one located approximately 0.17 below the conduction band, and the other deeper than 0.35 eV from either band edge. Furthermore, at least two kinds of recombination centers were introduced in this material by the irradiation, one at $E_c - 0.17$ being dominant at temperatures above 150–200°K and the other, deeper than 0.35 eV from either band edge, being dominant at temperatures below 150–200°K. The low temperature center was not significantly annealed in 1 h at 390°K, but at least partially annealed over longer times at room temperature (more highly lithium-doped silicon annealed completely at room temperature). The high temperature center was significantly annealed in 1 h at 380–400°K.

3. ESR Measurements

Electron spin resonance (ESR) measurements can yield additional detailed information about radiation-induced defects and under favorable conditions may separately determine the centers associated with τ degradation. Therefore, ESR measurements were investigated as a tool for the determination of the mechanisms for production and annealing of the divacancy in lithium diffused *n*-type silicon irradiated with 30-MeV electrons.

The divacancy, which is a predominant defect center in oxygen-lean silicon, is believed to have three electrical levels within the forbidden energy gap. The position of the Fermi level with respect to these levels determines whether or not the divacancy will be paramagnetic in the single negative charge state and therefore responsive to ESR measurements. The resistivity of the small ESR samples must be known very accurately to determine the Fermi level position. Also, correction factors have been obtained to compensate for the fact that the samples are small compared to the four-point resistivity measurement probes.

The first ESR experiments were performed on float-zone silicon samples containing approximately 10^{16} Li/cm³ that were irradiated at room temperature to fluences of 1×10^{16} , 2×10^{16} , and 3×10^{16} 30 MeV *e*/cm² and stored in liquid nitrogen after irradiation. No resonances were observed in any of the samples when searched over several magnetic field directions and over a range of 60 G. This indicated that either the divacancies had a lower introduction rate than expected, because of the effect of lithium (so that they were below the noise level of the measurements), or significant annealing had occurred during the irradiation. To eliminate the latter possibility, three more samples containing 10^{17} Li/cm³ were irradi-

ated in small fluence steps to fluences of 2.8×10^{17} , 3.2×10^{17} , and 4.0×10^{17} 30 MeV *e*/cm² at 77°K and subsequently stored at this temperature.

Examination of these samples at 20°K in the ESR spectrometer again indicated no paramagnetic resonances. Either the introduction rate of the divacancies was still too low, or they were non-paramagnetic. If the former is true, and it is believed that this is at least partially the case, this implies that the divacancy introduction rate at 77°K is much lower for lithium-doped silicon than for phosphorus-doped silicon. With regard to the possibility that the divacancies were not in a paramagnetic state, the resistivity of Sample 4 was determined after irradiation and it was found to have fewer than 10^{12} carriers/cm³, which locates the Fermi level so far into the forbidden gap that nearly all the divacancies would be in the neutral charge state and hence not paramagnetic at the 20°K ESR measurement temperatures. Thus, the difficulty of obtaining an ideal fluence becomes obvious. If the fluence is too low, not enough divacancies are formed to be observed above the noise, while if the fluence is too high, the Fermi level is too low to provide the right charge state for the divacancies to be observed through ESR. It is, therefore, quite easy to miss this relatively narrow window.

4. Infrared Absorption Studies

To obtain further information on the divacancy and the reason for the absence of a resonance in the ESR measurement, some preliminary infrared absorption studies were made. Infrared absorption is less sensitive than ESR and requires a minimum of 10^{16} defects/cm² for the observation of bands. The technique, however, has the advantage of not requiring the divacancy to be in a particular charge state for observation of the divacancy absorption band. The sample studied had received the highest fluence of all samples considered (4×10^{17} *e*/cm²) and was investigated after it had remained at room temperature for several days. The spectra obtained indicated no absorption based at or near the expected wavelength of 1.8 μ m, leading to the conclusion that fewer than 10^{16} divacancies/cm³ were present. There was, however, an indication of a band at 1.4- μ m wavelength that may be the result of a lithium-divacancy interaction associated with the room temperature anneal.

5. Neutron Activation Analysis

Neutron activation analysis has been used in an attempt to determine the lithium density in silicon and to

compare this with the amount of electrically active (singly ionized) lithium as determined by electrical resistivity and Hall measurements. The equipment was capable of detecting as little as 10^{-9} g of lithium, which is about an order of magnitude less than the quantity of lithium measured in the experiments on highly lithium-doped silicon. There was a limitation of sample size due to the equipment geometry and hence the low lithium content samples were subject to experimental uncertainties. For the low lithium content samples, the activation analysis results agreed with those obtained from electrical resistivity measurements. For the high lithium content samples, however, the activation analysis estimates ranged from values equal to the resistivity estimates to values much larger than indicated by the resistivity measurements. This leads to the conclusion that lithium might exist in highly lithium-doped silicon as precipitates, and that under certain conditions, this precipitated lithium may become available for participation in radiation-induced defect annealing.

F. Model Pore Electrode Studies, G. L. Juvinall

1. Introduction

Studies of the electrode surface reactions involved in the oxidation and reduction of the alkaline silver electrode are an important part of the JPL battery advanced development effort. A fundamental understanding of these reactions is important to the continuing effort to design and fabricate more reliable, versatile spacecraft batteries. This understanding is also necessary for the proper interpretation of the inflight battery data obtained during the course of JPL/NASA missions. The portion of the battery advanced development effort described here was performed at Brigham Young University under contract to JPL.⁶ Other work included in this task has been described previously.⁷

2. Discussion

Electrolyte resistance as a function of pore size and the penetration of silver oxides into the pores of silver electrodes have been studied by means of model pore electrodes. Earlier work has utilized model pore electrodes composed of bundles of 7-19 silver wires in holders of both circular and hexagonal design. The circular electrode design shown in Fig. 8 is easier to construct, and can be made with more accuracy than

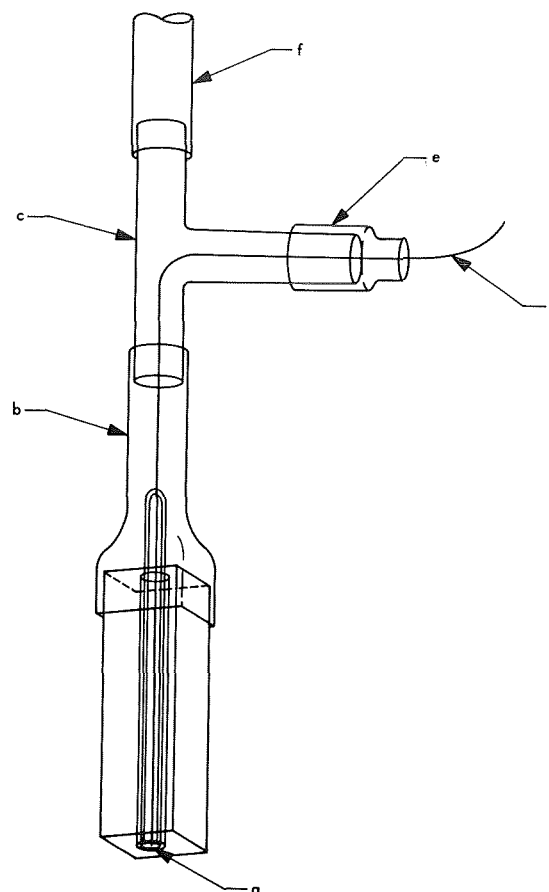


Fig. 8. Model pore electrode assembly

the hexagonal design. In addition, the rectangular plexiglass body of the electrode holder permits quantitative measurements of silver oxide penetration into the pores to be made with a cathetometer.

The hole [(a) in Fig. 8] is made by drilling the rectangular portion of plexiglass with a steel drill at low speed. A short length of Tygon tubing (b) connects the plexiglass body with a polyethylene T-joint (c). A silicone rubber seal was placed on the plexiglass-Tygon joint. The bundle of wires is fused to a silver wire (d) that exits through a septum cap (e) on the stem of the T-joint. Another length of Tygon tubing (f) is attached to the T-joint to facilitate electrode rinsing and meniscus height adjustment. Five electrodes of this design were constructed.

The electrode was immersed in the electrolyte to a precalculated depth that allowed all the electrodes to have the same current-to-area ratio in contact with the electrolyte. The electrode remained in the electrolyte during the oxidation-reduction cycling and was removed

⁶Dr. Eliot Butler is the principal investigator.

⁷SPS 37-39, Vol. IV, pp. 19-22; SPS 37-43, Vol. IV, pp. 66-69; SPS 37-53, Vol. III, pp. 47-49; SPS 37-56, Vol. III, pp. 119-120; and SPS 37-59, Vol. III, pp. 109-111.

only after a series of cycles. The reference electrode was a commercial calomel electrode. The growth of the oxides in the pore was followed with the aid of a cathetometer, which also was used to measure accurately the length of electrode immersed in electrolyte. A total of over 250 oxidation-reduction cycles were made in these experiments.

3. Results

Representative constant-current oxidation curves are illustrated in Fig. 9. The oscillations observed between P_1 and P_2 were present in each oxidation. Small in number and magnitude at the beginning of a series of cycles, their size and number increased quickly during the first 10–15 cycles and gradually thereafter. After a continuous series of about 30 oxidation-reduction cycles, a maximum of seven oscillations was observed. Through a series of from 2 to 5 cycles, an oscillation peak appeared first as

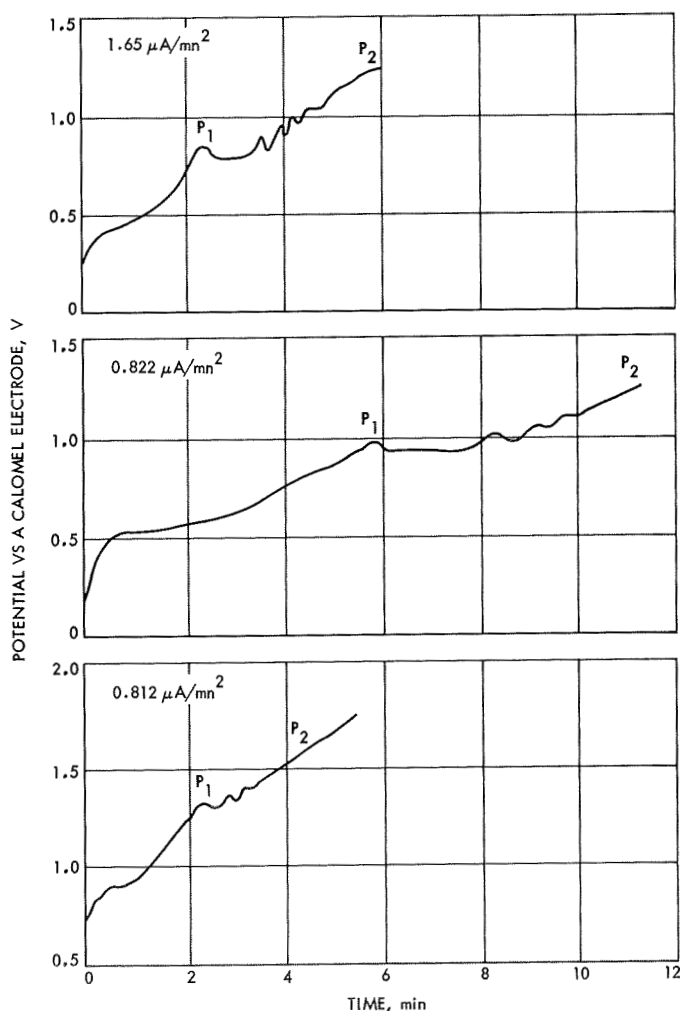


Fig. 9. Representative oxidation curves

an inflection, then a plateau, and finally in the shape shown in the bottom plot of Fig. 9.

In Fig. 10, the potential of the peak of the first oscillation achieved during oxidations at the aforementioned currents is plotted against the pore cross sectional area of each of the five electrodes. Only a small change in potential is observed between the two largest areas. However, as the area decreases, the slope of the curve increases in steepness.

The growth of the silver oxides into the pore was followed by means of a cathetometer. Figure 11 shows the maximum depth of penetration of each electrode after it was oxidized for 3 min at a current per surface area exposed of $1.65 \mu\text{A}/\text{mm}^2$. The oxide penetration values are plotted against the pore cross sectional area.

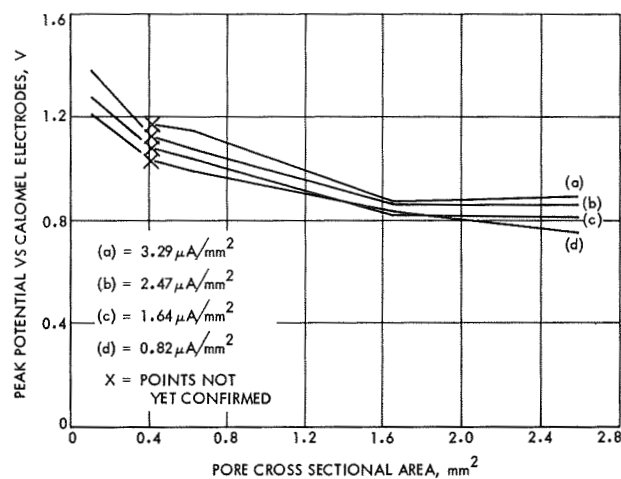


Fig. 10. Electrode potentials vs pore cross-sectional area

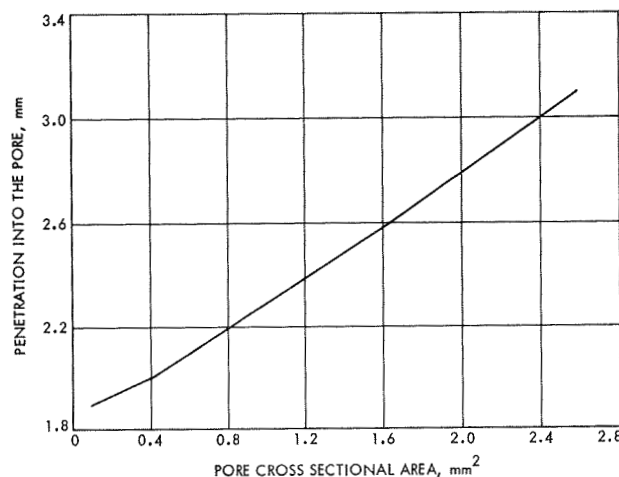


Fig. 11. Oxide penetration vs pore cross-sectional area

4. Conclusions

The oscillations that have been observed in all oxidations of the model pore electrodes appear to be related to the potential oscillations observed by Dignam, Barrett, and Nagy (Ref. 1) in the course of their studies of planar electrodes. The reaction producing Ag_2O proceeds smoothly up the pore in the constant-current oxidation. The potential required to keep the current constant increases nearly linearly as is expected because of the electrolyte resistance increase as the reaction gets farther into the pore. Eventually the potential at the open end of the pore, not subject to the same IR drop as that deep inside the pore, is high enough to cause oxidation of Ag_2O to AgO . If the electrolytic oxidation process is regarded as essentially crystal nucleation accompanied by an appreciable nucleation overpotential, followed by crystal growth, then at constant current in a pore an oscillation of potential should occur (Ref. 2). The potential increases until nucleation can occur, but then the current can be kept constant by a lower potential. The process is repeated as crystal nuclei form deeper in the pore, at successively higher potentials.

The increase in magnitude and number of peaks with cycling seems to indicate a change in crystal structure of the Ag_2O formed on the electrode.

It has been observed that if the electrode is left open-circuit in the reduced state in contact with the electrolyte for about 12 h or more, it tends to revert to its original behavior (fewer and smaller oscillations). These results suggest that the newly reduced silver may be in a metastable crystalline configuration. Aging or maturing of the electrode allows a more stable configuration to develop.

References

1. Dignam, M. J., Barrett, H. M., and Nagy, G. D., *Can. J. Chem.*, Vol. 47, p. 4253, 1969.
2. Fleischman, M., Lax, D. J., and Thirsk, H. R., *Trans. Faraday Soc.*, Vol. 64, p. 3128, 1968.

G. Development of a Long-Life, High Cycle Life, 30-A-h, Sealed AgO-Zn Battery,

R. E. Patterson, and R. S. Bogner

1. Introduction

To capitalize on the inherent advantages of the AgO-Zn system, JPL is currently engaged in development activities for improving its stand-life and cycle-life

characteristics. ESB, Inc., is presently under contract to JPL to develop a 30-A-h high cycle life, sealed AgO-Zn battery capable of performing at least 100 24-h cycles at 50% depth of discharge after a 6-mo wet-charged stand. The cell designs and test plan for the two-phase development program were discussed in SPS 37-59, Vol. III, pp. 121-124. This article discusses test results of the Phase I cells through 60 50% depth of discharge cycles and of the Phase II cells through 4 mo of charged stand.^s

2. Phase I Development Cells and Tests

a. Design 7. Design 7 was added to Phase I to test the effect of relative order of components wrapped on negative plates when compared to Design 5. The relative order of wrapping for both Designs 5 and 7 is shown in Table 4.

b. Cycling tests before charged stand. The first cycle discharge data and the second cycle charge data are presented in Table 4 for each of the seven cell design groups. Design 5 was the only group that failed to give satisfactory performance. During the first two cycles, it was found that the seven designs have unequal capability of supporting charge at 1.5 A and discharge at 15 A. For this reason, a two-step charge, two-step discharge was performed that improved minimum cell performance and, therefore, reduced the spread between minimum and maximum cell performance (Table 5).

c. Nondestructive tests during charged stand. Cell open-circuit voltages were read and recorded twice monthly during the 4-mo charged stand. All cell voltages were between 1.85 and 1.86 V at the end of the 4-mo stand. Following this period, ac impedances of cells were measured and compared to the values observed before and after formation charge. All cells showed a large increase in ac impedance from formation charge to the end of the 4-mo charge stand.

d. Cycling performance through 60 autocycles. Following the 4-mo charged stand, the cells were placed on automatic cycle control at 50% depth of discharge, 22-h charge, 2-h discharge. Design 5 cells failed to deliver 50% of rated capacity on their first autocycle. Depth of discharge for this group changed to 33% and cycling was successful. End-of-discharge voltage is decreasing

^sFarris, C. D., *High Cycle Life Sealed Silver Oxide-Zinc Cell Development*, Report E-16-70, ESB, Inc., Sept. 14, 1969 to Feb. 14, 1970 (prepared for JPL).

Table 4. Phase I: 70 cell experimental design^a

Parameter	KOH 90% saturated with ZnO, %	ZnO/Ag weight ratio	Wrap order, (+) to (-)	Positive wrap		Negative wrap				Combined
				1 EM470 4 FSC	1 EM470 4 RAI2291 2 FSC	1 EM476I 6 RAI2291	1 EM476I 6 RAI2291	1 EM470 2 FSC ^c 4 RAI2291	1 EM470 4 RAI2291 2 FSC	
Contour of negative plate Wedge ^b	45	1.2:1	1							3 RAI2291
			2							1 EM476I
			3							4 RAI2291
Standard	45	1.2:1	A-h ^d				40.6			
			V ^e				1.46			
			A-h ^f				40.6			
Standard	41	1.2:1	A-h	38.5	40.1		41.8	15.1	37.1	
			V	1.47	1.45		1.47	1.40	1.48	
			A-h	41.4	39.7		46.6	23.3	44.0	
Standard	41	1.2:1	A-h	38.5	40.1			15.1	32.7	40.6
			V	1.48	1.46			1.40	1.47	1.47
			A-h	38.9	40.2			21.6	38.9	39.8
Standard	41	1.4:1	A-h							40.6
			V							1.47
			A-h							39.8
Design pack pressure, oz/in. ²				80	40		40/2	80	12	12
Design number				2	3		1	5	7	6
Discharge rate, A				15	15		15	15	14	8
Discharge test end voltage, V				1.25	1.25		1.25	1.25	1.30	1.30

^aMean variable: cycle 1 discharge capacity/midpoint voltage/cycle 2 charge capacity.

^bThicker at plate top; pack pressure value less overall.

^cFSC was positioned with the smooth side toward the positive plate.

^dCycle 1 discharge capacity.

^eMidpoint voltage during cycle 1 discharge.

^fCycle 2 discharge capacity.

Table 5. Phase II: cycles 1-2 performance mean values

Parameter	Design number (sample size: 19)					
	2	3	4	5	6	7
Plate wrapped	+	+	-	-	Both	+
Order of separation, + to -	1 EM470 4 FSC	1 EM470 2 FSC 4 RAI	1 EM476I 6 RAI	1 EM470 2 FSC 4 RAI	1 EM476I 3 RAI 1 EM476I 4 RAI	1 EM470 4 RAI 2 FSC
Cell nominal capacity, A-h	25	28	31	28	28	28
Cell pack pressure, oz/in. ²	80	40	6	40	12	40
Cell S/N	1-19	23-41	45-63	67-85	89-107	111-129
Cycle 1						
Charge Formation, A-h						
7 mA/in. ² to input or 2.1-V first cell	37.8	42.4	41.2	42.2	39.3	40.6-45.1
7 mA/in. ² to 1.97-V first cell after seal	1.6	1.3	7.3	—	6.3	5.1-0.6
Total	39.4	43.7	48.5	42.2	45.6	45.7
Impedance (ac), before formation, mΩ	74	50	38	48	34	50
Impedance (ac), after cycle 3 charge, mΩ	39	38	42	57	47	47
Discharge, A-h						
52 mA/in. ² to 1.30-V first cell	35.8	39.6	46.4	30.7	40.3	44.0
Mid-voltage during discharge, V	1.52-1.53	1.48-1.51	1.50	1.46-1.50	1.48-1.50	1.50-1.51
Cycle 2						
Charge, A-h						
Step 1 to 2.0-V first cell	35.1	38.6	49.0	29.6	47.6	29.6
Step 2 to 1.98 V/cell	2.6	2.3	1.0	4.4	2.2	9.1
Total	37.7	40.9	50.0	34.0	49.8	38.7
Discharge, A-h						
52 mA/in. ² to 1.30 V/cell	39.2	38.6	47.6	40.9	42.2	38.7
Mid-voltage during discharge, V	1.50-1.52	1.49	1.48-1.50	1.48-1.50	1.46-1.50	1.49-1.50
Cycle 3						
Charge, A-h						
Step 1 to 2.0-V first cell	28.3	35.9	33.5	30.6	31.4	31.8
Step 2 to 1.98 V/cell	7.1	2.3	5.9	7.5	7.0	5.4
Total	35.4	38.2	39.4	38.1	38.4	37.2
Discharge current (52 mA/in. ²), A	6.4	7.2	8.0	7.2	7.2	7.2
Charge current (7 mA/in. ²), A step 1	0.86	0.97	1.08	0.97	0.97	0.97
Step 2 (4 mA/in. ²)	0.50	0.55	0.62	0.55	0.55	0.60

on all groups except Design 2 (performing at 1.50 V/cell) while end-of-charge current is gradually increasing. To determine the degree of cell degradation as a function of cycles performed, a full depth cycle (100%) is performed on each group every 25 cycles. Fig. 12 presents capacity decay through 50 cycles as determined by three

full depth cycles per group. Design 2 cells have the least capacity loss while Design 3 cells (41% KOH) have the greatest capacity loss through 50 cycles. Groups with greatest capacity loss correspond to groups with greatest fall-off of end-of-discharge voltage. Extrapolation of the data presented in Fig. 12 indicates that Designs 1 (wedge

CODE	KOH, %	ZnO/Ag WT RATIO	PACK PRESS., 2 oz/in.	ORDER WRAP, + TO -	PLATE WRAP	DESIGN	CODE	ZnO/Ag WT RATIO	PACK PRESS., 2 oz/in.	ORDER WRAP, + TO -	PLATE WRAP	DESIGN
●	45	1.2:1	80	1 EM 470 4FSC	+	2	□	1.2:1	2	1 EM 476I 6RAI	-	1 45% KOH
×	41						■					
○	45	1.2:1	40	1 EM 470 4RAI 2FSC	+	3	◇	1.4:1	12	1 EM 476I 6RAI	-	4 41% KOH
◻	41							1.2:1	2			
△	45	1.2:1	80	1 EM 470 2FSC 4RAI	-	5 ^a	◆	1.4:1	12	3 RAI 1 EM 476I 4RAI	BOTH	6 41% KOH
▲	41							1.2:1				

^a DESIGN 5 DISCHARGED DURING CYCLING (100% AND 50% DOD) AT 5.0 A RATHER THAN 7.7 A USED ON OTHER DESIGNS.

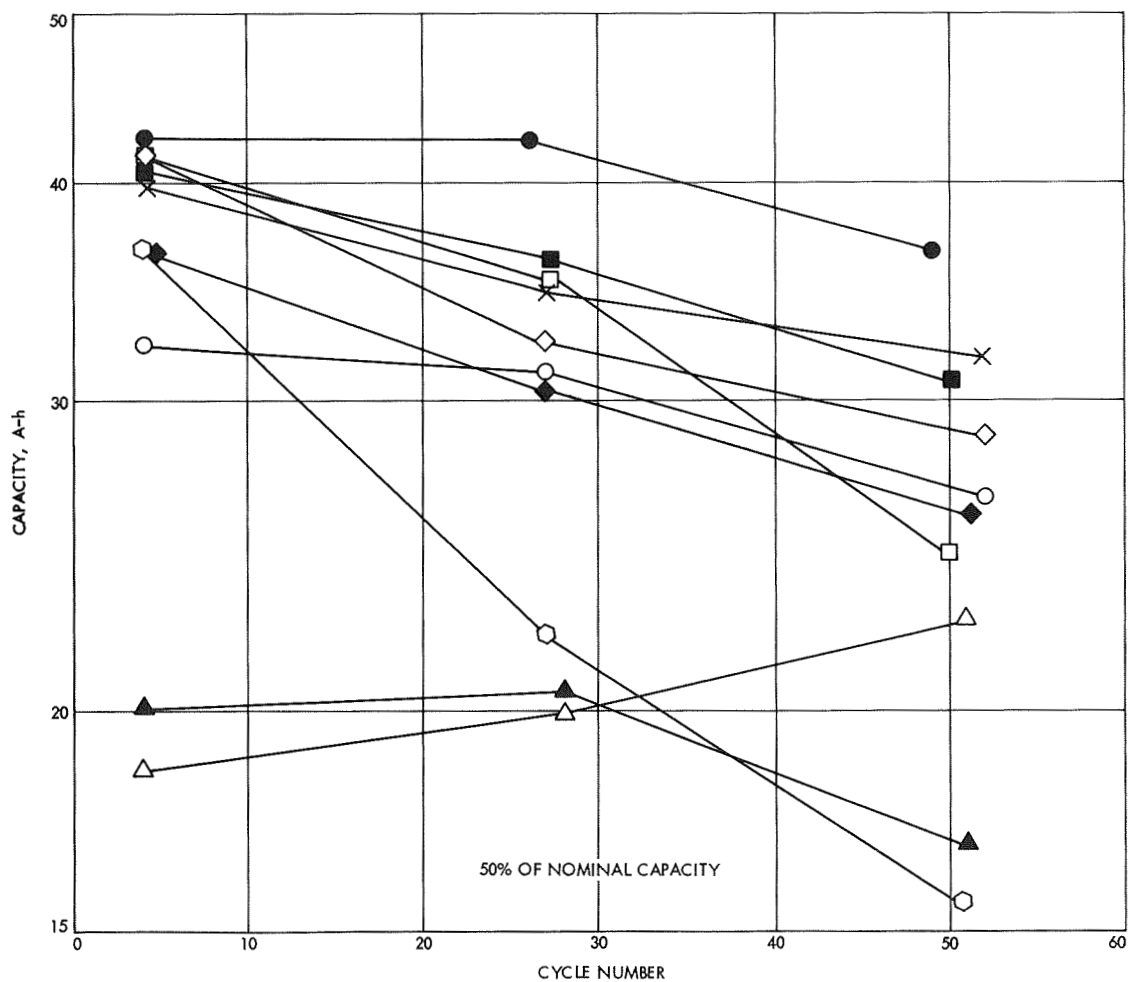


Fig. 12. High cycle life AgO-Zn battery capacity vs cycle number

negative), 2, 3 (45% KOH), 4, and 6 will satisfactorily perform 90 50% depth-of-discharge cycles.

3. Phase II Development Cells and Tests

a. Initial performance before charged stand. Mean performance data through cycle 3 is presented in Table 5 for each of six Phase II designs. Cycle 1 discharge capacities were comparable to those of Phase I (Tables 4 and 5). Phase II, Design 5 capacity data indicate that the reduction in pack pressure from 80 to 40 oz/in.² (Phase I to Phase II, Design 5) was beneficial.

b. Nondestructive tests during charged stand. Following Cycle 3 recharge, cells were placed on charged stand at 70 ±3°F. The open-circuit voltage of a Design 4 cell dropped to 1.57 V after 3 mo of charged stand while all other cells have remained above 1.85 V through 4 mo of charged stand. Upon completion of the stand period, the cells will be cycle tested.

4. Conclusions

In conclusion, the following were determined:

- (1) Two-step charging (10 mA/in.² to 1.97 V, then 5 mA/in.² to 1.97 V) and two-step discharging (100 mA/in.² to first cell to 1.25 V, then 33 mA/in.² to 1.25 V/cell) is effective in reducing cell-to-cell performance variability.
- (2) Improved performance of Design 7 cells at 12 oz/in.² compared to Design 5 cells at 80 oz/in.² (see Table 4) gives evidence that 80 oz/in.² pack tightness in a negative wrap is excessive.
- (3) Design 2 cells give the best performance of the Phase I cells.
- (4) At the cell level, Phase I, Designs 1 (wedge negative), 2, 3 (45% KOH), 4, and 6 appear to be capable of satisfying the design goals.

H. Heat-Sterilizable Battery Development,

R. Banes

1. Introduction

A program to develop sterilizable Ag-Zn cells is being conducted in conjunction with ESB, Inc., under contract to JPL. The program is divided into two main sections: one for sterilizable cells and the other for sterilizable, high-impact-resistant cells. The first section contains developments of a 70-A-h primary cell capable of 4 100% cycles, a 10-A-h primary cell capable of 4 100% cycles,

a 25-A-h secondary cell capable of 90 60% cycles, and a 48-A-h secondary cell capable of 400 60% cycles. The other section comprises developments of a 5-A-h primary cell capable of 4 100% cycles and a 25-A-h secondary cell capable of 90 60% cycles after sustaining a high-impact shock of 4000 g.

2. Discussion

The status of these developments is discussed in the summary that ensues.

a. 70-A-h primary cells. Data have been obtained for a battery containing six heat-sterilized, 70-A-h primary cells of a modified design. The battery was tested in vibration following a 6-mo open-circuit stand. The vibration levels for the test that was done in each of the three axes are given in Table 6.

After the wet stand that simulated the cruise portion of a spaceflight and the vibration testing, the battery delivered the full rated capacity.

In another test, control cells of this same type that had not been heat sterilized were deep discharged following five electrical cycles and then heat sterilized. Of the three cells so treated, only one survived the heat sterilization without leakage. The other two had been modified to allow visual observation of the cell during test. These modifications made the cells marginal for heat sterilization. The one cell that survived gave 0.35 A-h/g of silver on the cycle following the heat sterilization. This can be compared with 0.33 A-h/g for the cycle just prior to sterilization.

The conclusions from these data are that: (1) There is now available a high energy density primary 70-A-h cell

Table 6. Vibration levels for 70-A-h primary cells

Sinusoidal ^a (1/2-octave/min sweep)		Random ^b	
Frequency, Hz	Level, g (rms)	Frequency, Hz	Level, g ² /Hz
5-17	(0.5-in. double amplitude)	20-2000	0.315
17-50	5.0		
50-100	15.0		
100-2000	35.0		

^aIncreasing frequency sweep only.
^bTotal wide-band acceleration level was 25 g (rms). Duration of test was 9 min in each axis.

capable of providing satisfactory electrical performance after heat sterilization, up to 6-mo open-circuit stand, and severe vibration testing. (2) Electrical performance of this cell is satisfactory when heat sterilization follows five electrical cycles. (Previously, it was necessary to sterilize the unformed cells.)

b. 25-A-h secondary cells. Additional data have been obtained for the 25-A-h secondary cell (90 cycles of 60% depth of discharge), which is the prototype of cells being evaluated for the *Viking* lander. Five cells, with plate packs potted into the bottoms of the cell jar, were cycled following 8-mo open-circuit stand. The extra potting was a modification intended to provide vibration support. The capacity loss rates of these cells were 1.1%/mo or less, which is normal for this cell without the additional potting. All losses were recovered on the next 100% depth-of-discharge cycle. The conclusion from these data is that the added potting, even immersed in the electrolyte solution, has no detrimental effect on capacity during open-circuit stand.

3. Concluding Remarks

The design of impact-resistant cells incorporates a Zr sheet in the positive plate and an etched Ag sheet in the negative plate. A new fabrication technique has been developed for the negative electrode. This will eliminate the 325°C sintering procedure which was shown to reduce the tensile strength of the Ag sheet by 72%. A computer program has been developed to evaluate designs for shock survival and is being used to evaluate the present design. This evaluation will be compared with cell shock test results.

A study of the variables in the design of an extended life cell (400 cycles at 60% depth of discharge) is being done using a fractional factorial experiment. In cycle tests conducted to 100% depth of discharge, only the ratio of negative-to-positive active material has been shown to be significant up to 53 cycles. The cells having the median ratio used in the experiment (1.2:1.0) had better capacity retentions than those with ratios of 0.9:1.0 and 1.5:1.0. The development of this cell will be continued to investigate other design parameters.

I. Integrated Toggle Relay Driver, C. W. T. McLyman

1. Introduction

The objective of the integrated toggle relay driver task is to increase reliability and to reduce the weight and

volume of spacecraft power-conditioning hardware. There are a number of areas of circuit and functional element technology which, if improved, could greatly affect overall power system size and performance for future *Mariner* class, long-life spacecraft.

2. Relay Drivers

Relay driver circuits are in particular need of improvements in reliability. In the *Mariner* Venus 1973 power subsystem, there are 20 identical relay drivers (one is shown schematically in Fig. 13). Each driver consists of the following 30 discrete components:

- (1) Dual Transistors (1)
- (2) Capacitors (4)
- (3) Diodes (12)
- (4) Resistors (12)
- (5) Relay (1)

In the *Mariner* Mars 1969 spacecraft, the relay drivers occupied a 2.5- × 5-in. area on the circuit board, excluding the relay. On the *Mariner* Mars 1971 spacecraft, changes were required in the power conditioning electronics that necessitated new packaging concepts to meet the increased component packaging density requirements. Welded module construction was used in the *Mariner* Mars 1971 power conditioning electronics for the relay drivers to meet volume constraints. Figure 14 shows both the *Mariner* Mars 1971 welded module relay driver and the integrated circuit version described herein.

A program was started to further reduce the size and increase the reliability of the relay drivers. Because there are 20 identical relay drivers, they were ideal for thick-film technology. It was decided to keep all capacitors external to the thick-film circuit because of the special screening they require.

The stress level on the components in the thick-film design is very low because all of the components are mounted on an alumina substrate which is a good heat conductor. This distributes temperature uniformly throughout the device and eliminates hot spots. The circuit used in the thick-film design is shown enclosed by dotted lines in Fig. 13. Figure 15 shows both the discrete components used in the *Mariner* Mars 1971 module and the integrated-circuit package for a comparison of the relative sizes.

3. Current Status

Three of the integrated-circuit units have been built and are undergoing life and performance testing. If the testing continues successfully, a second version of the integrated circuit will be designed and fabricated in which the semiconductor elements are all quad-redundant. This should further increase the reliability of this critical spacecraft element.

J. Long-Term Life Test of the Spare Mariner Venus 1967 Power Subsystem

Hardware, A. Krug

1. Introduction

The startup of the long term life test of the spare Mariner Venus 1967 power subsystem has begun, following a delay from the expected mid-December start caused by two failures within the flight hardware.

2. Failure 1

The first failure was discovered during the initial checkout of the contractor-prepared test procedure. The indication was an apparent out-of-tolerance condition on the 400-Hz, 3-phase inverter output when the Gyros On command was given. An oscilloscope check of the 400-Hz inverter output disclosed a distorted waveform with a frequency of approximately 600 Hz. A quick review determined that the failure was not associated with any testing to that point. The detailed failure analysis performed by the contractor isolated the failure to a feedback diode (1N459A) used in the 3-phase divider of the power synchronizer. This circuit divides the 2.4-kHz signal by 6 and provides the A, B, and C phases to drive the 3-phase inverter. The part analysis performed by JPL disclosed an overstress defect beneath the gold bond probably caused by an electrical overstress.

3. Failure 2

The second failure was noted in the final testing preparatory to the start of the life testing. This failure was characterized by the inability to obtain the proper telemetry voltage output on the battery voltage telemetry transducer. The output of the transducer was 0.4 V with all input voltages between 30 and 40 V. The output should have varied between 0 and 3 V in proportion to input voltages between 23 and 40 V. The detailed failure analysis by the contractor determined that two diodes of the same type (1N459A) as found in the first failure were

at fault. The part analysis performed by JPL again disclosed an overstress defect beneath the gold bond.

4. Discussion

Efforts to isolate the cause of failure have been unsuccessful to date. The two circuits in which the failures occurred are electrically unrelated, and are physically located in separate cases of the Mariner Venus 1967 power subsystem. The contractor's failure analysis on the first failure indicated a possible source of the electrical overstress. However, testing of an identical module at JPL has failed to confirm this overstress. There is no apparent reason for the overstress in the battery voltage telemetry circuit. The effort to find the cause (or causes) of the failures is continuing.

The flight hardware was repaired after each of the failures and retested. The long term life testing was begun in the middle of March 1970. No further anomalies in the flight hardware have been noted.

K. RTG Radiation Test Laboratory, R. W. Campbell and M. Reier

1. Introduction

In SPS 37-52, Vol. III, pp. 46-47, the need and overall objectives for a radioisotope thermoelectric generator (RTG) radiation test laboratory were discussed. The construction, facilities, and capabilities of the laboratory constructed to meet these needs were presented in SPS 37-59, Vol. III, pp. 124-127. This article will report on the overall progress of the program carried out in this facility since its initial assembly and early experimental work.

2. The Laboratory

Both a *Standard Operating Procedure* and a *Safety Analysis Report* on the use of SNAP-19 or SNAP-27 fuel capsules in the laboratory have been written and have received all necessary JPL approval. Copies have been printed and forwarded to the Albuquerque operations office of the Atomic Energy Commission (AEC).

A decision was made to receive the SNAP-27 fuel capsule assembly under a no-cost contract with the AEC rather than through the AEC-NASA interagency agreement as originally proposed. It is anticipated that the final arrangements for this contract will be worked out

shortly. It is at this time that we should expect final approval of the *Standard Operating Procedures* and *Safety Analysis Report*.

3. Facilities

a. Simulated PuO_2 source. The design of a radioactive source to simulate the radiation from a 1575-W (thermal) PuO_2 fuel capsule assembly has been reported in SPS 37-59, Vol. III, pp. 111-115. The sources required to simulate fuel that is 2 yr old since processing have been purchased and received. Depleted U^{238} and lead plates have been fabricated to simulate the self-shielding of the fuel and the attenuation of the fuel capsule container. The completed simulated source has been used for both fresh fuel (no Th^{228} in the simulation) and 2-yr-old fuel (10 mCi of Th^{228} in the simulated source) measurements. Orders have been placed for additional Th^{228} in 20- and 40-mCi quantities so that used singly or together, fuel ages of 0, 2, 3, 4, 5.5, 7, 9, and 13 yr can be duplicated directly.

b. NE-213 scintillation spectrometer for neutrons. The calibration of the NE-213 liquid organic scintillation spectrometer for neutrons has now been completed by Battelle's Pacific Northwest Laboratory. The system uses gamma-ray discrimination based on pulse rise time and a block diagram of the system is shown in Fig. 16.

With the use of this electronics system, the response of both 1.5- and 2-in.-diam NE-213 scintillators was mea-

sured for Van de Graaff generated mono-energetic neutrons. Pulse-height spectra have been obtained for both scintillators. Each scintillator was calibrated with its circular plane perpendicular to and parallel to the neutron beam, and at 10 different values of neutron energy. These values ranged from 0.73 to 5.0 MeV at 0.5-MeV intervals. All spectra were obtained with the lowest possible electronic bias level. The response was then determined as the integral of each pulse-height spectrum above bias levels corresponding to selected electron equivalent energy values.

Calibration of the electron equivalent energy scale was based on the location of the Compton edge for the Cs^{137} , 0.662-MeV gamma-rays. The location of the half-peak edge was set equal to 1.04 times the energy of the Compton edge. Linearity and zero of the whole system (with the exception of the photomultiplier tube and base) was checked with a pulse generator at the input of the double delay line amplifier. With 256 channels for full range, the linearity and zero were both within one channel. The neutron flux was monitored by a standard long counter.

Compromises were required between efficient neutron-gamma discrimination and the settings of low-level discriminators. For the smallest pulses (in a spectrum from mixed sources), the ability to distinguish between neutron and gamma pulses deteriorates. If only large pulses were accepted, the neutron-gamma discrimination could be

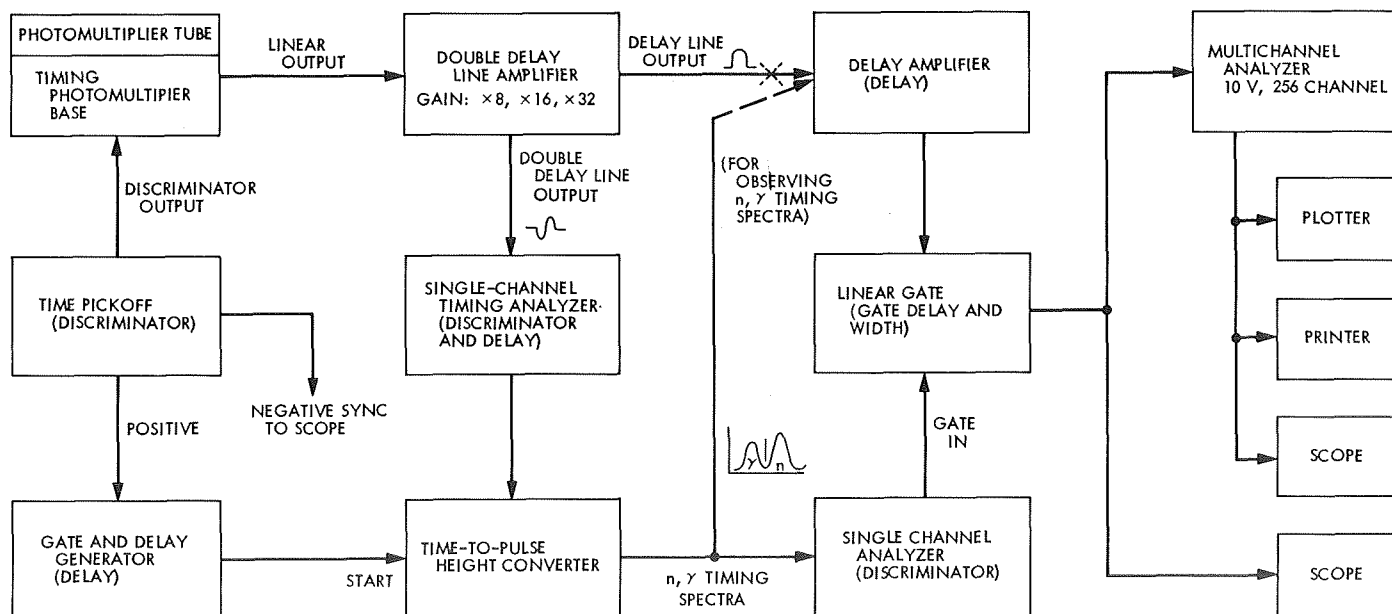


Fig. 16. Neutron spectrometer

quite sharp, but no data were obtained for the smaller pulses. If the discriminators were set as low as desired (by other criteria), the neutron-gamma discrimination deteriorated somewhat. Considerable emphasis was placed on achieving low bias levels with acceptable neutron-gamma discrimination. (One measure of the neutron-gamma discrimination was to note the fraction of Cs^{137} gamma events counted despite gamma discrimination. For the bias settings adopted, this fraction was about 1%. This value is also a crude estimate of the neutron events not counted.)

A small computer program was written to integrate the proton recoil spectra for each neutron energy above specified bias levels (e.g., 50, 75, 100, 150 keV electron equivalent energy).

4. Experiments

a. Spectrum measurements of SNAP-15A heat source. The gamma-ray data from a SNAP-15A source have been completely analyzed. The data in the vicinity of the photopeaks were analyzed by least-squares fit techniques to a Gaussian and a straight-line function. This technique linearizes the function by expanding it in a Taylor Series with respect to each of the parameters and truncating the series after the linear terms. This was done using a computer code called GAUSS III (obtained from Phillips Petroleum, Idaho). All of the corrections to the data have been applied and the errors have been calculated. These corrections included: (1) detector efficiency, (2) source to detector distance, (3) source container attenuation, and (4) source self-absorption. In general, the intensities of the unattenuated lines are in excellent agreement with the published results on Tl^{208} , Pb^{212} , and Bi^{212} , and daughter products that result from the decay of the impurity, Pu^{238} . It is felt that the Pu^{238} data represent a significant improvement over previously published measurements.

b. New technique for plutonium impurity concentrations. The gamma-ray measurements of the SNAP-15A heat source discussed above, have led to a new technique for the determination of the Pu^{238} impurity at the time of processing. The intensity of the gamma rays that result from Th^{232} decay may be used to calculate the amount of Pu^{238} originally in the sample if it is known when it was processed. The 2.615-MeV line was used since its intensity (100% of Tl^{208} decays) and the α, β branching ratio of Bi^{212} appear to be very well established. In addition, the source self-shielding corrections are smallest for that line.

The value obtained was 1.79 ± 0.07 parts/ 10^6 of Pu^{238} in the fuel at the time of processing. This agrees with the estimate of 1–2 parts/ 10^6 by Mound Laboratory. With this technique, it is felt that a Pu^{238} concentration in the 1–2-parts/ 10^6 level can be measured with an accuracy of better than 10% even if the fuel is only several months old since the time of processing. This capability is particularly important in a flight program where accurate knowledge of the impurity content of the delivered fuel must be known to assess its long-term radiation characteristics.

c. Photon response of solid-state detectors. Photon response measurements have been conducted for all of the solid-state detectors. Lithium-drifted silicon detectors of depletion depths of 1, 2, 3, 5 mm, and totally depleted surface-barrier silicon detectors of thicknesses of 30, 100, 300, 500, and 1000 μm were investigated. Of these, the 3-mm and 1000- μm detectors proved to be too noisy to allow useful data to be taken. Limited work has been done with the 30- μm detector because of its extremely low gamma sensitivity.

Energy response calibration of the detectors was made on the basis of Hg^{203} spectra taken in a vacuum chamber. The 194- and 264-keV conversion electron peaks of Hg^{203} showed prominently in all cases.

Radioisotopic sources used and their associated energies are:

- (1) $\text{Hg}^{203} - \gamma = 0.279$ MeV.
- (2) $\text{Cs}^{137} - \gamma = 0.662$ MeV.
- (3) $\text{Mn}^{54} - \gamma = 0.835$ MeV.
- (4) $\text{Co}^{60} - \gamma_1 = 1.173$ MeV, $\gamma_2 = 1.332$ MeV.
- (5) $\text{Na}^{24} - \gamma_1 = 1.369$ MeV, $\gamma_2 = 2.754$ MeV.

Because these detectors are very transparent to gamma rays (but opaque to electrons), shields were used to remove electrons emitted by the gamma sources and arising from gamma-ray interactions in the surrounding material. Only gamma rays plus electrons from gamma interaction in the shield itself are counted; these two events must be separated analytically.

Each of the response measurements for a given detector and gamma source were made for four different kinds of absorber materials shielding the detector. The

absorbers consisted of aluminum, graphite, Lexan, or lead sheets and were approximately 2×3 in. Thicknesses were set at the range of the maximum-energy electron generated by a particular source. Sensitivities above 50, 75, 100, and 150 keV were determined and were found to cover a range from 5×10^{-3} to 20×10^{-2} counts/gamma-cm², including counts of electrons from the shield. Some problems still exist with these data and complete results will be obtained when comparisons can be made with a calculational effort.

Response measurements with the totally depleted and lithium-drifted silicon detectors exposed to 0.73- and 1.38-MeV mono-energetic neutrons were completed. The efficiency of the detectors for counting 0.73-MeV neutron events as a function of increasing depletion depth varied from about 5×10^{-4} to about 10^{-2} counts/n-cm² for a 75-keV bias. In general, the neutron efficiency seems to be about two orders of magnitude less than the gamma efficiency for the neutron energies used so far. Total neutron fluences for the exposures have been kept under 10^7 n/cm². These efficiencies seem high and are still being investigated.

d. Geiger-Mueller counter. The gamma-ray response of an EON-6213 Geiger-Mueller tube for mono-energetic gamma-rays transversely incident to the counter axis was determined. Gamma-ray energies varied from 0.514 to 1.27 MeV and yielded counts per incident gamma ray of from 8.5×10^{-4} to 20×10^{-2} , respectively. An additional response point at 2.7 MeV will be determined shortly. An attempt is now being made to predict these results analytically. Gamma-ray interactions in the counter are being treated by Monte Carlo techniques and the resultant electrons followed into the counting chamber by the use of range-energy relations. Once an electron had entered the chamber, it was assumed to be counted with 100% efficiency. The inadequacy of range-energy relations in the treatment of this problem is recognized and an improved treatment is being developed. The 100%

counting efficiency for electrons in the chamber is undoubtedly high, particularly for a small 0.3×0.09 -in. counting chamber where wall and end effects play such an important part. A more suitable electron counting efficiency is under investigation. The initial calculations result in higher gamma counting rates than would be expected from these two effects.

e. Channeltrons. Experimentation with Bendix CEM-4010 Channeltrons has been delayed considerably because the vacuum envelopes containing the Channeltrons were found to be quite gassy. This caused considerable after pulsing of the tubes. Plans are now being made to use them in a continuously operating vacuum system.

f. SNAP-27 gamma-ray spectrum. The gamma-ray spectra obtained from a SNAP-27 fuel capsule are now being analyzed. Compton scattering in both the fuel and the capsule gives rise to a large gamma-ray continuum as does the spontaneous fission process. Photopeak analysis of data obtained with a Ge(Li) spectrometer system does not, therefore, give us adequate information. Several approaches are being used to acquire the additional information concerning the continuum. A first approach uses a photopeak analysis of the line spectrum only and converts this information to number of gamma-ray lines emitted by the fuel itself. This is done by correcting the data for counter detection efficiency and source-to-detector distance and then calculating the appropriate fuel self-shielding and capsule attenuation. One can then use this information to normalize the entire gamma spectrum emitted by the fuel and from this compute the continuum spectrum as emitted from the fuel capsule.

A second approach will use an unfolding or stripping technique to treat both the line and the continuum spectrum. A code for such an analysis is being developed for JPL by NUS Corp. Spectra obtained from a NaI(Tl) spectrometer will also be unfolding with a similar code written by NUS for the Goddard Spaceflight Center.

XI. Guidance and Control Analysis and Integration

GUIDANCE AND CONTROL DIVISION

A. Support Equipment for a Strapdown

Navigator, R. E. Williamson

1. Objectives

The objectives of this effort were to provide a test van, data acquisition equipment, prime power, test support, and system engineering for a strapdown electrostatic gyro aerospace navigator (SEAN). The objectives were completed in February 1970 with the conclusion of the test program for the SEAN system.

2. Support Equipment

a. Van system. The van (Fig. 1) was used for over-the-road testing to provide a dynamic environment for the SEAN system. Several modifications were made to the van for installation and operation of the SEAN equipment. The modifications included: wiring the van for 60- and 400-Hz power; installation of air conditioning, lighting, seating, intercom, altimeter port, and optical sighting ports; and fabrication of structures for mounting the equipment in the van. Shock and vibration isolators were provided for the equipment consoles.



Fig. 1. Test van

The installation of the SEAN system and support equipment in the van is shown in Fig. 2. The view is looking from the front towards the rear. At the front is the inertial

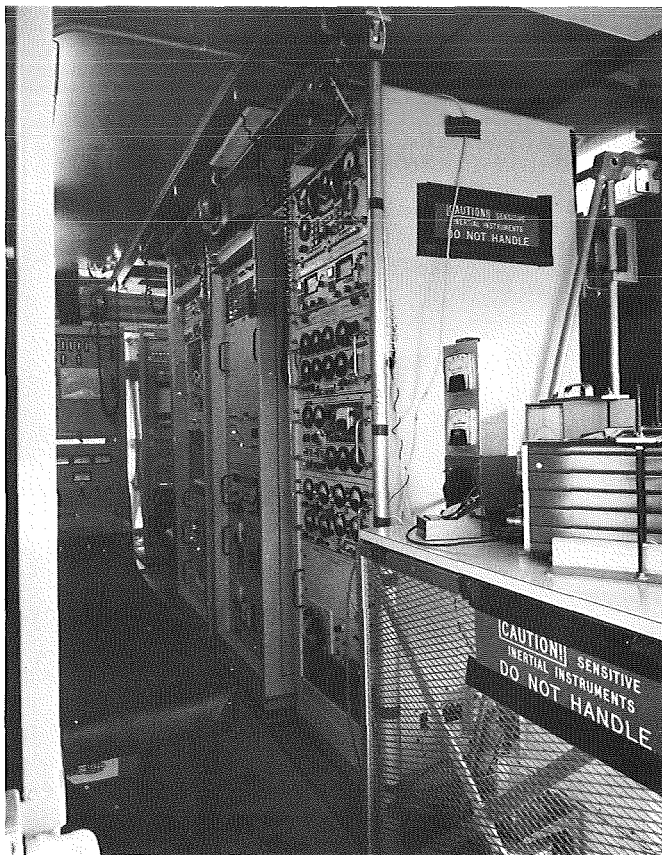


Fig. 2. SEAN system installed in van

measurement unit (IMU) and its protective cage. Next is the IMU control console, followed by the computer, adapter, and display console, and the computer accessories console. In the rear on the left is the uninterruptible power system and on the right the analog recording system. Shown in Fig. 3 are the magnetic tape transports for the digital recording system and the altimeter, which were mounted over the right rear wheel cover.

b. Power system. Power for road operations was provided by the trailer-mounted engine generators shown in Fig. 1. A 30-kV-A, 400-Hz generator was used for SEAN system power and two 5-kV-A, 60-Hz generators were used for air conditioning and miscellaneous test equipment power.

An uninterruptible power system (UPS) was provided primarily to meet the constraints of aircraft testing. The flight test phase of the SEAN program was deleted, however. The UPS, operating from either a 60- or 400-Hz source and a battery pack will provide constant, transient-free, 400-Hz power to the SEAN system. The unit, oper-

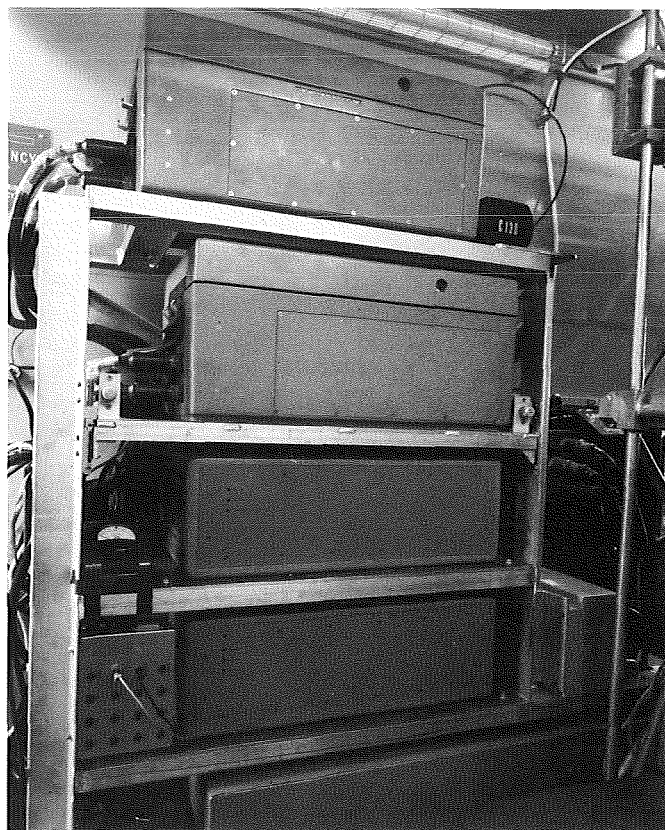


Fig. 3. Tape transports and altimeter installed in van

ating from the batteries, was to be used to provide uninterrupted power during the transitions between ground power and aircraft power.

c. Data acquisition. The data acquisition system provided the capabilities required to evaluate and troubleshoot the SEAN system.

Digital recording system. A digital magnetic-tape recording system was provided to record computer system outputs (position, velocity, and navigation time), navigation computations, IMU data (gyro and velocity meter counts), and other parameters as required. The data provided a means of evaluating the navigator performance.

The recording system consisted of two flight-environment-qualified digital magnetic-tape recorders and a tape control unit. The tape control unit interfaced with the SEAN digital computer memory where it obtained system data and formatted it for recording on the tapes in an IBM-compatible format. The control unit also performed the functions of tape transport control, error detection, and self-test.

Analog recording system. An oscillographic recording system was provided to monitor analog functions such as IMU temperatures and temperature control circuits, gyro status, power supply voltages, and vibration levels. Accelerometers were mounted inside the IMU and at the base of the IMU such that the vibration levels at the gyros and vibration levels of the van could be recorded.

Two oscillographic recorders were used: an eight-channel thermal writing recorder, and a 36-channel optical-writing recorder.

d. Photo recorder. A 35-mm, remotely controlled, photo recorder was used during road trips to photographically record the computer adapter display output information (latitude, longitude, speed, azimuth, and time). The camera was controlled by the SEAN computer to photograph the display panel every 5 s.

e. Altimeter. An altitude computer was used to provide digitized altitude information. The altimeter was planned primarily for flight tests, but also was used during the van road tests.

3. System Integration and Test Support

The SEAN system and support equipment were installed in the van in November 1969. Checkout and integration testing were completed during the first part of December 1969. Test and maintenance support was provided for the van operations and road tests from December to the completion of tests in February 1970.

B. Contacts to Photoconducting Cadmium

Sulfide, R. J. Stirn

1. Introduction

This article presents more recent results from the investigation of metal-cadmium sulfide (CdS) work functions at non-zero current densities. For earlier results and a discussion of the phenomena observed, refer to SPS 37-58, Vol. III, pp. 138-145. Basically, what is observed is a lowering of the Schottky barrier associated with a blocking contact (cathode) on photoconducting CdS crystals when the incident light intensity is increased. This allows the additional current needed in the bulk to be supplied by the contact. This behavior is surprising because normal Schottky barriers have constant characteristics (e.g., contact resistance) determined by the metal electronegativity, semiconductor electron affinity and doping, and possibly by the surface state distribution. It is understood in the following discussion that tunneling currents are not im-

portant for the reasons discussed in SPS 37-58, Vol. III, p. 142.

2. High-Field-Domain Results

With some changes in the optical dewar system, the low end of the temperature range was extended from 155 to 80°K. The results of the high-field-domain technique in the range from 155 to 295°K (SPS 37-58, Vol. III) were found to hold at the 80°K point also. That is, the calculated work functions continued to decrease linearly with temperature and the sample photocurrents continued to vary linearly with optical excitation density. The significance of the latter point is that the sample current is *not* contact-limited as would be expected for metal contacts having barrier heights greater than ~ 0.4 eV at zero current density.

Additionally, it was found that the value of the field within the domain (E_H) decreased at 80°K from the values measured at higher temperatures for all six metals used as contact materials. Actually, the peak value of E_H occurs around 180°K, which agrees with recent theoretical predictions.¹ The reasons for the temperature dependence of E_H have to do with the mechanism of field ionization of hole traps in the CdS used in this investigation. Field ionization of hole traps is necessary for the presence of a region of negative differential conductivity which, in turn, allows for the presence of the stationary high-field domains used in the analysis. Since field ionization of impurity levels (Poole-Frenkel effect) is analogous to Schottky emission of carriers from an electrode, there is an exponential dependence on the reciprocal temperature for the ionization. In addition, the competing process, the capture of carriers by the hole traps, has a temperature dependence because of the importance of the phonon distribution on the capture cross section. Since this phenomenon does not relate to the contact analysis directly, no details will be presented here.

3. Current Kinetics

The mechanism of the barrier lowering is believed to be due to a redistribution of electrons and holes very close to the cathode. An accumulation of positive charge of sufficient density (about 10^{13}cm^{-2}) within the region of the barrier maximum will account for the observed amount of lowering. One possible mechanism for storage of holes is that of storage within hole traps with energy levels lying in the forbidden band gap. However, it can be shown that

¹Dussell, G. A., and Böer, K. W., "Field Quenching as a Mechanism of Negative Differential Conductivity in Photoconducting CdS" (to be published in *Phys. Stat. Sol.*)

the required density of traps is too high for the doping characteristics of the crystals. Another possibility, the storage of free holes within the required distance from the cathode ($< 100 \text{ \AA}$), did not seem feasible because the hole image force should cause the free holes to enter the metal where they would immediately recombine with free electrons. However, it now appears that the apparent difficulties can be solved. Part of the necessary positive space charge can be shown to be contributed by uncompensated donors whose energy levels are below the Fermi level in the bulk away from the barrier region. In the region very near the cathode, these donors are ionized by the higher electric fields there, becoming positively charged centers. In addition, the free holes moving toward the cathode are shielded from the metal interface by the presence of these donors and other Coulomb repulsive centers with energy levels deeper in the forbidden band. A random and fluctuating fraction of the free holes will be prevented from reaching the cathode and these holes, along with the ionized donors, will provide enough positive space charge to lower the metal-CdS work function by the required amount.

It was to further investigate the barrier lowering within the narrow region near the cathode where the high-field-domain analysis cannot be applied (a region smaller than the mean-free-path distance for electrons), that the kinetics of the current upon sudden application of a voltage step were studied. In SPS 37-58, Vol. III, a preliminary display of the current as a function of time was given, as well as a model explaining the structure seen. The main features of the current-time dependence after the application of a voltage step were an initial large drop in current, a current minimum occurring at around 1 ms, and a slow rise to an equilibrium value.

The measuring apparatus has since been greatly improved and the current behavior studied at different light intensities and temperatures. The earlier measurement was severely limited by noise and a relatively long time constant in the sample circuit. By introducing thin shielded cabling into the optical dewar system and grounding the shields at a common point to the oscilloscope ground, the stray capacity was lowered from about 80 to about 2 pF. It was also discovered that most oscilloscope plug-in amplifiers take many tens of microseconds to recover from saturation, depending upon the sensitivity setting. The saturation results from the initial very large voltage pulse across the capacitance voltage divider formed by the sample-scope capacities. This problem was minimized by the use of a Tektronix 3A9 oscilloscope plug-in, which has a recovery time of a few microseconds.

It was found after these changes were made that the total change in current magnitude was now about one order of magnitude rather than the more than three orders shown in the SPS mentioned above. However, the general features remain, and in fact, a weak *second* minimum is now detectable. This minimum in the current reflects the presence of the donor levels mentioned above, from which electrons are swept out by the applied step voltage until those levels in the barrier region are ionized.

Typical current-time curves are shown in Figs. 4 and 5 for the case of a gold electrode with an applied voltage of 145 V and for two different temperatures. Measurements

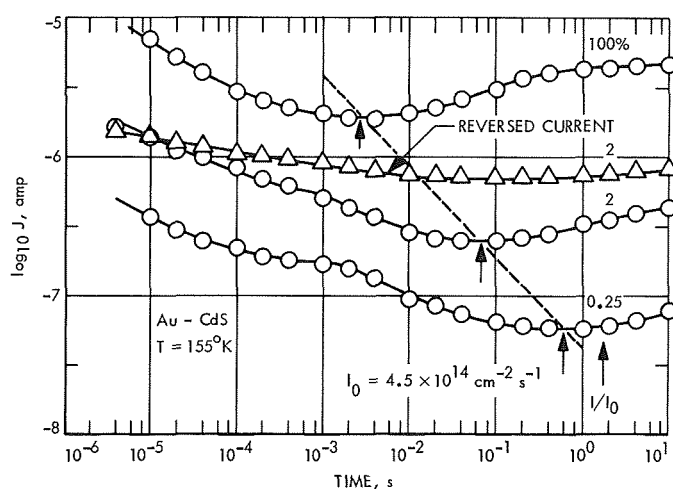


Fig. 4. Time-dependent current upon application of 145 V at 155°K

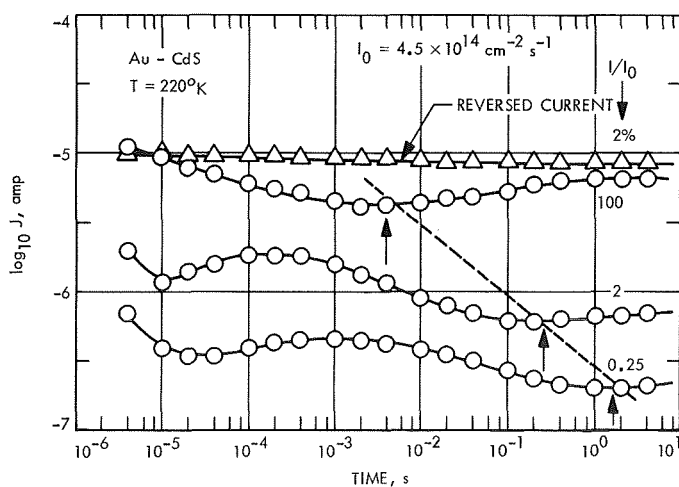


Fig. 5. Time-dependent current upon application of 145 V at 220°K

at room temperature also were performed but are not shown here, since the higher thermal excitation of traps masks out most of the structure in the curves. Also, one can see from the figures that the higher the light excitation, the less structure appears in the curves.²

Also shown in the figures is the current in the opposite direction (gold positive) for the lowest light intensity used (2% of I_0). This emphasizes the effect of the opposite contact, which is evaporated indium and was made to be as ohmic as possible. There appears to be a slight barrier associated with the indium contact, but it clearly is much smaller than the copper-associated barrier and can have no effect on the reverse current (gold negative).

A very definite trend in these curves is the movement of the minima toward longer times as the light intensity and temperature decreases. Additional measurements are to be taken at about 50 and 300 V (the latter voltage is high enough to insure the formation of a high-field domain after equilibrium is reached), which should show the field dependence of the kinetics. The analysis of the voltage and light dependence of the current kinetics will then be incorporated into a quantitative model for the barrier lowering in cooperation with the theoretical group at the University of Delaware.

C. Plasma-Immersion-Probe Tube in Metal-Ceramic Envelope,

K. Shimada and P. L. Cassell

1. Introduction

Plasma-immersion-probe tubes (Refs. 1 and 2) have been conveniently employed for evaluating electron-emission capabilities of various metals in contact with alkali-metal vapor. A similar tube has been effectively used to study the effect of oxygen that coadsorbs with cesium vapor on the electron emission of tantalum electrodes (Refs. 2 and 3). A conventional tube is made in a glass envelope, which is simple to construct, and in which it is easy to maintain a large volume ($\approx 100 \text{ cm}^3$) of plasma where sample electrodes are placed. The glass tube is limited in operating temperatures mainly by the glass-to-metal seals (which are particularly susceptible to reactions with cesium vapor) and by limitations in the current-carrying capacity of the lead-throughs.

²Both effects are due to the fewer number of unionized (filled) traps available for the field ionization relative to the free carrier concentration, since the higher light excitation and temperature cause increased generation of free carriers.

We have developed a tube in a metal-ceramic envelope to reduce these limitations and to increase the capability of the tube as a test vehicle for electron-emission studies. The outside wall, which serves as a vacuum envelope, is made of stainless steel; the inside wall, which provides an insulating surface for the plasma, is an alumina cylinder. A pair of discharge electrodes, the sample probes, and two sapphire view ports are mounted on demountable flanges with copper gaskets. This tube can be baked out and operated at much higher temperatures ($\approx 400^\circ\text{C}$) than the glass tubes. Also, interchangeability and repairability of the electrodes is provided by making them demountable. These features are nonexistent in a glass tube since the glass parts have to be cut open and fused again to complete the repair; such an operation contaminates the whole tube.

During the first phase of our experiments with the metal tube, argon gas was used (in place of cesium) to demonstrate the tube's capability to contain a plasma. The plasma was examined with a movable Langmuir probe that was insulated with a layer of metal oxide. Results indicated that the argon plasma was successfully maintained in the tube and that its volume was large enough to accommodate as many as three sample probes for the emission study. Results also indicated that the thin metal-oxide that was formed as an insulator for a Langmuir probe was quite satisfactory in providing a good insulation without distorting the plasma conditions near the probe.

2. Tube Construction

The main body of the tube is 33 cm long and 7.6 cm in diameter (Fig. 6). One end of this stainless cylinder is closed with a flange which supports the discharge anode; the other end is provided with a sapphire window for observing the interior of the tube. Five side ports are also provided for mounting electrodes. The discharge cathode, which is a coiled tungsten wire, is inserted into the port that is farthest from the discharge anode. During the preliminary test of this tube, a movable Langmuir probe was inserted into one of the ports and an additional sapphire window was mounted on the other. One other port was provided with a thermocouple gauge and the other was blanked off. For future tests with cesium, these ports will be provided with sample probes to be investigated. The metal cylinder provides a bakeable vacuum envelope inside of which a ceramic cylinder with a 0.25-cm wall thickness is snugly fitted to insulate the metal wall from the plasma. Other metallic parts, such as the copper holders for the discharge cathode, are insulated by lucalox tubing whenever possible to reduce leakage currents along the wall.

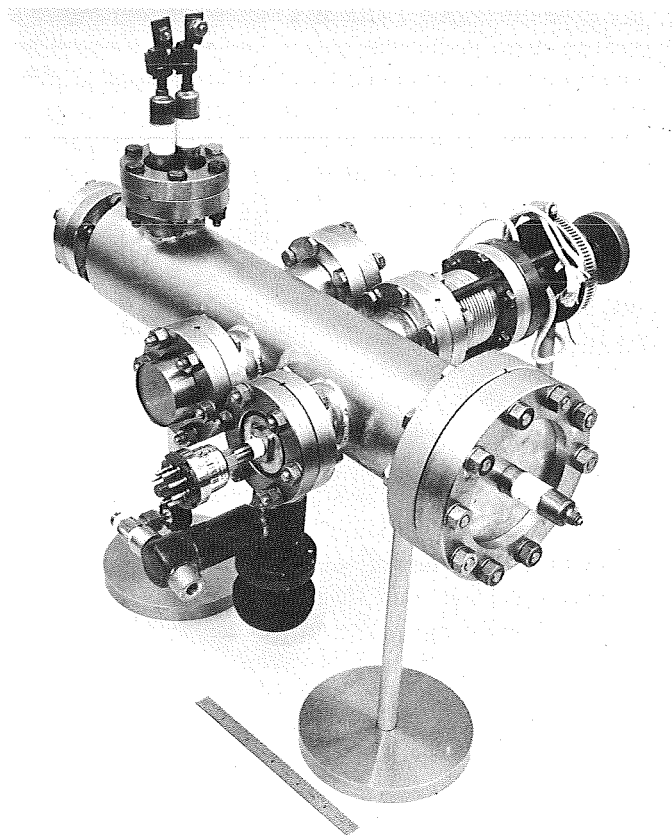


Fig. 6. Metal-ceramic plasma-immersion-probe tube

The movable probe—a needle probe—used in this phase of the work is unique in its construction, and its salient features are described below. It is made of tantalum wire (diameter = 0.9 mm). The active surface of the probe is normal to the radius of the tube, having a circular area of 0.64 mm². The probe is movable radially from the axis of the tube out to 18.5 mm off-axis by an external micrometer that is connected through a vacuum-tight flexible bellows. The cylindrical sides of the probe are covered with a thin tantalum-oxide layer (≈ 0.05 mm) for insulating the probe from the plasma. The oxide is formed by a simple process of heating the stretched wire in an oxidizing atmosphere. The layer was subsequently examined by a scanning electron microscope and by a hardness tester. The layer adheres tenaciously to the metal but it has pores or microcracks (< 0.01 mm wide) which lower the resistance of the insulation to the order of 1000 Ω when the wire is wetted by ordinary tap water. However, we believe that the pores do not contribute spurious probe areas in the plasma since none of them offer line-of-sight paths to the metal surface through which charged particles could reach the probe. This method of probe insulation was developed at JPL as a simple way to form a thin layer of insulator directly on

the probe. The layer is thin enough—compared with the probe diameter—that the sampled plasma by the probe was not perturbed by the insulator as would occur with thick insulators.

The tube was chemically cleaned, washed, and dried before pumping; outgassed at approximately 150°C for 1 h; and ultimately pumped with a diffusion pump. It then was filled with argon gas of a purity of 99.996% at pressures between 0.2 and 1 torr during the tests. For future tests with cesium, more rigorous processing of the tube will be followed.

3. Experiments

As mentioned above, argon gas was used during the initial tests to demonstrate the formation of a positive column that has properties desirable for a future use of the tube as an emission test vehicle.

Items of importance which were examined were: (1) formation of a glow discharge in a metal envelope; (2) the effects of electrical grounding of various electrodes to the tube envelope; (3) properties of the movable Langmuir probe as a plasma probe; (4) uniformity of plasma densities in the positive column; and (5) electron temperatures in the positive column. The results of these tests are described below.

After pumping the tube to a pressure less than 10^{-3} torr, it was flushed with dry argon prior to final filling with argon gas. The argon pressure suitable for satisfactory formation of the discharge was between 1.0 and 0.2 torr. Although the discharge cathode was not heated externally, it became incandescent because of ion bombardment when the power consumed in the discharge was increased above 10 W. The circuit used for the measurement is shown in Fig. 7. The grounding of any electrode either shifted the plasma in an unsymmetrical shape or caused a local breakdown, and, therefore, all the electrodes were floated except the envelope of the tube for the remainder of this experiment. These phenomena were attributed to an abnormal potential distribution in the tube that occurred when any one of the electrodes were grounded.

Plasma densities and electron temperatures were determined from the volt-ampere curves (probe characteristics) of the Langmuir probe. As shown in Fig. 7, a somewhat unconventional probe circuit was used in which the probe power supply was connected to the cathode rather than to the discharge anode. This connection was preferable since the electronic power supply need never be driven negatively during an entire voltage sweep of the probe. A

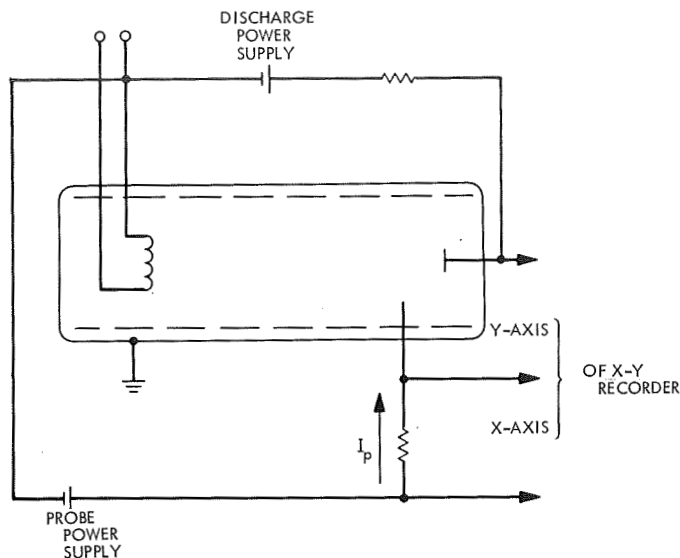


Fig. 7. Test circuit

probe guard ring that was provided to eliminate the leakage currents into the probe circuit was not used during these measurements since the leakage was negligible with argon gas.

Probe characteristics (Fig. 8) were obtained with the probe tip at the axis of the tube and at a distance $d = 18.5$ mm from the axis. These curves exhibit saturation ion currents, indicated by negative currents at large negative probe voltage, and show the increase in electron current as the probe potential increases positively. However, saturation was not observed in the electron current since local breakdown of argon occurred at the probe tip before electron saturation was reached. Such behavior is expected with most Langmuir-type probes. We found decreasing ion currents and more-negative floating potential (at which the probe current becomes zero) as the probe is removed from the axis. These results are in agreement with expectations.

From these curves, the ion densities and the electron temperatures were determined. Since the probe current I_p is the difference between the saturated ion current I_{is} and the electron current, and since the electron current is reduced by a Boltzmann factor from the saturation value I_{es} , I_p can be written as

$$I_p = -I_{is} + I_{es} \exp\left(\frac{-eV}{kT_e}\right) \quad (1)$$

where V is the potential difference across the ion sheath shown in Fig. 9, T_e is the electron temperature (a Max-

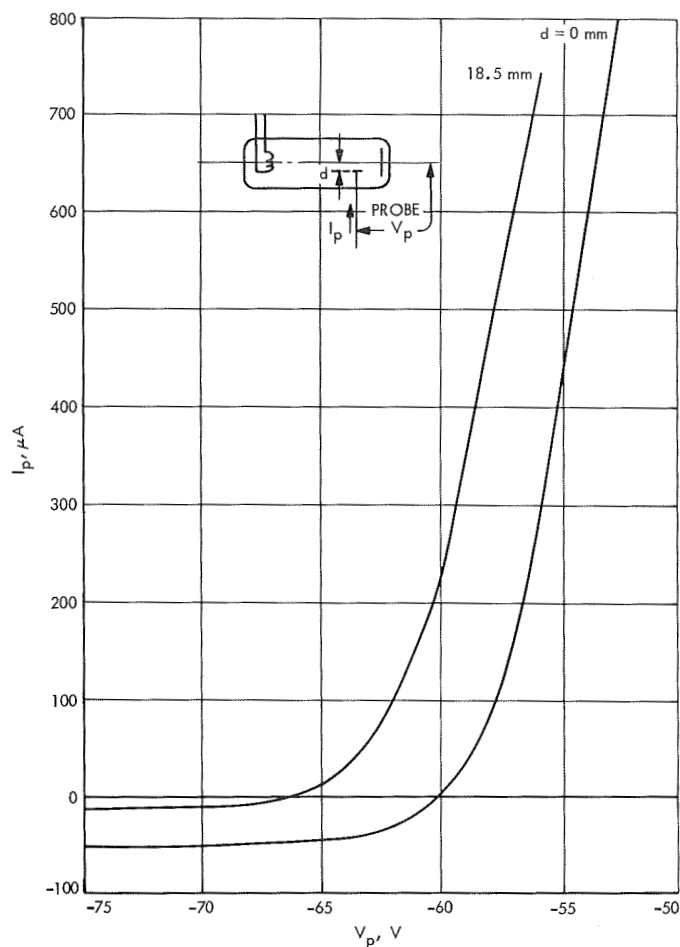


Fig. 8. Probe characteristics in argon gas (pressure = 1 torr, discharge current = 25 mA)

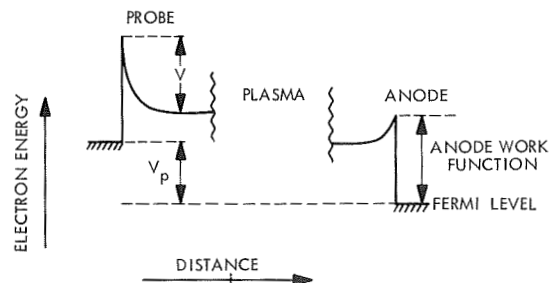


Fig. 9. Electron energy diagram near the probe

wellian velocity distribution is assumed in the plasma), e is the electron charge, and k is the Boltzmann constant. Therefore, for large values of V (≈ 15 V), the probe current I_p nearly equals the saturation ion current I_{is} . At an argon pressure of 1 torr and with a probe tip diameter of 0.9 mm, the perturbation of a plasma by the probe is considered to be small enough (SPS 37-59, Vol. III, pp. 151-155) so that the saturated ion-current density is given in terms of an

ion density n_i at the sheath edge and a linear average velocity $\bar{v}_i$ as follows:

$$\frac{I_{is}}{S} = \frac{1}{4} n_i \bar{v}_i \quad (2)$$

where S is the active area of the probe tip. If the velocity distribution of ions is assumed Maxwellian, which is usually the case in similar plasmas, the velocity $\bar{v}_i$ is given by

$$\bar{v}_i = \left(\frac{8kT_i}{\pi M} \right)^{1/2} \quad (3)$$

where M is the mass of an argon ion and T_i is the ion temperature which is nearly equal to the gas temperature ($\approx 300^\circ\text{K}$). Calculated ion densities were $4.65 \times 10^{12}/\text{cm}^3$ at the axis and $0.99 \times 10^{12}/\text{cm}^3$ at 18.5 mm off the axis of the tube.

Electron temperatures can also be determined from Eq. (1). By rearranging Eq. (1), one obtains

$$\ln(I_p + I_{is}) = -\frac{eV}{kT_e} + \ln I_{es} \quad (4)$$

A semilog plot of $I_p + I_{is}$ versus V is shown in Fig. 10. A straight-line relationship was obtained along 1.5 decades of the graph, indicating that the Boltzmann relation (Eq. 1) is valid. The electron temperature T_e , which was determined from the slope, was approximately $22,000^\circ\text{K}$. The value was nearly twice that which was reported in Ref. 4. The discrepancy may be caused by the fact that our plasma was somewhat abnormal; the plasma drop was larger ($\approx 30 \text{ V/cm}$) than that in a normal glow discharge. Since our main concern here is to show the feasibility of using a metal-ceramic structure for a plasma-immersion-probe tube and not to make critical studies of the plasma parameters during this phase of the work, present results are considered satisfactory.

4. Conclusion

A plasma-immersion-probe tube that will be used for determining the electron emission from cesiated metals was fabricated in a metal envelope in place of a glass envelope of the type commonly used in the past. The tube was so constructed that it can be operated at temperatures as high as 400°C , and all electrodes can be interchanged without destroying the tube. To provide an insulating wall for the plasma, a ceramic cylinder was fitted inside the envelope.

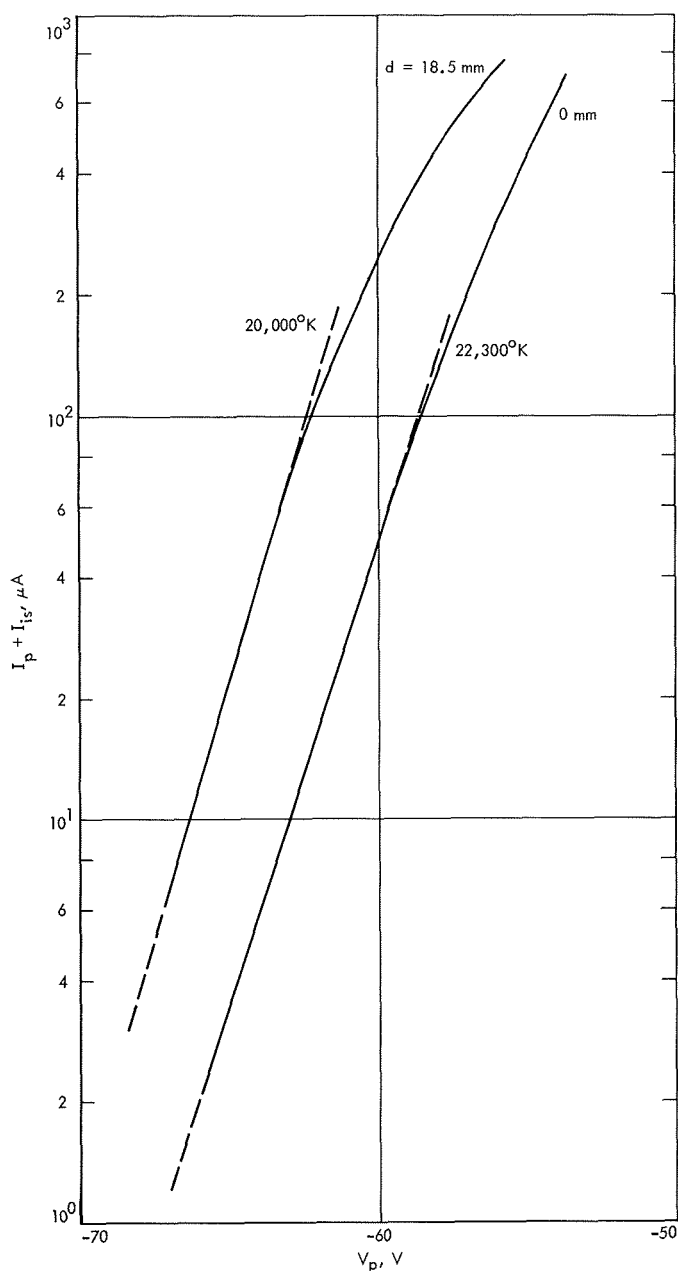


Fig. 10. Electron current vs probe voltage

Experiments were conducted with argon gas (instead of cesium vapor) to demonstrate that the tube can contain plasma. It was shown that the plasma density and electron temperature were in satisfactory agreement with expectations. The plasma was examined by a movable Langmuir probe that has unique insulation; the probe was developed in our laboratory. The probe was made of a 0.9-mm-diameter tantalum wire insulated with a thin ($\approx 0.05 \text{ mm}$) tantalum-oxide layer formed by heating the wire in an oxidizing atmosphere. This type of insulation was used to

avoid perturbation of the plasma near the probe such as would occur with a relatively thick insulation normally provided by glass-coating or ceramic-sleeving of the probe. Test results show that the metal tube successfully contained the plasma, and that the probe is mechanically and electrically satisfactory.

During a subsequent phase of the work, the tube will be provided with electron-emitting probes and charged with cesium for evaluating oxygen-cesium-coadsorbed tantalum as an efficient emitter material in practical thermionic converters.

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XII. Spacecraft Control

GUIDANCE AND CONTROL DIVISION

A. Valve Evaluation Program for Use in Nitrogen Attitude Control Gas Systems, J. D. Ferrera

1. Introduction

The *Mariner* spacecraft is a three-axis stabilized vehicle that utilizes a mass expulsion system for attitude control. The mass expulsion system consists of high-pressure, cold nitrogen gas, a titanium tank pressurized to 2650 lb/in.², a single-stage pressure regulator, which reduces the pressure to 15 lb/in.², and 6 coaxial, self-relieving solenoid valves. Two such systems, acting in parallel, are flown on each spacecraft. The *Mariner* missions have typically been 1 yr in duration. The solenoid valves used meet the leakage requirement of less than 3 cm³ (std)/h and provide a thrust of 5 to 10 lb $\times$ 10⁻³.

In addition to the *Mariner* spacecraft, the *Viking* Orbiter, which is a Mars orbiting spacecraft, and the Thermoelectric Outer-Planet Spacecraft, which will swing by most of the outer planets, are being worked on. The valve performance requirements for these new spacecraft are much more severe than for the *Mariner* spacecraft. The additional requirements include: leakage rates of less than 1 cm³ (std)/h, thrust ranges of 10 to

120 lb $\times$ 10⁻³, operational life of 8 to 12 yr, and the ability to survive a substantially lower temperature environment. Since the *Mariner* valves cannot meet all these new requirements, a program has been initiated to find and qualify new valves.

2. Approach

An industrial search letter was sent to over 20 companies listing the valve requirements and requesting information on any valves which might be applicable. No valves were found that satisfied the new requirements. However, as a result of this survey, valves that came close to meeting the requirements were procured from five vendors for evaluation. These five valves are listed in Table 1 along with vendor-supplied functional data. These valves will all undergo an identical evaluation test in order to determine the applicability of any of these valves for the proposed missions. The evaluation test program is outlined in Table 2. In addition to these subtests, a functional test, leakage test, and failure analysis, as required, will be performed between each subtest. The response checks, as part of the functional tests, include both opening and closing times as well as opening and

Table 1. Vendor-supplied valve data

Parameter	M-Tron 10090	Carleton 1958001	Wright 15457	Hydraulic Research 3900600	Sterer 35360
Valve type	Coaxial, solenoid, non-self relieving	Coaxial, solenoid, non-self relieving	Coaxial, solenoid, non-self relieving	Torque motor, non-self relieving	Solenoid, self relieving
Valve seat	Hard	Soft	Soft	Hard	Hard
Coil resistance (70°F), Ω	140	350-380	320	115	145
Operating voltage, Vdc	26	12-36	28	22.5-25.5	26 ± 6
Weight, lb	—	0.135	0.25	0.25	—
Operating temperature range, °F	-200 to 200	—	-40 to 125	—	-100 to 300
Internal leakage	$<0.1 \text{ cm}^3 \text{ (std)/h}$, gaseous N_2 , 15 psig	$<6 \times 10^{-5} \text{ cm}^3 \text{ (std)/s}$, He	$<6 \times 10^{-5} \text{ cm}^3 \text{ (std)/s}$, He, 60 psid	$<0.5 \text{ cm}^3 \text{ (std)/h}$, gaseous N_2 , 15 psig	$<1.0 \text{ cm}^3 \text{ (std)/h}$, gaseous N_2 , 30 psig
Pressure:					
Operating, psig	15-50	0-300	0-60	30	25
Proof, psig	—	400	90	85	—
Burst, psig	—	—	240	550	—
Relief, psig	—	—	—	—	40-60
Flowrate vs ΔP	$3 \times 10^{-4} \text{ lb/s}$, gaseous N_2 , 15 psia, 2 psid	22,000 $\text{cm}^3 \text{ (std)/min}$, gaseous N_2 , 15 psig	1.14 $\text{ft}^3 \text{ (std)/min}$, air, 35 psig	$5 \times 10^{-5} \text{ lb/s}$, NH_3 , 30 psig, 5 psid max	1.2 $\text{ft}^3 \text{ (std)/min}$, gaseous N_2 , 15 psig
Response:					
Opening time, ms	10 max at 26 Vdc	—	15 max	10 max at 25 Vdc	15 max
Closing time, ms	5 max at 26 Vdc	—	15 max	10 max at 25 Vdc	3 max
Pull-in voltage	—	10-11.5 Vdc	20 Vdc max	15 Vdc max	60 mA
Drop-out voltage	—	2.0 Vdc	5 Vdc min	2 Vdc min	60 mA
Fluid	Gaseous N_2	—	Gaseous N_2 , He, CF_4	N_2H_4 , gaseous N_2 , NH_3	N_2H_4 , gaseous N_2 , NH_3

Table 2. Valve evaluation test summary

Subtest	Description
1. Functional test	Response and coil resistance, contamination check, and leakage tests at 0, 10,000, 20,000, 30,000, and 40,000 cycles
2. Flow versus ΔP test	Measure flow rate at various inlet pressures; record pressure drop across valve (ΔP)
3. Vibration test	Shake in three planes at 5- to 10-g level; perform functional and leakage tests (no cycling) between planes
4. Temperature cold test	Perform functional test over temperature range from ambient to -30°F
5. Life test	Cycle valves to at least one million cycles; perform functional and leakage tests at every 100,000 cycles
6. Disassemble and inspect	As applicable

closing voltages (currents). The leakage tests will be performed with nitrogen at 15 lb/in.² using a water displacement method. The cold temperature test is required principally to check the performance of those valves containing soft seat material. The contamination check will be performed prior to the first and sixth subtests only.

3. Status

All test valves have been procured, cleaned, and assembled on a common manifold as shown in Fig. 1. A test console has been built which contains the nitrogen gas supply and associated pressure gages and all the electronics necessary to perform the functional test and life cycle on 12 valves simultaneously. Initial functional tests are now being performed. A test plan for the first five subtests will be written and, together with all test data, will be included in a final report.

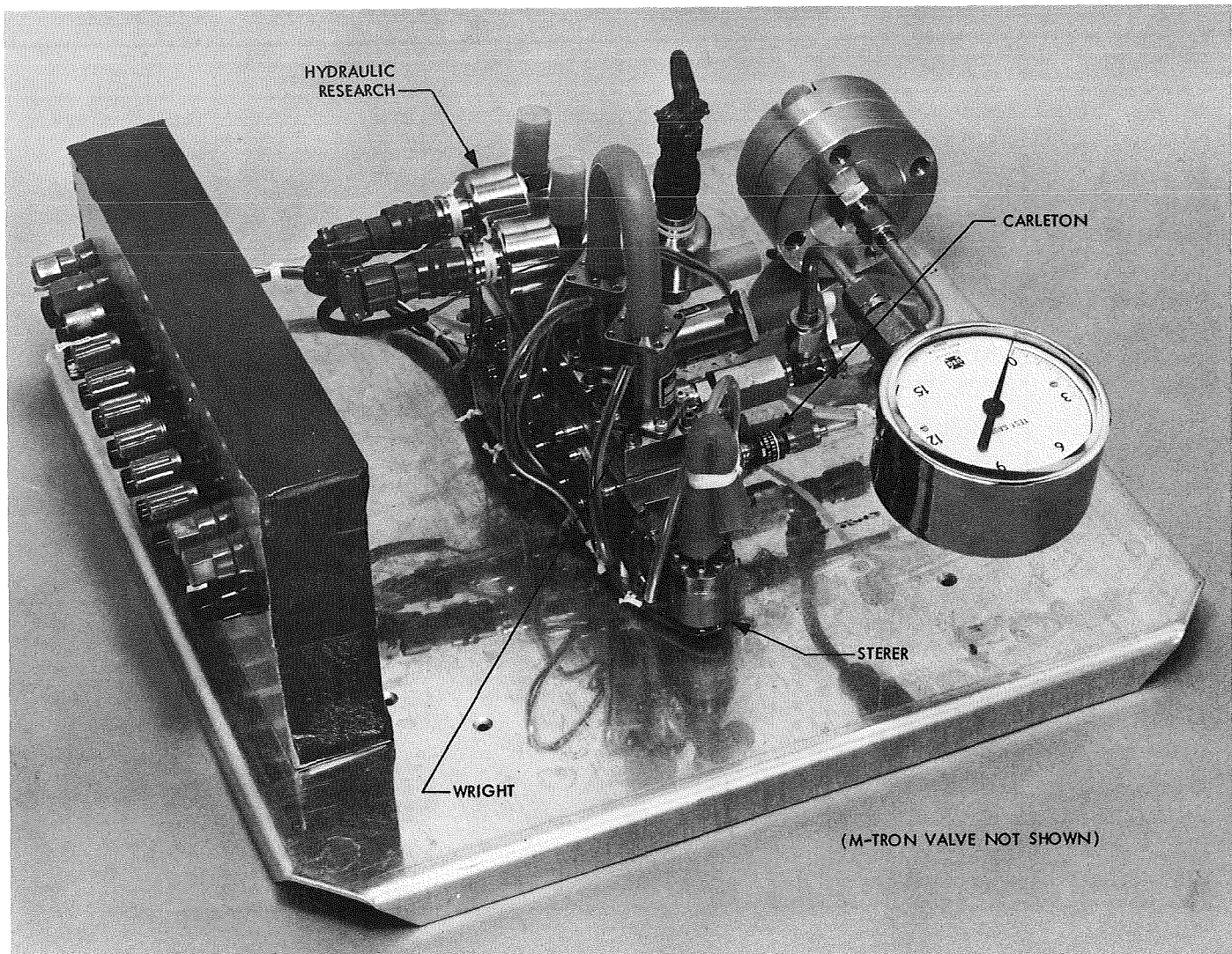


Fig. 1. Valve test assembly

B. An Analysis of TOPS Science Package Pointing Error Due to Structural Vibration of the Supporting Booms, *L. L. Schumacher*

1. Introduction

The current Thermoelectric Outer-Planet Spacecraft (TOPS) design can be divided into the following major units:

- (1) Primary bus structure containing the propulsion and electronic equipment.
- (2) The radioisotope thermoelectric generator (RTG) boom structure.
- (3) The science boom structure.

The various science instruments and the television camera mounted on the science boom structure have very exacting pointing requirements. The object of this analysis was to determine the nature of the science package motion due to the structural vibration of the supporting booms so that an evaluation of instrument pointing errors can be made.

A structural dynamic spacecraft model was developed and simulated on the UNIVAC 1108 computer using the Continuous System Simulation Language.

2. Model Development

Figure 2 shows the physical model used for the analysis. Torsional equations of motion describe the mass-

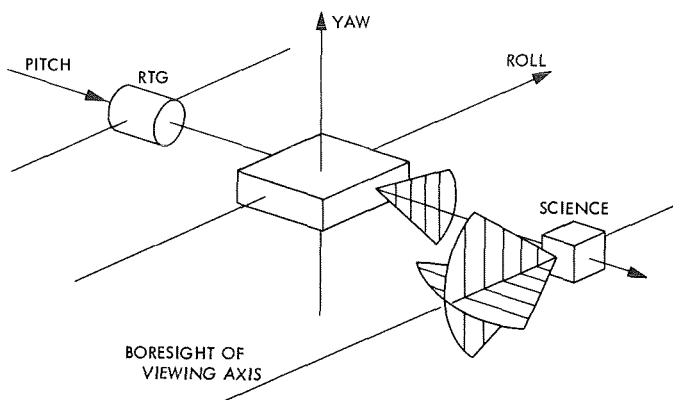


Fig. 2. TOPS structural model

motion about the pitch axis, and cantilever equations of motion describe the mass-motion about the yaw and roll axes.

The equations of motion are developed under the following assumptions:

- (1) Each axis is dynamically decoupled from the other axes.
- (2) The booms are assumed weightless, rigid, and are attached to the spacecraft bus with a hinge-like spring containing a small amount of damping.
- (3) The spacecraft composite center of mass does not move in inertial space so that any translation of the boom tip mass centers can be expressed as a rotation about an axis perpendicular to the plane of motion.

The shaded area in Fig. 2 illustrates the uncoupled motion pattern of interest in this investigation.

Figure 3 shows the worst-case inertial angular displacements of the bus, RTG, and the science package from a typical momentum wheel impulse.

The pointing error is the difference in the inertial spacecraft bus displacement and the inertial science package displacement. The peak pointing error can be observed from Fig. 3 to be 2×10^{-8} rad.

It can be shown that the maximum expected angular error for an arbitrary viewing axis in the science package is given by

$$\theta E (\text{MAX}) = 3^{1/2} \theta P (\text{MAX}) \quad (1)$$

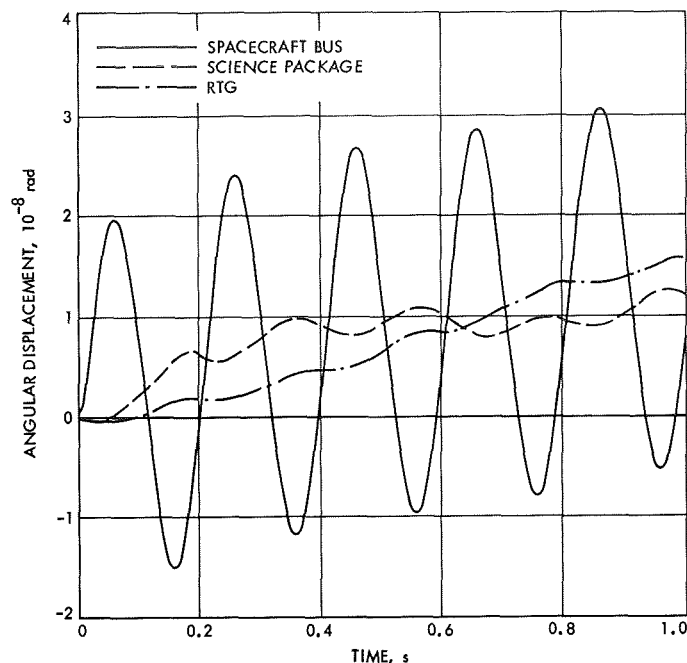


Fig. 3. Angular displacement of major spacecraft masses in yaw

where

$\theta E (\text{MAX})$ = maximum pointing error of the pointing axis

$\theta P (\text{MAX})$ = maximum pointing error in any of the observed axes

In this case, $\theta E (\text{MAX}) = 3.4 \times 10^{-8}$ rad.

3. Conclusion

Equation (1) illustrates that the worst-case angular displacement from some arbitrary pointing axis in the science package is very small (3.4×10^{-8} rad). It is concluded that if no major structural modifications are made in boom stiffness or in the three major mass values, then the pointing inaccuracy of an arbitrary axis in the science package due to natural vibrations of the booms can be ignored.

C. Two-Dimensional Molecular Gas Leakage Flow, F. G. Roselli-Lorenzini

1. Introduction

A simple way of preventing exposure of flight equipment to hard vacuum in space is to enclose it in pressurized containers filled with some gaseous mixture. The

mixture will be inert relative to the concerned equipment. An extreme, but not unlikely case, will be a single species gaseous environment. The pressurized system will generally be sealed to prevent the gas to escape in space. Since a perfect, zero-leakage sealing system is almost a technical impossibility, a decay on the amount of gas due to leakage should be accounted for.

The laws governing the gas leakage flow rates are largely empirical due to the imperfect knowledge of the flow path geometry, the gas state parameters in the leaking stream, and the energy losses experienced by the gas in the stream. The analytical treatment of the phenomena involved will require some exemplification. A preliminary analysis, dealing with the pressure decay in pressurized containers due to leakage, has been performed under the following assumptions:

- (1) The external pressure is zero.
- (2) The gas flow through the leakage path is purely molecular.
- (3) The gas species involved behave as perfect gas.
- (4) The gas is flowing through one leak port only at any one time.
- (5) The leak port area is constant.

A way of measuring the mixture leak rate is to measure the leak rate of one of the mixture components, used as a tracer. The tracer might be helium in mass spectrometers, a radioactive gas such as C^{14} (CO) or Kr 85 in measuring systems using ionization chambers or Geiger-type detectors and Freon or metallic-bound-chain hydrocarbons in interferometry-based systems. Thus, in the analysis, one of the mixture gas species is singled out as the species whose leak rate is measurable or specifiable. The parameters associated with the remaining species of the mixture are written relative to the values of the same parameters as associated with the tracer.

2. Analysis

The adopted procedure is to compute the leakage port area using a given set of initial conditions and then introduce this value in the integral expression for the pressure in the vessel, the mixture ratios, and the mass flow rate through the port. The conventional expressions of the kinetic theory of the gases for the partial pressure, the average molecular velocity, and the molecular flow rate through a unit area for each single species of the mixture are used. The required set of initial data is the following:

- (1) The initial conditions P_0, T_0 .

- (2) The initial mixture volumetric composition, expressed by the mixture ratios r_{0i} , relative to the detectable j th species.
- (3) The allowable or measured mass leak rate of the j th species, $\dot{M}_{gj}$.
- (4) The molecular weights of the single gaseous species in the mixture, m_i .
- (5) The volume of the pressurized container, V .

The result is a set of time functions of a general exponential form. A computer program¹ tabulates and plots the time dependence of the relevant parameters, i.e., the mixture ratios, the total mass flow rate, and the total pressure in the vessel. More specifically, the program solves the following system of equations, which results from the analytical development of the above assumptions:

$$\bar{v}_i = \left(K_1 \frac{T_0}{m_i} \right), \quad i = 1, 2, \dots, N$$

$$A = K_2 \dot{M}_{gj} \frac{T_0}{P_0} \left(1 + \sum_{i=1}^N r_{0i} \delta_{ij} \right) \left(\frac{1}{K_1 m_j N_a} \right)^{1/2}$$

$$r_i = r_{0i} \exp \left[- \frac{A (\bar{v}_i - \bar{v}_j) t}{4V} \right], \quad i = 1, 2, \dots, N$$

$$n_{j0} = \frac{N_a P_0 T_s}{V_{ms} P_s T_0} \left(1 + \sum_{i=1}^N r_{0i} \delta_{ij} \right)^{-1}$$

$$\begin{aligned} \dot{M}_{gm} &= \frac{aA n_{j0} \bar{v}_j}{4} \exp \left(- \frac{\bar{v}_j A t}{4V} \right) \\ &\times m_j \left(1 + m_j^{-3/2} \sum_{i=1}^N m_i^{-3/2} r_{0i} \right. \\ &\times \exp \left[- \frac{A (\bar{v}_i - \bar{v}_j) t}{4V} \right] \delta_{ij} \Big) \end{aligned}$$

$$\begin{aligned} P &= P_0 \exp \left(- \frac{\bar{v}_j A t}{4V} \right) \left(1 + \sum_{i=1}^N r_{0i} \right. \\ &\times \exp \left[- \frac{A (\bar{v}_i - \bar{v}_j) t}{4V} \right] \delta_{ij} \Big) \left(1 + \sum_{i=1}^N r_{0i} \delta_{ij} \right)^{-1} \end{aligned}$$

where

$\bar{v}$ = molecular average velocity

δ_{ij} = Dirac delta

N_a = Avogadro number

A = leak port area

¹Written by G. Winje, JPL Computation and Analysis Section.

n_{j0} = initial molecular density of the j th species

$\dot{M}_{gm}$ = total mass flow rate

a = atomic mass unit

P = total pressure in the vessel

t = time variable

The K_r ($r=1,2$) are combinations of universal and conventional constants whose value depends upon the selected system of units. They are given by the expressions:

$$K_1 = \frac{8K}{\pi}$$

$$K_2 = \frac{4V_{ms}P_s}{aN_aT_s}$$

where

K = Boltzmann constant

V_{ms} = molar volume at standard conditions

P_s = standard pressure

T_s = standard temperature

3. Application

The analysis has been applied to the specific case of the *Mariner* Mars 1971 scan actuator. The actuator outer shell (Fig. 4) is a pressurized container filled with a 90/10 mixture of dry nitrogen and helium. The container free volume is 500 cm³, the initial pressure 20 psi. The estimated average operating temperature is normal (298°K). The specified helium leak rate, measured with a mass spectrometer, is 3.35×10^{-9} g/s. The results are shown in Figs. 5, 6, and 7. In particular, Fig. 7 shows that a substantial positive pressure will still exist in the vessel after 50 yr of operation.

4. Conclusions

The applicability of the present analysis to real cases is severely restricted by the simplifying assumptions listed in the introduction. A further step to the achievement of

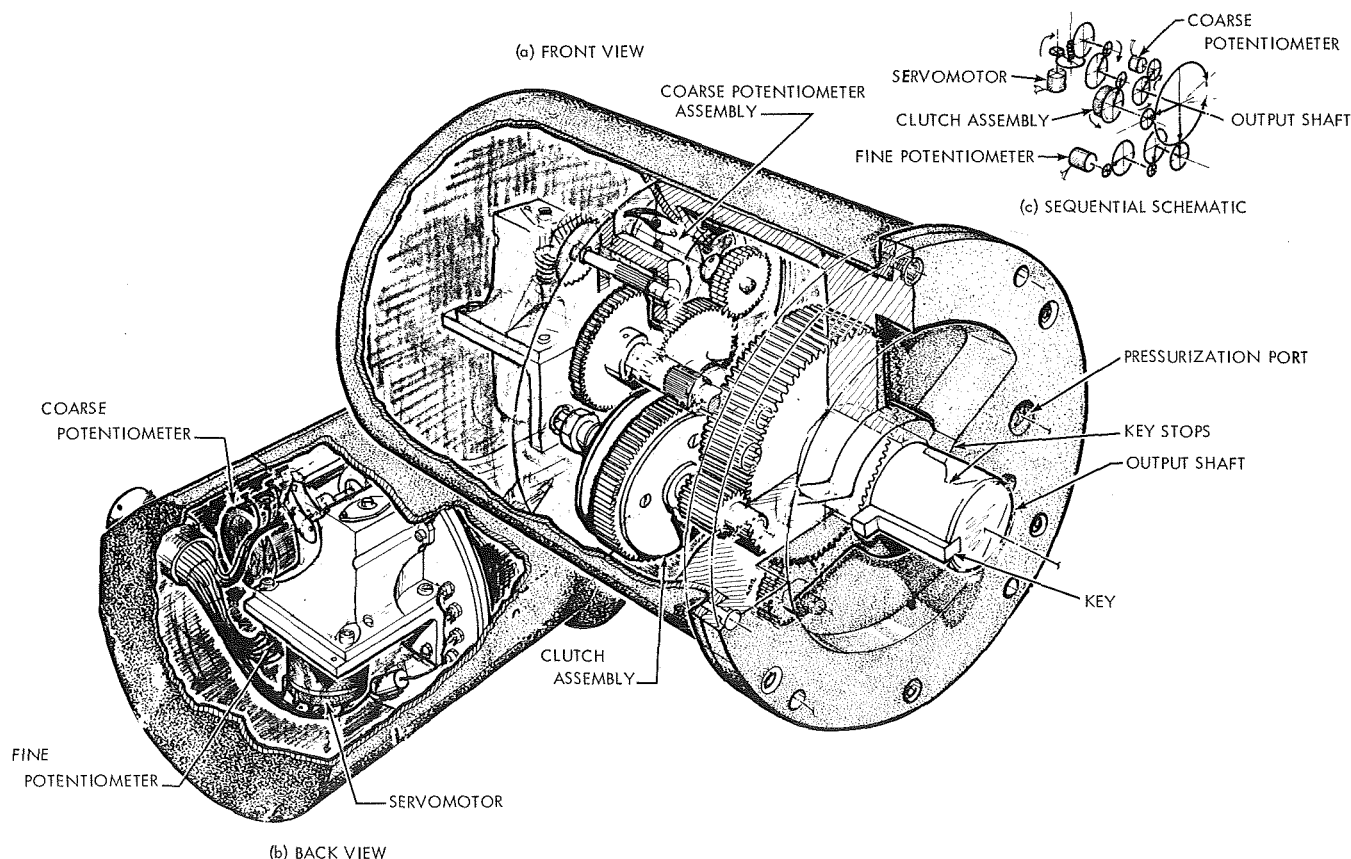


Fig. 4. Schematic of scan actuator

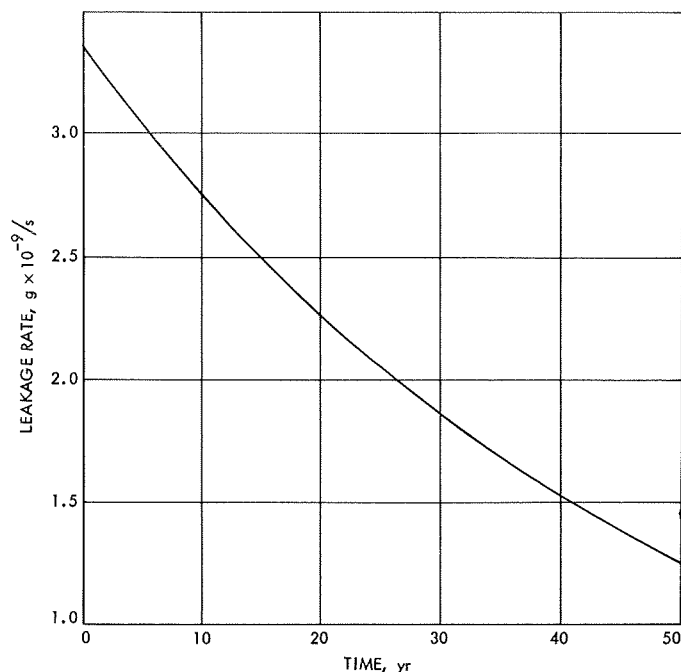


Fig. 5. Total mass leakage rate vs time

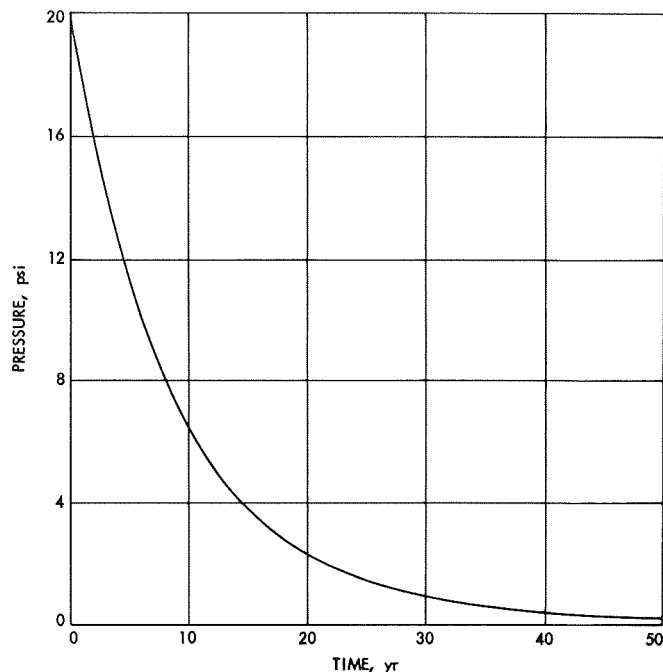


Fig. 7. Total pressure decay

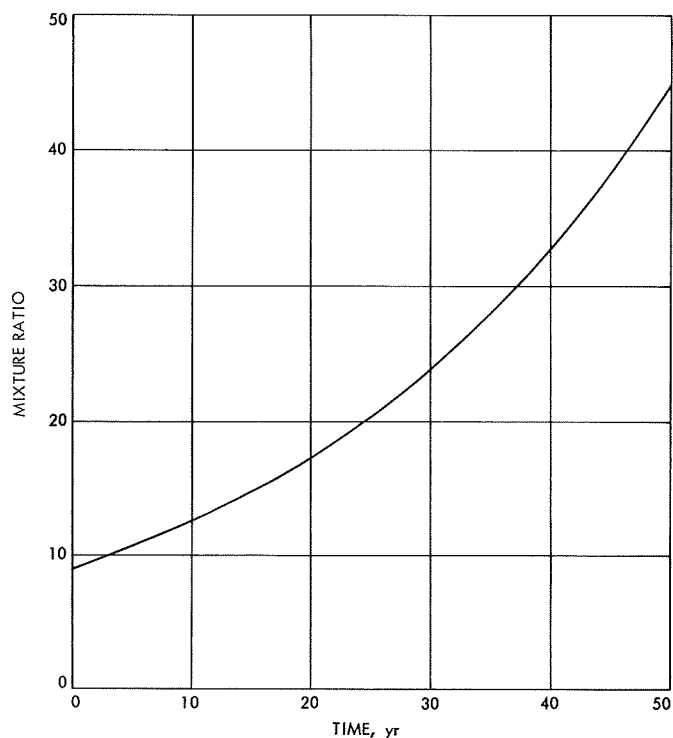


Fig. 6. N₂/He mixture ratio vs time

a suitable mathematical model of the phenomenon will be the extension to a three-dimensional case (i.e., assuming a three-dimensional leak path geometry) and to a sta-

tistically distributed variation of the leak path geometric parameters. This analysis is now under consideration.

D. Solar Electric Spacecraft Thrust Vector Control System Mechanization, W. E. Crawford and G. E. Fleischer

1. Introduction

This article describes a modified mechanization of the Solar Electric Propulsion Spacecraft (SEPS) translator (or gimbal) feedback compensation, and shows that it can be made equivalent to the compensation network originally proposed by Mankovitz.² System gain values are determined which, in fact, achieve that equivalence.

2. Mechanization

Figure 8 shows the general form of the proposed feedback circuitry. The feedback signal is derived from the voltage-controlled oscillator (VCO) output directly rather than from a potentiometer measuring translator (or gimbal) position. In this way, it is clear that a substantial improvement in feedback signal-to-noise ratio can be obtained at the expense of having a direct indication of translator (or gimbal) motion. Since the latter consideration is not necessary for proper control loop operation

²1975 Jupiter Flyby Mission Using a Solar Electric Spacecraft, March 1, 1968 (JPL internal document).

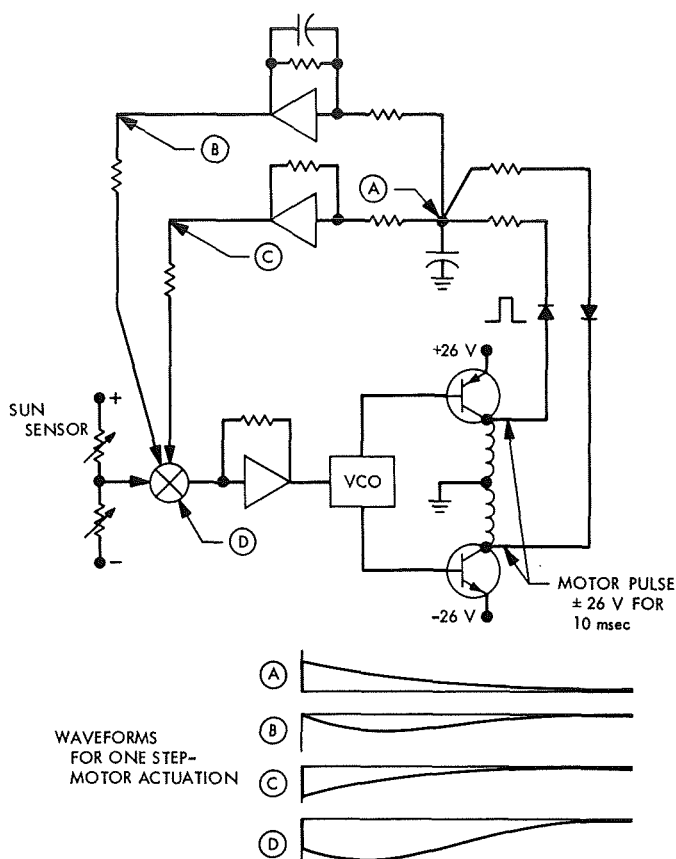


Fig. 8. Proposed mechanization of SEPS translator controller and compensation

(absolute knowledge of actuator output position is not required except for telemetering purposes), it is sacrificed for the former.

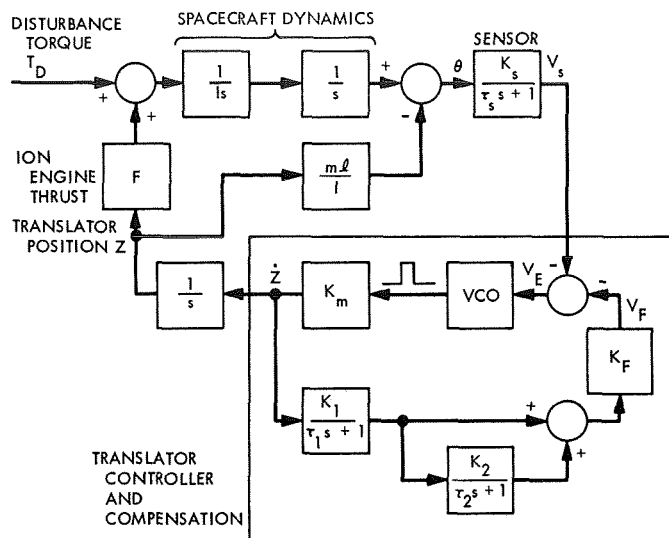
3. Transfer Function Analysis

A transfer function block diagram of this approach is pictured in Fig. 9a. Shown is a single axis translator control loop, simplified to eliminate the high-frequency dynamics of the actuator mechanism but including the effect of translator-spacecraft interaction (tail-wags-dog effect). The gains K_1 and K_2 , as well as the time constants τ_1 and τ_2 , are to be determined for achieving control loop response identical to that of the previous configuration (Fig. 9b).

From Fig. 9b, the transfer function is given by

$$\begin{aligned} \frac{V_F(s)}{Z(s)} &= K_F \frac{(\tau_A s + 1)}{(\tau_B s + 1)} - K_F \frac{1}{(\tau_p s + 1)} \\ &= \frac{K_F s [\tau_A \tau_p s + (\tau_A + \tau_p - \tau_B)]}{(\tau_p s + 1)(\tau_B s + 1)} \end{aligned}$$

(a) PROPOSED CONFIGURATION



(b) PREVIOUS CONFIGURATION

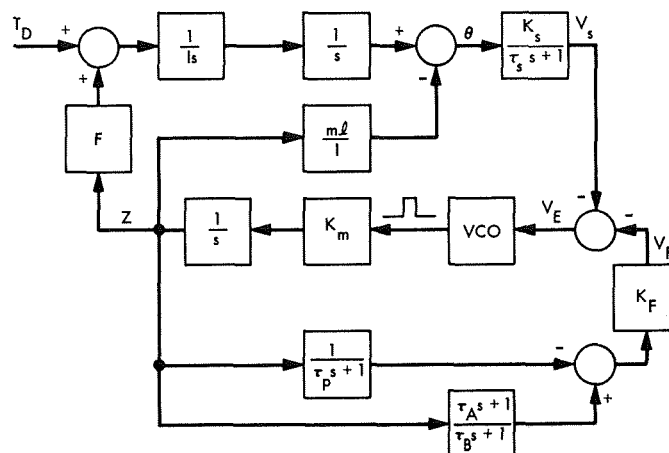


Fig. 9. Transfer function block diagram

Nominally, the parameter values may be assumed as

$$\begin{aligned} \tau_A &= 50 \text{ sec} \\ \tau_B &= 500 \text{ sec} \\ \tau_p &= 1000 \text{ sec} \end{aligned}$$

Therefore,

$$\frac{V_F(s)}{Z(s)} = \frac{K_F s (50000s + 550)}{(1000s + 1)(500s + 1)}$$

for the system of Fig. 9b.

The configuration in Fig. 9a may be similarly developed as

$$\frac{V_F(s)}{Z(s)} = s \left\{ K_F \left[\frac{K_1}{(\tau_1 s + 1)} + \frac{K_1 K_2}{(\tau_1 s + 1)(\tau_2 s + 1)} \right] \right\}$$

or

$$\frac{V_F(s)}{Z(s)} = \frac{K_F s [K_1 \tau_2 s + K_1 (K_2 + 1)]}{(\tau_1 s + 1)(\tau_2 s + 1)}$$

Thus, for example, if we let $\tau_1 = \tau_p = 1000$ and $\tau_2 = \tau_B = 500$, then

$$K_1 \tau_2 = 50,000$$

$$K_1 = 50,000/500 = 100 \text{ V/ft/sec}$$

and

$$K_1 (K_2 + 1) = 550$$

$$K_2 = 550/100 - 1 = 4.5$$

On the other hand, one could interchange τ_1 and τ_2 values so that, for

$$\tau_1 = 500 \text{ and } \tau_2 = 1000,$$

$$K_1 = 50,000/1000 = 50 \text{ V/ft/sec}$$

$$K_2 = 550/50 - 1 = 10$$

While there may be little to choose between the two sets of parameter values, one possible reason for making $\tau_1 = 500$ sec is that there is a slight advantage in capacitor voltage ratings.

E. Minimum Redundancy Transformation Between Body and Inertial Frame for a Typical Strapped-Down Inertial Navigation System, A. Holick

1. Introduction

For the application of electrically suspended gyros (ESGs) to either long life spacecraft or to more conventional aerospace inertial navigators (SPS 37-59, Vol. III, pp. 128-130; SPS 37-52, Vol. III, pp. 52-55; SPS 37-56, Vol. III, pp. 133-135), it is desirable to make the best possible use of the information available from the gyros. Equations that take advantage of the inherent redundancy in a two-ESG inertial reference have been developed. These equations convert the direction cosine data obtained from the six-gyro pickoffs (three per gyro) into attitude

information. Previously, the data from two pickoffs on each gyro had to be given to form the attitude matrix. It is now possible to derive this matrix when data from two pickoffs on one gyro and one or the other are given. These new equations make use of the fact that the angle between the spin axes of the two gyros is known and is changing only slowly. The new equations require only slightly more complex software and do not affect the hardware at all. These changes enhance system reliability and performance with only a small increase in cost for the more complex software.

2. Approach

The transformation M_{BI} , between body and inertial frame, is a vital part of strapped-down inertial navigation systems and must be computed continuously on-board. Let the inertial reference frame be derived from two ESGs. If the spin axis falls within an approximately 44-deg cone about a particular pickoff axis, then the corresponding direction cosine measurement cannot be made. Since these cones are not overlapping, two out of three direction cosines are always theoretically available from each gyro for computing the transformation. However, experience has shown that one of the four measurements may be corrupted sporadically by intolerably high noises (the probability of multiple transient failures is very small). Equations for computing M_{BI} from three rather than four direction cosines are given below. It is assumed that the count data which is produced by the gyro pickoffs has been converted to direction cosine form by use of pattern calibration data.

The problem of finding missing direction cosines can be approached rather straightforwardly by using orthogonality relations and a prior knowledge of the angle between the two spin vectors. This approach is taken first to illustrate the basic problem, but proves to be too complicated for mechanization, especially in the presence of pickoff misalignments. Let $\{x_{11}, x_{12}, x_{13}\}$ and $\{x_{21}, x_{22}, x_{23}\}$ denote the direction cosines of the spin vectors of gyros 1 and 2 in the pickoff frames. The components in the common body frame are, for instance, $\{-x_{11}, -x_{12}, -x_{13}\}$ and $\{x_{21}, x_{22}, -x_{23}\}$. Suppose $\{x_{11}, x_{13}\}$ and $\{x_{21}, x_{22}\}$ are measured at a particular time and x_{22} is found to fall outside the acceptance limits so that it is to be replaced by some function of the correct measurements $\{x_{11}, x_{13}, x_{21}\}$. There are three equations available:

$$x_{11}^2 + x_{12}^2 + x_{13}^2 = 1 \quad (1)$$

$$x_{21}^2 + x_{22}^2 + x_{23}^2 = 1 \quad (2)$$

$$-x_{11}x_{21} - x_{12}x_{22} + x_{13}x_{23} = \cos \alpha \stackrel{\Delta}{=} a \quad (3)$$

where α is the angle between the spin vectors. The signs in Eq. (3) stem from the particular realization of the body frame.

Then, x_{22} can be computed from the quadratic equation

$$x_{22}^2 + \frac{2x_{12}(a + x_{11}x_{21})}{1 - x_{11}^2}x_{22} + \frac{a(a + 2x_{11}x_{21}) + x_{13}^2(x_{21}^2 - 1) + x_{11}^2x_{21}^2}{1 - x_{11}^2} = 0$$

with a singularity for $x_{11} = 1$ and with a sign ambiguity to be resolved in the on-board computer. It can be shown quite generally that three direction cosines suffice to determine M_{BI} . Not all three measurements must come from the

same gyro, of course. It is well known that any change of orientation of a body with one fixed point is equivalent to a rotation about a fixed line through that point (Euler's theorem). One may describe body attitude in terms of the direction cosines $\{\ell, m, n\}$ of this rotation line plus the rotation ϕ required for matching body and inertial frame. The parameters $\{\ell, m, n, \phi\}$ constitute a once-redundant description of body orientation since $\ell^2 + m^2 + n^2 = 1$.

Define

$$\left. \begin{aligned} g_1 &= \ell \tan \phi/2 \\ g_2 &= m \tan \phi/2 \\ g_3 &= n \tan \phi/2 \end{aligned} \right\} \quad (4)$$

Then the transformation M_{BI} can be expressed in terms of the three Euler-Rodriguez parameters g_1, g_2, g_3 :

$$M_{BI} = \frac{1}{1 + g_1^2 + g_2^2 + g_3^2} \begin{bmatrix} (1 + g_1^2 - g_2^2 - g_3^2) & 2(g_1g_2 + g_3) & 2(g_1g_3 - g_2) \\ 2(g_1g_2 - g_3) & (1 - g_1^2 + g_2^2 - g_3^2) & 2(g_1 + g_2g_3) \\ 2(g_1g_3 + g_2) & 2(g_2g_3 - g_1) & (1 - g_1^2 - g_2^2 + g_3^2) \end{bmatrix}$$

But rather than solving this problem in detail, we consider the slightly more complicated case where the pickoff axes are misaligned. To comprehend the complexity of the situation, we define the operational modes, depending on which measurements are available. Table 3 shows the 18 possible combinations. The modes are labeled with a three-digit number, the first of which designates the gyro from which two measurements are available. Obviously, the modes $\{2, i, j\}$ are obtained from $\{1, i, j\}$ by adding 1 to the first index of the x_{ij} , module 2. Hence, attention may be restricted to the first nine modes in Table 3.

Let $\{y_{11}, y_{12}, y_{13}\}$ and $\{y_{21}, y_{22}, y_{23}\}$ denote the actually measured direction cosines, and $\{x_{11}, x_{12}, x_{13}\}$ the corresponding corrected ones. The transformation matrices between both frames are given in Table 4, where C^1 and C^2 are defined by

$$x^i = C^i y^i$$

with

$$x^i = (x_{i1}, x_{i2}, x_{i3})^T$$

$$y^i = (y_{i1}, y_{i2}, y_{i3})^T$$

$$i = 1, 2$$

Table 3. Measurement modes

Mode	Available measurements						Variables	Variable to be eliminated
	y_{11}	y_{12}	y_{13}	y_{21}	y_{22}	y_{23}		
111		1	1	1			$x_{11} \ x_{22} \ x_{23}$	x_{22}
112		1	1		1		$x_{11} \ x_{21} \ x_{23}$	x_{21}
113		1	1			1	$x_{11} \ x_{21} \ x_{22}$	x_{22}
121	1		1	1			$x_{12} \ x_{22} \ x_{23}$	x_{22}
122	1		1		1		$x_{12} \ x_{21} \ x_{23}$	x_{21}
123	1		1			1	$x_{12} \ x_{21} \ x_{22}$	x_{22}
131	1	1		1			$x_{13} \ x_{22} \ x_{23}$	x_{22}
132	1	1			1		$x_{13} \ x_{21} \ x_{23}$	x_{21}
133	1	1				1	$x_{13} \ x_{21} \ x_{22}$	x_{22}
211	1				1	1	$x_{12} \ x_{13} \ x_{21}$	x_{12}
212		1			1	1	$x_{11} \ x_{13} \ x_{21}$	x_{11}
213			1		1	1	$x_{11} \ x_{12} \ x_{21}$	x_{12}
221	1			1		1	$x_{12} \ x_{13} \ x_{22}$	x_{12}
222		1		1		1	$x_{11} \ x_{13} \ x_{22}$	x_{11}
223			1	1		1	$x_{11} \ x_{12} \ x_{22}$	x_{12}
231	1			1	1		$x_{12} \ x_{13} \ x_{23}$	x_{12}
232		1		1	1		$x_{11} \ x_{13} \ x_{23}$	x_{11}
233			1	1	1		$x_{11} \ x_{12} \ x_{23}$	x_{12}

Table 4. Transformation matrices for the various modes

Measurement modes	Transformations between pickoff frames	
	C ¹	C ²
111	A ₁	B ₁
112	A ₁	B ₂
113	A ₁	B ₃
121	A ₂	B ₁
122	A ₂	B ₂
123	A ₂	B ₃
131	A ₃	B ₁
132	A ₃	B ₂
133	A ₃	B ₃

where

$$A_1 = \begin{bmatrix} 1 & 0 & 0 \\ c_{21}^1 & 1 & -c_{23}^1 \\ c_{31}^1 & -c_{32}^1 & 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} 1 & -c_{12}^1 & c_{13}^1 \\ 0 & 1 & 0 \\ c_{31}^1 & -c_{32}^1 & 1 \end{bmatrix} \quad A_3 = \begin{bmatrix} 1 & -c_{12}^1 & c_{13}^1 \\ c_{21}^1 & 1 & -c_{23}^1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 1 & -c_{12}^2 & -c_{13}^2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad B_2 = \begin{bmatrix} 1 & 0 & 0 \\ c_{21}^2 & 1 & -c_{23}^2 \\ 0 & 0 & 1 \end{bmatrix} \quad B_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ c_{31}^2 & -c_{32}^2 & 1 \end{bmatrix}$$

and c_{ij}^k is the small misalignment.

Note that misalignments are taken into account only for the measured components. Consider, for instance, the mode 111; there is no need for knowing y_{11} , y_{22} , y_{23} and one can set for simplicity

$$x_{11} = y_{11}, x_{22} = y_{22}, x_{23} = y_{23}$$

Computation of the missing (corrected) direction cosines is performed iteratively. The concept is illustrated next for mode 111. Given $\{y_{12}, y_{13}, y_{21}\}$, find all the direction cosines, x_{ij} , in the corrected pickoff frames.

First, we look at Eq. (1) as a nonlinear equation for x_{11} and solve it iteratively with the Newton-Raphson procedure (the index i denotes iteration cycle):

$$x_{11}^{i+1} - x_{11}^i = -\frac{f_1}{f'_1}$$

$$f_1 = (x_{11}^2 + x_{12}^2 + x_{13}^2 - 1)^i$$

$$f'_1 = 2x_{11}^i$$

$$x_{12}^i = c_{21}^1 x_{11}^i + y_{12} - c_{23}^1 y_{13}$$

$$x_{13}^i = c_{31}^1 x_{11}^i - c_{32}^1 y_{12} + y_{13}$$

Then Eq. (2) is solved iteratively for x_{23} after elimination of x_{22} using constraint (3):

$$x_{23}^{i+1} - x_{23}^i = -\frac{f_2}{f'_2}$$

$$f_2 = (x_{21}^2 + x_{22}^2 + x_{23}^2 - 1)^i$$

$$f'_2 = 2 \left(x_{23}^i + c_{13}^2 x_{21}^i - \frac{(x_{22}^i - c_{12}^2 x_{21}^i)(x_{13}^i - c_{13}^2 x_{11}^i)}{c_{12}^2 x_{11}^i - x_{12}^i} \right)$$

$$x_{21}^i = y_{21} - c_{12}^2 x_{22}^i + c_{13}^2 x_{23}^i$$

$$x_{22}^i = \frac{a + x_{11}^i (y_{21} + c_{13}^2 x_{23}^i) - x_{13}^i x_{23}^i}{c_{12}^2 x_{11}^i - x_{12}^i}$$

3. Equation Switching Logic

The mechanization equations are grouped into six sets. Each of the first nine modes in Table 3 can be implemented with two out of these six sets plus some additional equations common to all modes. The algorithms for the remaining modes $\{2, i, j\}$ are obtained by simple indexing as described in Subsection 2. The pattern used to relate

Table 5. Modes and equation sets

Mode element ^a	Equation set					
	I	II	III	IV	V	VI
a	1	1	1	1	1	1
b	1	2	3	— ^b	— ^b	— ^b
c	— ^b	— ^b	— ^b	1	2	3

^aModes are comprised of a 3-digit label {a, b, c}.
^bThis entry can be 1, 2, or 3 depending on measurement mode.

equations and modes can be seen from Table 5. For mode {1 1 1}, for instance, we need the equation sets I and IV.

4. Mechanization Equations

The equations used to implement the operational modes {1, i, j} are given below. They are to be combined in accordance with Table 5. The upper index *i* denotes iteration cycle.

Equation set I

$$x_{11}^{i+1} - x_{11}^i = -\frac{f_1}{f'_1}$$

$$f'_1 = 2x_{11}^i$$

$$x_{12}^i = c_{21}^1 x_{11}^i + y_{12} - c_{23}^1 y_{13}$$

$$x_{13}^i = c_{31}^1 x_{11}^i - c_{32}^1 y_{12} + y_{13}$$

Equation set II

$$x_{12}^{i+1} - x_{12}^i = -\frac{f_1}{f'_1}$$

$$f'_1 = 2x_{12}^i$$

$$x_{11}^i = y_{11} - c_{12}^1 x_{12}^i + c_{13}^1 y_{13}$$

$$x_{13}^i = c_{31}^1 y_{11} - c_{32}^1 x_{12}^i + y_{13}$$

Equation set III

$$x_{13}^{i+1} - x_{13}^i = -\frac{f_1}{f'_1}$$

$$f'_1 = 2x_{13}^i$$

$$x_{11}^i = y_{11} - c_{12}^1 y_{12} + c_{13}^1 x_{13}^i$$

$$x_{12}^i = c_{21}^1 y_{11} + y_{12} - c_{23}^1 x_{13}^i$$

Equation set IV

$$x_{23}^{i+1} - x_{23}^i = -\frac{f_2}{f'_2}$$

$$f'_2 = 2 \left(x_{23}^i + c_{13}^2 x_{21}^i - \frac{(x_{22}^i - c_{12}^2 x_{21}^i)(x_{13}^i - c_{13}^2 x_{11}^i)}{c_{12}^2 x_{11}^i - x_{12}^i} \right)$$

$$x_{21}^i = y_{21} - c_{12}^2 x_{22}^i + c_{13}^2 x_{23}^i$$

$$x_{22}^i = \frac{a + x_{11}^i (y_{21} + c_{13}^2 x_{23}^i) - x_{13}^i x_{23}^i}{c_{12}^2 x_{11}^i - x_{12}^i}$$

Equation set V

$$x_{23}^{i+1} - x_{23}^i = -\frac{f_2}{f'_2}$$

$$f'_2 = 2 \left(x_{23}^i - c_{23}^2 x_{22}^i + \frac{(x_{21}^i + c_{21}^2 x_{22}^i)(x_{13}^i + c_{23}^2 x_{12}^i)}{x_{11}^i + c_{21}^2 x_{12}^i} \right)$$

$$x_{21}^i = -\frac{a + x_{12}^i (y_{22} - c_{23}^2 x_{23}^i) - x_{13}^i x_{23}^i}{x_{11}^i + c_{21}^2 x_{12}^i}$$

$$x_{22}^i = c_{21}^2 x_{21}^i + y_{22} - c_{23}^2 x_{23}^i$$

Equation set VI

$$x_{21}^{i+1} - x_{21}^i = -\frac{f_2}{f'_2}$$

$$f'_2 = 2x_{21}^i + c_{31}^2 x_{23}^i$$

$$- \frac{(x_{22}^i - c_{32}^2 x_{23}^i)(x_{11}^i - x_{13}^i c_{31}^2)}{x_{12}^i + c_{32}^2 x_{13}^i}$$

$$x_{22}^i = -\frac{a - x_{13}^i (y_{23} + c_{31}^2 x_{21}^i) + x_{11}^i x_{21}^i}{x_{12}^i + c_{32}^2 x_{13}^i}$$

$$x_{23}^i = c_{31}^2 x_{21}^i - c_{32}^2 x_{22}^i + y_{23}$$

where, for all sets,

$$f_1 = (x_{11}^2 + x_{12}^2 + x_{13}^2 - 1)^i$$

$$f_2 = (x_{21}^2 + x_{22}^2 + x_{23}^2 - 1)^i$$

F. State Determination for Disturbed Limit Cycle Operations via Nonlinear Filtering,

A. K. Bejczy

1. Introduction

Many operations of a spacecraft attitude-control system exhibit the features of a particular dynamical behavior called limit cycle operation. There are cases when limit cycle operations in the spacecraft attitude-control system are (or can be) integrated into the overall spacecraft control problem by simple conventional means. There are cases, however, when the desire for a more precise and more reliable control performance warrants a dynamically fine-structured handling of limit cycle operations in the spacecraft attitude-control system. This, in turn, requires detailed information on the state of the limit cycle operation in question. When the model for limit cycle dynamics is known in terms of ordinary differential equations, then the problem of state determination is relatively straightforward even though it is, in general, not a trivial study on integrating ordinary differential equations. For many practical cases, however, the dynamical model for limit cycle operations in the spacecraft attitude-control system must be a stochastic model because of the inevitable random forces acting on the system. This fact adds new features to the state determination problem.

The present article is addressed to the question of determining the state of a disturbed limit cycle operation, given noisy observations on the state. This state determination problem is treated as a filtering problem using the conceptual and technical framework of modern filter theories with an additional constraint: the filter algorithm should be feasible for implementation on a limited capability flight computer. For convenience, the state determination problem is treated in the continuous time domain.

2. The van der Pol Model

A fairly large class of limit cycle operations is, in the deterministic case, described by the van der Pol equation (Ref. 1). Assuming that the acting random forces are independent Wiener processes, the stochastic state space

version of the van der Pol model is

$$\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} = \begin{Bmatrix} x_2 \\ -x_1 + \epsilon x_2 (1 - x_1^2) \end{Bmatrix} + \begin{Bmatrix} s_1 \\ s_2 \end{Bmatrix} \quad (1)$$

where the dot denotes time derivative, ϵ is a given constant, and s_1 and s_2 formally denote additive gaussian white noise with given covariance

$$R = \begin{pmatrix} R_{11} & 0 \\ 0 & R_{22} \end{pmatrix}$$

Let the noisy observations y_1 and y_2 on the states x_1 and x_2 be given as

$$\begin{Bmatrix} y_1 \\ y_2 \end{Bmatrix} = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} + \begin{Bmatrix} v_1 \\ v_2 \end{Bmatrix} \quad (2)$$

where v_1 and v_2 formally denote additive gaussian white noise with specified covariance

$$Q = \begin{pmatrix} Q_{11} & 0 \\ 0 & Q_{22} \end{pmatrix}$$

The *a priori* knowledge (initial conditions) on x_1 and x_2 is assumed to be given in terms of a normal distribution $N(\cdot, \cdot)$ with specified mean α_i and variance β_i :

$$x_1(0) = N(\alpha_1, \beta_1)$$

$$x_2(0) = N(\alpha_2, \beta_2)$$

Now, the task is to form a sequential filtering estimate on x_1 and x_2 in some appropriate "optimal" sense, using the information given by Eqs. (1) and (2).

3. A Constant Gain Nonlinear Filter

The filtering problem for the system given by Eqs. (1) and (2) is a nonlinear filtering problem because of the nonlinear nature of Eq. (1).

In three previous communications,³ a new analytical (deterministic) technique was briefly described and successfully applied to construct simple nonlinear filter schemes and to evaluate their performance. According to that new technique, and postulating a constant filter gain

³SPS 37-59, Vol. III, pp. 132-137; SPS 37-60, Vol. III, pp. 112-118; and SPS 37-61, Vol. III, pp. 158-164.

matrix, the following nonlinear filter is constructed for the stochastic van der Pol system (Eq. 1) observed through noisy measurements y_i specified by Eq. (2):

$$\dot{z}_1 = z_2 + G_{11}(y_1 - z_1) + G_{12}(y_2 - z_2) \quad (3)$$

$$\dot{z}_2 = z_1 + \epsilon z_2(1 - z_1^2) + G_{21}(y_1 - z_1) + G_{22}(y_2 - z_2) \quad (4)$$

where G_{ij} are constant filter gains, assumed to be given. Furthermore, $z_1(0) = \alpha_1, z_2(0) = \alpha_2$ are given.

The time history of the performance of the filter specified by Eqs. (3) and (4) is described approximately (in terms of second order statistics) by the following set of *deterministic* differential equations:

$$\begin{aligned} \dot{B}_{11} = & -2G_{11}B_{11} + 2(1 - G_{12})B_{12} + G_{11}^2Q_{11} \\ & + G_{12}^2Q_{22} + R_{11} \end{aligned} \quad (5)$$

$$\begin{aligned} \dot{B}_{12} = & -(1 - G_{21})B_{11} + (\epsilon - G_{11} - G_{22})B_{12} \\ & + (1 - G_{12})B_{22} + G_{11}G_{21}Q_{11} + G_{12}G_{22}Q_{22} \end{aligned} \quad (6)$$

$$\begin{aligned} \dot{B}_{22} = & -2(1 + G_{21})B_{12} + 2(\epsilon - G_{22})B_{22} \\ & + G_{21}^2Q_{11} + G_{22}^2Q_{22} + R_{22} \end{aligned} \quad (7)$$

where B_{ij} are the elements of the filter error covariance matrix. The initial conditions for Eqs. (5-7) are: $B_{11}(0) = \beta_1, B_{12}(0) = 0, B_{22}(0) = \beta_2$, with β_1 and β_2 given.

The significant feature of Eqs. (5-7) is the fact that they form a *closed* system of equations that can be integrated in a *deterministic sense* once the filter gains G_{ij} are specified, since the noise parameters R_{ii} and Q_{ii} are assumed to be given. The fact that Eqs. (5-7) form a closed system of equations (that is, that they are decoupled from the differential equations for the mean filter error m) is a consequence of the system's structure described by Eqs. (1) and (2), which, in the framework of the new performance evaluation method, yields for the time history of the mean filter error in this problem: $\dot{m} = \Psi(m, B)m$. This, in turn, results in $m(t) \equiv 0$ since $m(0) = 0$. Hence, $\dot{B} = \Phi(B, m) = \Phi(B, 0)$.

Now, for the filter specified by Eqs. (3) and (4), appropriate optimal constant gains G_{ij}^* can be determined from Eqs. (5-7) by requiring that the steady-state value of the filter error covariance is minimized. This yields the following construction for the minimum steady-state variance

constant gain (MSSVCG):

$$G^* = \bar{B}^* H^T Q^{-1} \quad (8)$$

where $\bar{B}^*$ denotes the minimum steady-state variance, and H^T is the transpose of the Jacobian of the observation vector h . Once $\bar{B}^*$ is determined, the problem is solved.

$\bar{B}^*$ can be determined by substituting Eq. (8) into Eqs. (5-7) and solving the resulting Riccati system of *algebraic* ($\dot{B} = 0$) equations. From a computational point of view, however, it is more convenient to find $\bar{B}^*$ by solving the resulting Riccati system of *differential* ($\dot{B} \neq 0$) equations when Eq. (8) is substituted into Eqs. (5-7) such that the steady-state mark (the bar) over B is omitted.

4. Time-Varying Gain Nonlinear Filters

For the state determination problem under consideration, the known nonlinear filter theories yield an algorithm constructed of five coupled nonlinear *stochastic* differential equations. According to that algorithm, the filter gain and the (approximate) filter error covariance are *stochastic* quantities; they are computed on-line and are time-varying because of their parametric dependency on the estimated state.

To compare the structure and performance of the MSSVCG nonlinear filter described in the previous subsection to the structure and performance of other nonlinear filter algorithms for the problem under consideration, the equations resulting from the maximum principle least-squares (MPLS) nonlinear filter theory⁴ (Ref. 2) are:

$$\dot{z}_1 = z_2 + P_{11}Q_{11}^{-1}(y_1 - z_1) + P_{12}Q_{22}^{-1}(y_2 - z_2) \quad (9)$$

$$\begin{aligned} \dot{z}_2 = & -z_1 + \epsilon z_2(1 - z_1^2) + P_{12}Q_{11}^{-1}(y_1 - z_1) \\ & + P_{22}Q_{22}^{-1}(y_2 - z_2) \end{aligned} \quad (10)$$

$$\dot{P}_{11} = 2P_{12} - P_{11}^2Q_{11}^{-1} - P_{12}^2Q_{22}^{-1} + R_{11} \quad (11)$$

$$\begin{aligned} \dot{P}_{12} = & -(1 + 2\epsilon z_1 z_2)P_{11} + \epsilon(1 - z_1^2)P_{12} + P_{22} \\ & - P_{11}P_{12}Q_{11}^{-1} - P_{12}P_{22}Q_{22}^{-1} \end{aligned} \quad (12)$$

$$\begin{aligned} \dot{P}_{22} = & -2(1 + 2\epsilon z_1 z_2)P_{12} + 2\epsilon(1 - z_1^2)P_{22} \\ & - P_{12}^2Q_{11}^{-1} - P_{22}^2Q_{22}^{-1} + R_{22} \end{aligned} \quad (13)$$

⁴The MPLS nonlinear filter equations for the present problem are formally equivalent to applying linear filtering about the computed mean; they are also equivalent to the minimum variance filter equations for a linear expansion of the system's nonlinearities.

where P_{ij} are elements of the (approximate) filter error covariance matrix and $P_{ij}Q_{jj}^{-1}$ in Eqs. (9) and (10) are the filter gains. The initial conditions for Eqs. (9–13) are the same as those given in the previous subsection for the MSSVCG nonlinear filter equations (Eqs. 3–7).

Note that the basic difference between the MSSVCG and MPLS filters (Eqs. 3–7 and Eqs. 9–13, respectively) is that, in the former case, the filter gain is a precomputed constant matrix and the filter performance is given by deterministic differential equations (thus, it is also precomputed), while in the latter case the filter gain matrix and the filter performance expressed by the P matrix are stochastic quantities that must be computed “on-line” and evaluated by Monte Carlo techniques. The MSSVCG filter calls for the implementation of integrating two coupled nonlinear differential equations, while the MPLS filter calls for the implementation of integrating five coupled nonlinear differential equations.

5. Numerical Results

Several cases with different measurement configuration, noise parameters, and initial conditions were computed and simulated on the digital computer. To illustrate the overall features of the obtained results, two cases are presented. In both cases the parameter ϵ was equal to one. Furthermore, it was also assumed in both cases that $x_1(0) = \alpha_1 = 1$, $x_2(0) = \alpha_2 = 0$ and such that $\beta_1 = \beta_2 = 0$. Having zero values for the *a priori* ($t = 0$) variances β_1 and β_2 means that, for practical purposes, the knowledge of $x_1(0)$ and $x_2(0)$ can be regarded as exact. This *a priori* distribution corresponds to a situation when the initiation of the dynamical process can be regarded as perfectly known.

Case 1. The noise parameters are: $R_{11} = 0$, $R_{22} = Q_{11} = Q_{22} = 0.3$. That is, there is dynamical noise acting only on the x_2 coordinate but both coordinates (x_1 and x_2) are observed through noisy measurements. The minimum steady-state variance from Eqs. (5–8) is

$$\begin{pmatrix} \bar{B}_{11}^* & \bar{B}_{12}^* \\ \bar{B}_{12}^* & \bar{B}_{22}^* \end{pmatrix} = \begin{pmatrix} 0.267 & 0.162 \\ 0.162 & 0.534 \end{pmatrix} \quad (14)$$

and the corresponding values for the optimal constant gains are

$$\begin{pmatrix} G_{11}^* & G_{12}^* \\ G_{21}^* & G_{22}^* \end{pmatrix} = \begin{pmatrix} 0.88 & 0.54 \\ 0.53 & 1.78 \end{pmatrix} \quad (15)$$

Figure 10 depicts the theoretical and experimental time histories of the filter error covariance for the MSSVCG

and MPLS nonlinear filters. The theoretical MSSVCG curves are the solutions of Eqs. (5–7) with the G_{ij}^* values given by Eq. (15), while the theoretical MPLS curves are the mean values for $P_{ij}(t)$ from 200 simulated solutions of Eqs. (9–13). The experimental curves for the MSSVCG and MPLS filters were formed according to the standard empirical formulas for the covariance. (See also SPS 37-59, Vol. III, pp. 132–137.) The sample space again contained 200 Monte Carlo-type simulated solutions of Eqs. (3) and (4) with the gains given by Eq. (15) for the MSSVCG filter and 200 Monte Carlo-type simulated solutions of Eqs. (9–13) for the MPLS filter.

Here and in all subsequently quoted calculations, the Monte Carlo-type simulated solutions mean numerical integration of the relevant ordinary differential equations with stochastic inputs. The sampling time was made equal to the integration stepsize $\Delta t = 0.1$ s. The stochastic inputs for dynamical and measurement noise were obtained from a gaussian random number generator through the application of the equivalence claim

$$\Sigma' = \frac{\Sigma}{\Delta t}$$

where Σ and Σ' are the variance of a gaussian random process and random sequence, respectively, and Δt is the integration stepsize.

The curves in Fig. 10 formed from Monte Carlo simulations are depicted in 0.5-s time intervals with linear interpolation between these points. As seen in this figure, the MPLS filter yields a somewhat better performance than the MSSVCG filter (which, in fact, is expected). But the MPLS filter's performance is not substantially better than the performance of the much simpler MSSVCG filter when the performance of the two filters is measured by their respective variances. It is also noted that the theoretical MSSVCG variance curves *underestimate* somewhat the MSSVCG filter's performance in the sense that the obtained experimental variance values are lower (hence better) than the theoretical variance values for the MSSVCG filter. It is also noted that the theoretical and experimental covariance values of the MPLS filter agree almost completely.

Case 2. The noise parameters are now: $R_{11} = Q_{11} = 0$, $R_{22} = Q_{22} = 0.3$. That is, only the x_2 coordinate is observed through noisy measurements and only the x_2 coordinate is disturbed by dynamical noise.

Now, Eq. (8), substituted into the steady-state version of Eqs. (5–7), yields the rather interesting result that

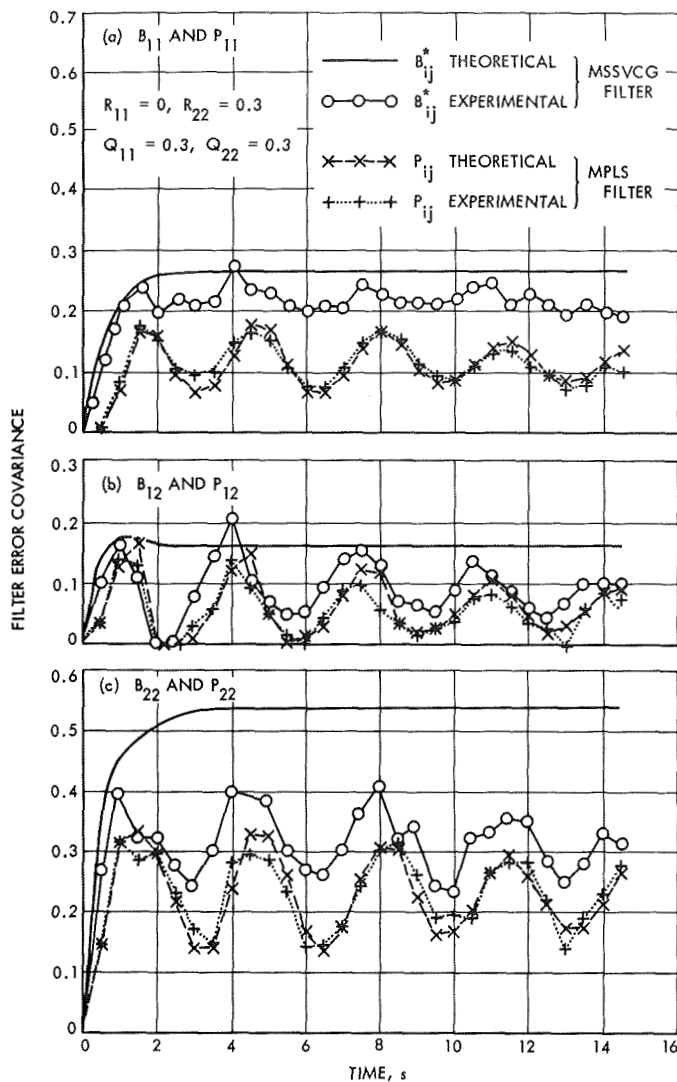


Fig. 10. Filter error covariance for Case 1

$\bar{B}_{12}^* = 0$ or $\bar{B}_{12}^* = 2Q_{22}$. But the last value would not yield positive real numbers for $\bar{B}_{11}^*$ and $\bar{B}_{12}^*$. Hence, for $\bar{B}_{12}^*$ only the zero value is acceptable which then yields for the minimum steady-state variance in this case:

$$\begin{pmatrix} \bar{B}_{11}^* & \bar{B}_{12}^* \\ \bar{B}_{12}^* & \bar{B}_{22}^* \end{pmatrix} = \begin{pmatrix} 0.73 & 0 \\ 0 & 0.73 \end{pmatrix} \quad (16)$$

The corresponding values for the optimal constant gains are

$$\begin{pmatrix} G_{12}^* \\ G_{22}^* \end{pmatrix} = \begin{pmatrix} 0 \\ 2.43 \end{pmatrix} \quad (17)$$

This means that the given noisy measurements on x_2 should be fed into the x_2 integration channel only, and

not at all into the x_1 integration channel! That is, in terms of the MSSVCG nonlinear filter algorithm, the "best" estimate on x_1 is obtained by simply integrating over the x_2 coordinate. (See Eq. 3.)

Figure 11 displays the theoretical and experimental time histories of the filter error covariance for the MSSVCG and MPLS nonlinear filters. The theoretical MSSVCG curves are solutions of Eqs. (5-7) with the G_{ij}^* values given by Eq. (17). The other curves are the same type of simulated solutions as explained for Case 1, but clearly with different gain values and noise parameters in the present case. The curves formed from Monte Carlo-type simulated solutions are again depicted in 0.5-s time intervals with linear interpolation between these points.

As seen again, the MPLS filter's performance is somewhat better (but not substantially) than the performance of the simple MSSVCG filter when measuring their performance by their respective variances. This is again not unexpected because of the structural restrictions involved in constructing the MSSVCG filter. It is also seen in Fig. 11 that the theoretical (analytical) MSSVCG values underestimate somewhat the filter's performance since the experimental variance values are lower than the theoretical values. This is so in greater extent in this case than it was in Case 1. In this regard it is noted, however, that Eqs. (5-7) do not and cannot reflect upon the state-dependent properties of the filtering problem which is a consequence of the involved approximations. Dealing with self-excited or relaxation oscillation problems where the state dependency of the dynamics is the essential element of the problem, it is rather surprising, however, that the approximate analytical equations (Eqs. 5-7) still provide a fair picture for the MSSVCG filter's performance. (Note in this regard that the experimental $\bar{B}_{12}^*$ curve of the MSSVCG filter oscillates about zero as a *mean* value, while the theory predicts $\bar{B}_{12}^* \equiv 0$.) It should also be noted how well the theoretical values (the relevant P_{ij} equations) agree with the experimental values for the MPLS filter. All of these results should be compared to the difficulties encountered when a much more complex filtering approach was applied to the same stochastic van der Pol problem (Ref. 3).

When comparing Fig. 11 to Fig. 10, it should be observed that employing (noisy) measurements on the x_1 coordinate in addition to the (noisy) measurements on the x_2 coordinate greatly improves the filter estimate on the state of the disturbed van der Pol system. This is rather interesting since, for another nonlinear filtering problem, increasing the dimensionality of the observation vector

did not significantly improve the filter estimate on the state (SPS 37-60, Vol. III, pp. 112-118).

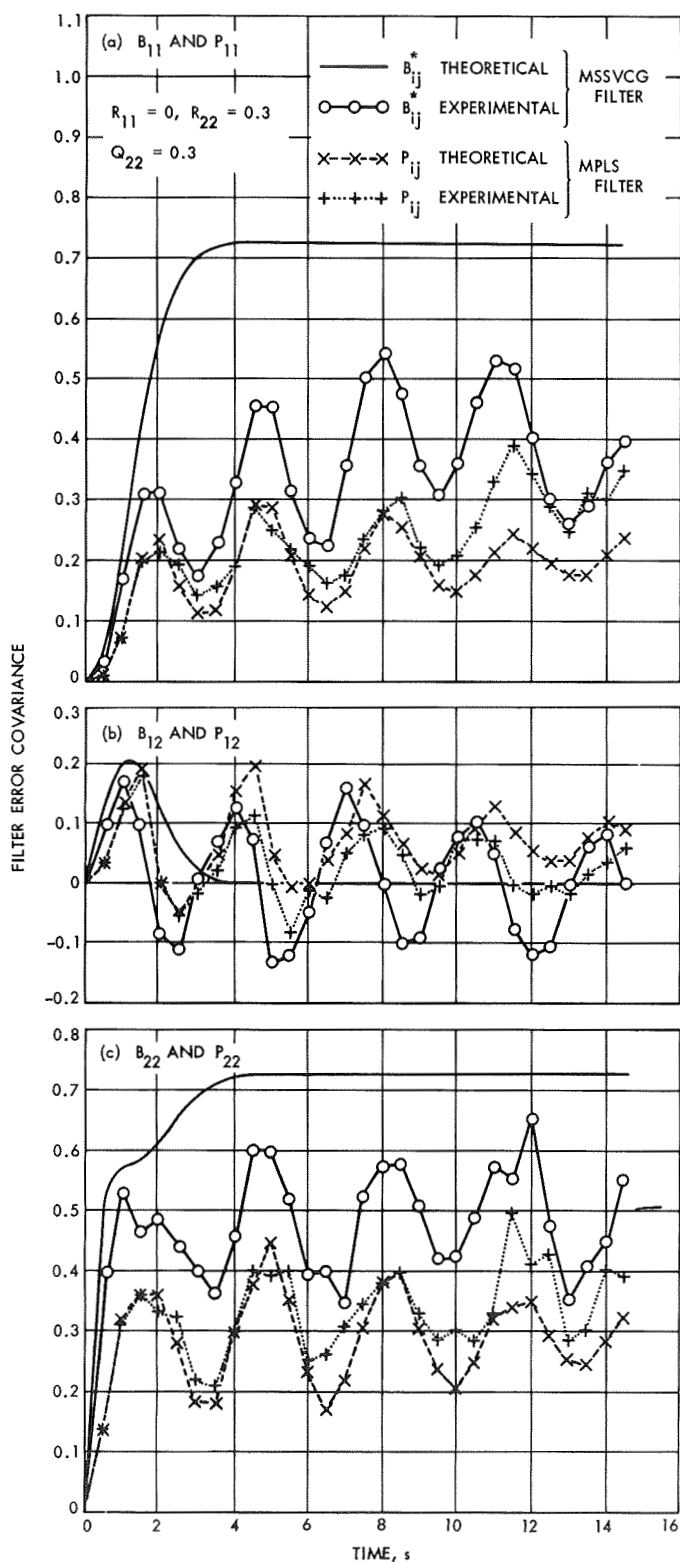


Fig. 11. Filter error covariance for Case 2

To compare the performance of the MSSVCG and MPLS nonlinear filters in terms of "actual" estimates, Fig. 12 depicts a sample case for the x_1 coordinate. (This sample case, selected arbitrarily, is one of the 200 realizations of the filter on the digital computer.) The σ channels centered about the estimated state are theoretical values of the standard deviations $\pm(\bar{B}_{11}^*)^{1/2}$ and $\pm(P_{11})^{1/2}$, respectively. As seen in Fig. 12, a substantial part ($\sim 90\%$) of the "true" trajectories are inside the σ channels of the respective filters. Figure 12 also gives a clear picture of the differences in the width of the σ channels of the MSSVCG and MPLS nonlinear filters. The width of the σ channels can be viewed as an immediate measure for the performance of the two filters.

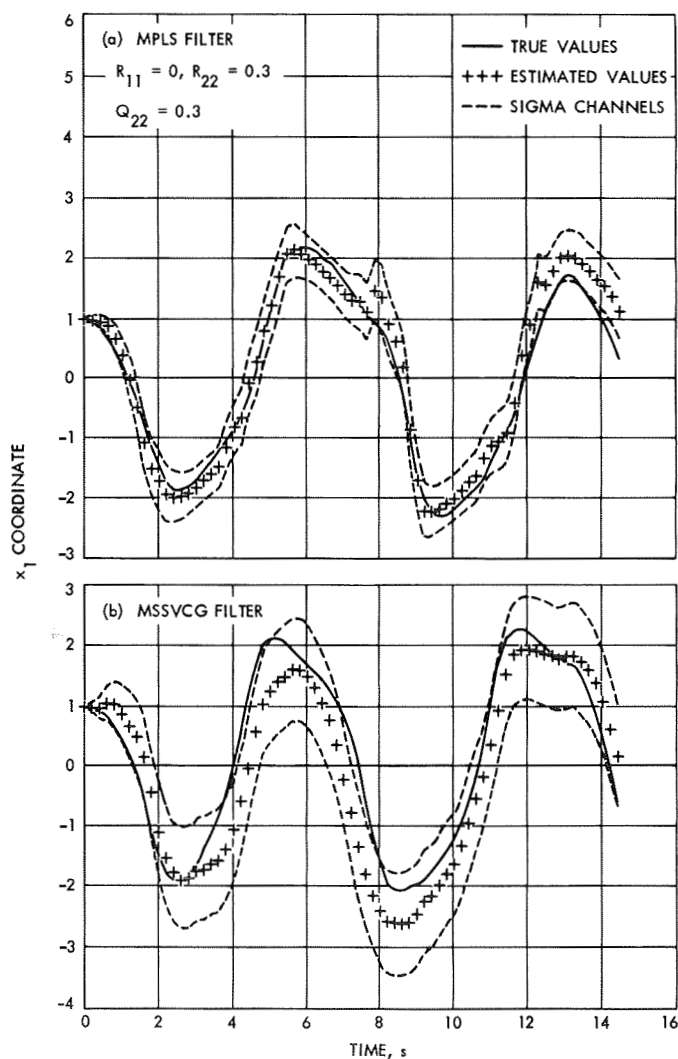


Fig. 12. A single realization of the filters for the x_1 coordinate (no observations on x_1)

6. Conclusions

(1) Comparing the experimental time histories of the variances of the MSSVCG and MPLS filters, it is seen that there is no significant difference between the actual performances of the two filters. In view of the significant differences between the structural complexities of the two filters, this fact is of great importance regarding the implementation of a suitable nonlinear filter for determining the state of a disturbed limit cycle operation of the van der Pol type. Clearly, the implementation of the simple MSSVCG filter is a much easier task than the implementation of the MPLS filter.

(2) The developed analytical techniques provide a fair approximate description of the MSSVCG filter's performance in parametric terms. Note in this regard that the significance of the determination of the minimum steady-state variance filter error should be viewed not only by itself (which in any case is only an approximation) but also in relation to the value of the optimal constant gain

for the filter the structure of which is fixed by Eq. (3). The optimal constant gain given by Eq. (8) provides the best performance for the filter specified by Eq. (3). Any other constant gain will worsen the performance (increase the variance) of the postulated constant gain nonlinear filter algorithm.

(3) The application of the simple MSSVCG nonlinear filter to self-excited (or self-sustained) oscillation problems in control systems provides improved knowledge on the state of the system.

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XIII. Materials

ENGINEERING MECHANICS DIVISION

A. Mesh Materials for Deployable Antennas,

J. G. Fisher

1. Introduction

High data rate transmission over great distances as required by the Thermoelectric Outer Planet Spacecraft (TOPS) Program necessitates large spacecraft antennas. The 14-ft-diameter high-gain antenna presently in the TOPS base line design cannot be accommodated within existing launch vehicle shrouds without furling. Since doubly curved rigid surfaces cannot be folded, a number of designs involving mesh reflecting surfaces supported on ribs have been developed. No matter what rib and deployment scheme is chosen, it is the mesh which establishes the precision of the reflecting surface, and so, in turn, the gain derived from the antenna under any set of conditions. An attempt has been made to sort out various mesh characteristics in order of their importance to surface precision, to understand the reasons for their relative influences, and to evaluate mesh samples procured from commercial sources.

2. Mesh Characteristics

Stretching a fabric or mesh over a set of radially arranged parabolic ribs will not produce a parabolic surface of rotation. Because of circumferential tension in the mesh,

it tends to chord between ribs. This effect can be reduced by increasing the number of ribs, at a weight penalty. In addition, because of radial tension, the mesh tends to chord in the radial direction as well, resulting in inward bulging of the reflective surface. This latter effect is most pronounced in deep paraboloids, and can be controlled only by adjusting the ratio of cross, or circumferential, tension to radial tension in the mesh. Since mesh tension tends to distort the supporting ribs, it is desirable that the mesh be as "soft" as possible, that is, have a low effective spring constant. From studies of the problem so far, it appears that this is the most important mesh characteristic.

The antenna is assembled with the ribs and mesh at the same temperature. If, in service, the temperature of the mesh changes in a manner different from that of the ribs, thermal distortion of the reflective surface due to changing tensions in the mesh is likely. Again, a "soft" mesh is desirable.

The reflectivity of the mesh surface to RF energy depends on a number of factors. From experiments described in SPS 37-61, Vol. III, pp. 99-106, it appears that electrical conductivity across the mesh is important. This implies not only conductivity through the individual mesh strands, but across the contacts between strands at mesh intersections.

Nonmetallic substrates from which an electrically conductive coating can flake off are at a disadvantage. This also applies if the substrate is metallic, but of poor electrical conductivity. For this reason also, metals such as aluminum and titanium which form tightly adherent protective oxides, or metals such as stainless steels which become passive in oxidizing media, may also present problems. The tests on bare Chromel-R discussed in SPS 37-61, Vol. III, are a case in point.

Advanced spacecraft antennas must operate for long periods in space. The mesh must resist deterioration from UV radiation, low temperature, and possibly from micrometeorite sputtering and solar wind impingement. If a nonmetallic substrate is to be used, it would appear important to have a continuous metallic coating.

3. Evaluation of Two Commercial Meshes

a. Copper-plated Dacron mesh. Thus far in the TOPS program, two types of mesh have been examined: copper-plated Dacron and gold-plated Chromel-R. The first, supplied by Lockheed Aircraft Co., consists of multifilament Dacron, electroless copper-plated and protected with a dip-applied silicone coating. The sample tested had been given no special handling, and might not be representative of carefully protected material. Figure 1 shows the construction. There is a single twisted strand in one direction, and two strands twisted together in the other. The twisting between crossovers lends diagonal rigidity to the mesh by

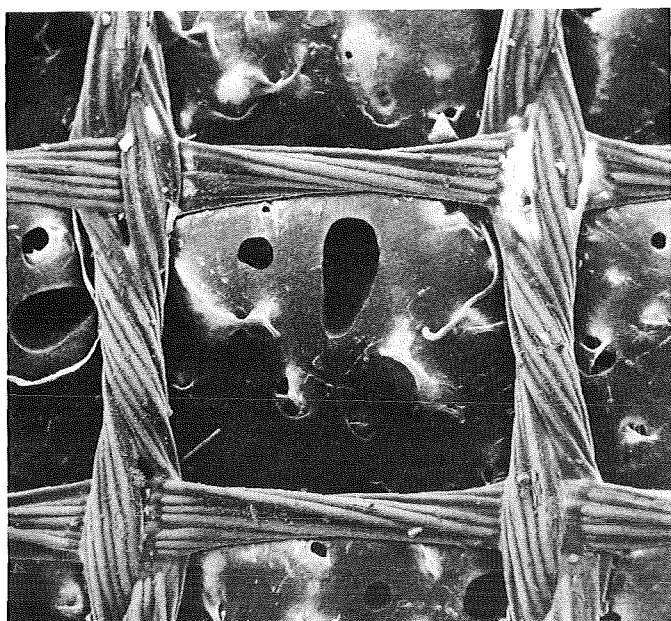


Fig. 1. Lockheed copper-plated Dacron mesh (49 $\times$)

reducing slippage. Due to the straight-through weave, the material is quite stiff in both warp and woof directions. Because plating is done on the finished mesh, electrical contact at crossovers is good so long as the electroless copper plating remains intact. Dacron is a dielectric, so cracks or flaking in the copper can be expected to have a serious effect on conductivity along the strands. Figure 2 shows the end of a single-twist strand, as cut with a sharp X-acto knife. The plating is mostly on the outside of the bundle and the copper deposit is brittle. Figure 3 shows that cracking of the copper and lifting off the Dacron is a genuine problem. The lifting occurred during manufacture, before the silicone dip had been applied. Cracking and lifting, such as is evident in Fig. 3, results in erratic reflectivity performance. Such behavior is reported in SPS 37-61, Vol. III.

Tests were made to determine the spring constant of the mesh in both longitudinal (parallel to the double strands) and transverse directions at room temperature. The results are shown in Table 1 for two sample widths. Presumably, the lower values for the narrower samples are caused by greater ease of deformation perpendicular to the axis of stress.

The effective thermal expansion and contraction of the mesh was measured in both longitudinal and transverse directions between 150 and -200°F , under two preloads

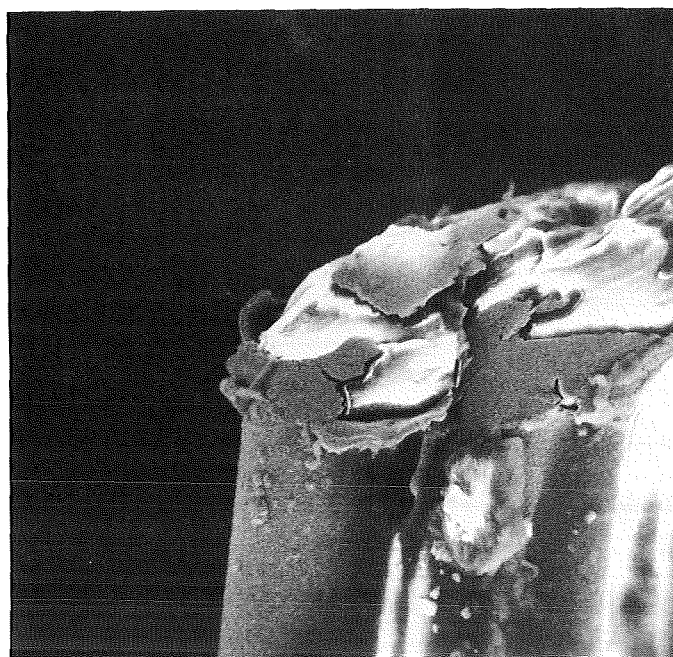


Fig. 2. Lockheed copper-plated Dacron mesh, knife-cut end (875 $\times$)

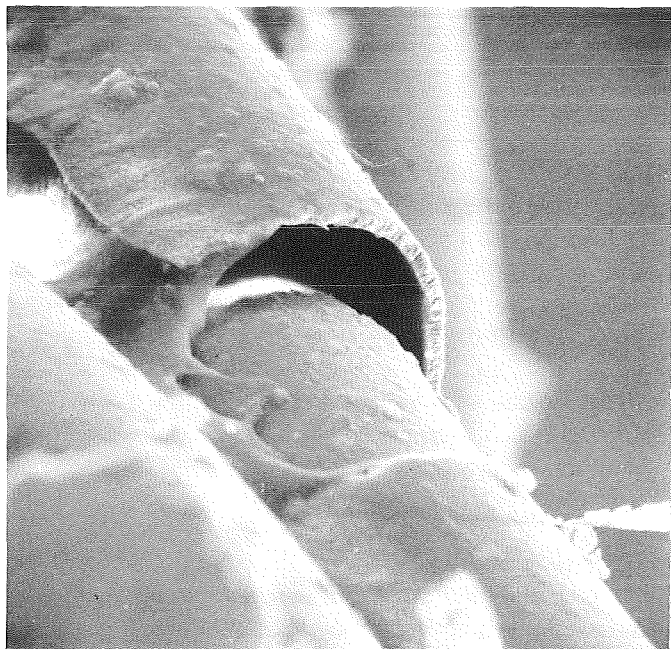


Fig. 3. Lockheed copper-plated Dacron mesh, showing lifted plate (2000 ×)

in tension. Table 2 shows the results. The direction of measurement appears to be less critical than in the case of tensile stretching. Note that if the mesh were completely restrained from shrinking, very large stresses would be set up. For example, use the 1450 lb/in./in. of width for the longitudinal direction (Table 1 above). Applying this to shrinkage from 70 to -200°F of 0.0074 in. under 200-g preload, the stress becomes $0.0074 \times 1450 = 10.7$ lbs/in. Such a loading is much more than the ribs can stand without unacceptable deformation.

Table 1. Effective spring constant of copper-plated Dacron mesh

Direction	Spring constant, lb/in./in.	
	4-in. wide sample	1-in. wide sample
Longitudinal	1450	433
Transverse	990	242

Table 2. Effective thermal expansion of copper-plated Dacron mesh

Direction	Thermal expansion, in./in. $^{\circ}\text{F}$			
	70 to 150°F range		-200 to 70°F range	
	200-g load	300-g load	200-g load	300-g load
Longitudinal	40×10^{-4}	57×10^{-4}	74×10^{-4}	83×10^{-4}
Transverse	35×10^{-4}	46×10^{-4}	74×10^{-4}	86×10^{-4}

b. Gold-plated Chromel-R mesh. The second mesh type examined is produced by Prodesco, and is a tricot-knit material made of 0.0005-in.-diameter Chromel-R wire, with 14 wires per strand. Chromel-R is a nickel base alloy containing nominally 20% chromium, and 3% each of iron and aluminum. It is highly passive in air in a manner similar to stainless steels, and commonly is used as an electrical resistance alloy. The sample evaluated was supplied by Radiation Incorporated, which had plated the mesh using an undercoat of 40- μ in. electroless nickel and a top layer of 5- μ in. electroless gold. Again the sample had received no special protection, and might not be representative of more carefully handled material. Figure 4 shows the appearance of the mesh at low magnification. The strands loop back and forth in an extremely devious path so that electrical continuity across the many contacts is the determining factor in effective electrical conductivity across the mesh surface. The importance of the gold plating is thus in improving the contact continuity. Being inherently noble, there is no oxide layer to increase resistance. Unplated mesh shows reflectivity losses roughly 10 times as high as mesh with gold plating (SPS 37-61, Vol. III).

The knit structure of the mesh results in an extremely compliant material. Stretching results primarily in re-orienting loops, as is seen in Fig. 5. With respect to soft spring action, the knit structure meets the primary requirement of a good antenna mesh material. However, the plating process is inadequate. As seen in Fig. 6, the plate

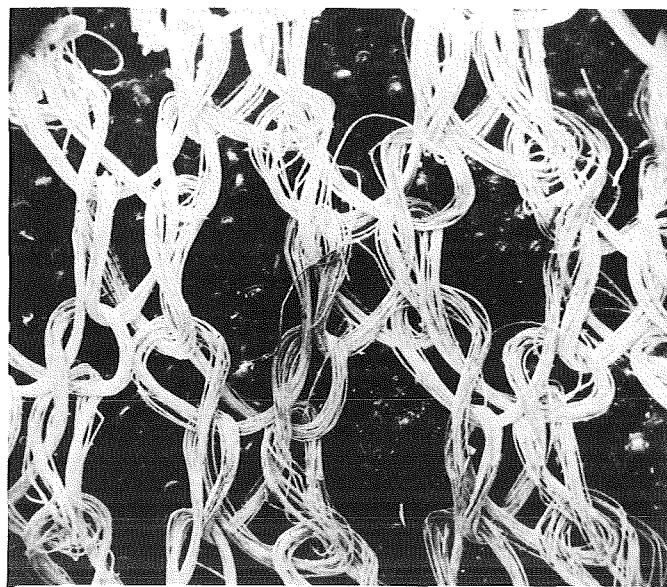


Fig. 4. Radiation, Inc., Chromel-R mesh, relaxed condition (18 ×)

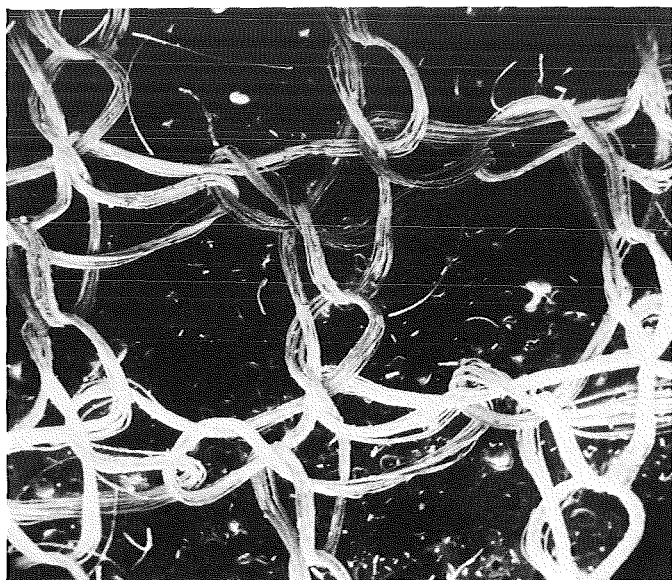


Fig. 5. Radiation, Inc., Chromel-R mesh, stretched condition (18 ×)

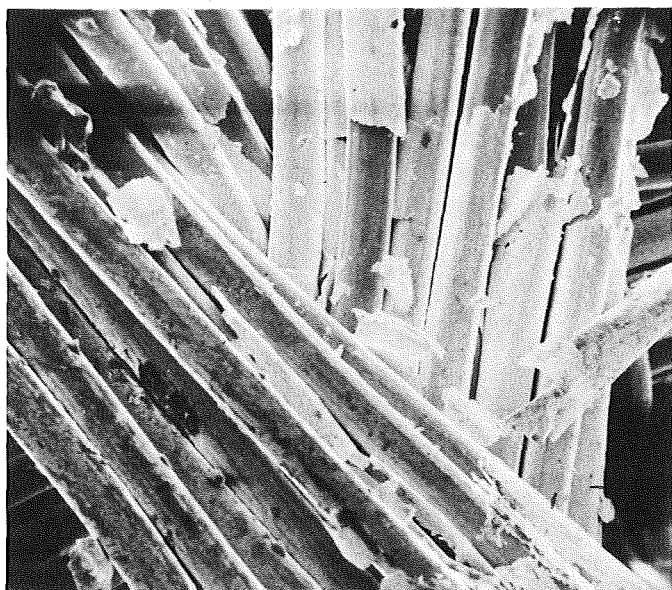


Fig. 6. Radiation, Inc., Chromel-R mesh, gold-plated (446 ×)

is brittle and nonadherent. Extensive flaking as the mesh works in service is inevitable. Nonadherence probably is due to the inherent passivity of the Chromel-R, and the difficulty of producing and maintaining an active surface in the plating bath during initial deposition. Compared to electroplating, electroless plating improves throwing power into the contact surfaces, but at a cost of increased plating brittleness. It should be noted that this brittleness

is not confined to material supplied by Radiation Incorporated. Other samples plated by other sources have been no better.

4. Conclusion

Many approaches to solving the antenna mesh problem are possible. To date only a few have been examined, and those in limited depth. One approach is to change the substrate to an inherently noble metal not dependent upon passivity for corrosion resistance. Plating would not be necessary since good electrical conductivity across contacts is provided by the base metal. A second approach is to change to a metal easier to activate and thus obtain improved adherence of a noble metal plating. A third approach is to change the coating process to produce an inherently ductile layer which will not crack as the mesh is flexed. With this third approach, adherence to the substrate is not critical since the ductile layer surrounds the substrate as a tube and cannot fall off.

The above approaches to solving the antenna mesh problem will be investigated. In addition, developments associated with the antenna ribs and central hub will be pursued.

B. Temperature Control Materials Technology,

J. R. Crosby and T. F. Moran

1. Multilayer Insulation Test Facility

The long-range objective of this task is to develop materials technology and fabrication processes necessary to optimize the performance characteristics of multilayer insulation thermal shield systems for use on spacecraft. Principal emphasis is placed on realistic fabrication and installation influences, and on thermal shield predictability and reproducibility.

The calorimeter for measuring the thermal properties of multilayer insulation has been designed, constructed, calibrated, and placed in operation. A detailed description of the calorimeter is reported in SPS 37-59, Vol. III, pp. 156-158. A sketch of the device is shown in Fig. 7.

From an energy balance of the power required to maintain a specified inner-wall temperature while radiating to a liquid nitrogen cold wall, a value of effective emittance for the insulation is determined. The purpose of the equipment and resultant testing is to determine possible factors that cause the order of magnitude difference in effective emittance which exists between theoretical and idealized

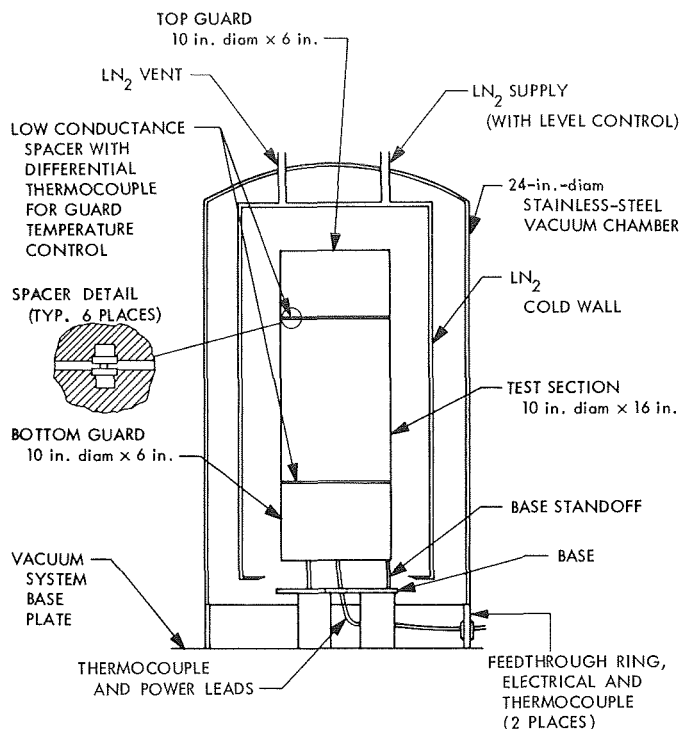


Fig. 7. Multilayer insulation thermal vacuum test setup

calorimeter data versus actual spacecraft insulation performance as determined from temperature control flight model tests. Much of this can be attributed to the method of application, handling, seams, stitching, patches, windows, corner, and edge effects, etc. It is these effects that are being evaluated as testing proceeds.

The apparatus has been run and calibrated. Refinements in instrumentation, recording capability, power circuits, LN₂ control, and shield mounting were all necessary to assure reproducibility and accuracy of the data. Figure 8a shows the effect of these improvements over the earlier data of Fig. 8b. The transient portion of the test time lasts approximately 12 h until steady-state conditions prevail; 24 h of steady-state operation usually comprises a test. It has been determined that reproducibility of data from test to test on the same shield is within $\pm 2.5\%$. At present, a comparison of thermal performance between the blanket configurations used on the *Mariner Mars 1969* and the *Mariner Mars 1971* systems is being made. *Mariner Mars 1969* utilized 20 layers of doubly aluminized $\frac{1}{4}$ -mil Mylar sheet with nylon net spacers. *Mariner Mars 1971* is using 15 layers of doubly aluminized $\frac{1}{8}$ -mil Mylar sheet with nylon net spacers. The resulting data will be compared with results attained from thermal model tests wherein, due to complexity, the uncertainty in the data is quite large. Thus, the calorimeter tests will be able to provide

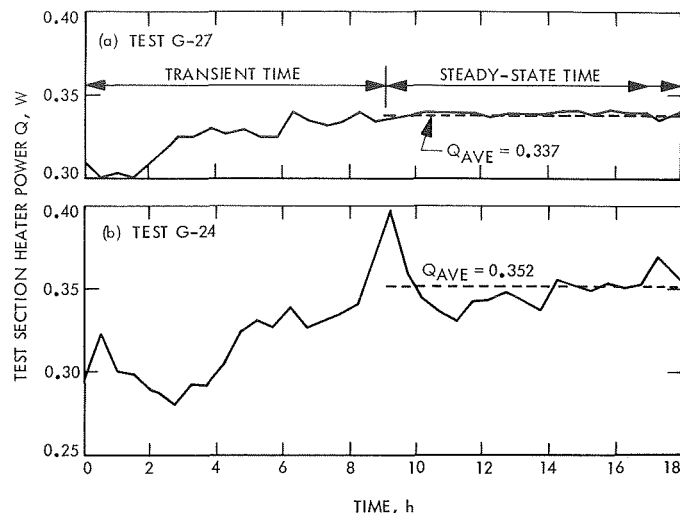


Fig. 8. Power history (typical tests)

a more meaningful input to existing and future project requirements.

2. Heat Pipe Materials Compatibility

A preliminary investigation on the formation of non-condensable gases in water/metal heat pipes was performed. A series of metals and alloys was tested to determine if the hydrogen gas generation noted by various investigators in water/stainless steel systems is peculiar to stainless alloys, also to develop a compatible water/metal system which precludes this problem. Metals evaluated included various stainless steel alloys, nickel, titanium, beryllium-copper, copper, brass, and bronze alloys.

The test procedure involved individually sealing the metal samples in a highly evacuated glass apparatus at 152°C for 279 h. The glass apparatus containing each immersed sample was provided with a break-seal and bulb appendage for use during mass spectrometer measurements for hydrogen, nitrogen, and oxygen performed after the thermal soak.

With the exception of the copper and several of the bronze alloys, all samples showed a significant generation of hydrogen gas occurring. Thus, fabrication of an all copper/copper alloy heat pipe was initiated. The heat pipe wall and mesh are made from commercially pure copper. All joining operations were performed by torch brazing using Sil-Fos 5, a fluxless copper-silver braze alloy. Currently, the heat pipe is undergoing final instrumentation with thermocouples. Operating conditions for the water/copper heat pipe will be 250–300°F at 125-psi pressure.

XIV. Applied Mechanics

ENGINEERING MECHANICS DIVISION

A. Development of Advanced Composite Liquid Propulsion Pressure Vessels, A. C. Knoell

1. Introduction

The purpose of this activity is to develop large-scale, advanced composite, filament-wound (FW) pressure vessels suitable for use in a space-storable liquid propulsion (OF_2 and B_2H_6) system. The total scope of the program consists of the design, analysis, fabrication, and testing of propellant and pressurant prototype vessels. The value of this activity rests in the development of an extremely lightweight, fuel-compatible, highly reliable tankage system for use on advanced spacecraft missions.

2. Design Development Activities

Preliminary design studies (see SPS 37-60, Vol. III, pp. 156-157) have been completed and a materials test and evaluation program was initiated. The design studies included development of suitable tank configurations and assessment of the effects of thermal shock, negative pressure loading during tank filling, and space environment on the structural performance of the tanks. The materials test and evaluation program includes both advanced composite and metal liner tensile, flexural, and shear specimens. Particular attention was given to boss-liner joint specimens and specimens designed to investigate pres-

surized gas leakage through metal joints and the strength of the composite-liner structural bond.

In support of these activities, a computer program was used to design the FW pressure vessels.

3. Program Results

The dome closures of the basic tank configurations described in SPS 37-60, Vol. III, were redesigned using the FW pressure vessel design program and the resulting tank configuration was adopted for final design layout. Essentially, the FW program develops the dome head-shape coordinates using planar-wound netting analysis theory and also determines wall thicknesses (and associated tank weights) by computing the number of planar and hoop wraps for a given internal tank pressure and ultimate lamina tensile strength. The overall tank dimensions reported in SPS 37-60, Vol. III, were maintained.

Liner-tank separation due to thermal (filling) shock was studied by computing the adhesive force required to maintain bonding between both components. It was found that an adhesive stress of approximately 55 psi was required in the transient case where the metallic liner and the boron-epoxy tank were at cryogenic and room temperature, respectively. In the steady-state case, where

both components reached cryogenic temperature, the required bonding stress was approximately half the transient value. These relatively low levels of bonding stress are well within the adhesive capability of the composite resin matrix; therefore, separation due to thermal shock should not be encountered.

A stability analysis of the propellant tank design was made to determine the resistance of the tank to buckling when subjected to a negative atmospheric pressure of 14 psi during fuel loading. It was found that the minimum tank buckling stress (governed by local crippling) was approximately 21,000 psi whereas the stress developed in the tank wall due to the pressure loading was approximately 1000 psi. The computed minimum buckling stress compares well with that derived from Ref. 1.

A brief study was made to determine the effects of space environments on the mechanical properties of advanced composite materials. Environments included in the study were radiation, cryogenic temperature, and vacuum. The types of radiation exposure considered were electrons, protons, neutrons, gamma rays, X-rays, and ultraviolet. The radiation sources considered were those from the belts of Jupiter and earth, galactic cosmic radiation, solar radiation, and radiation generated from a radioisotope thermoelectric generator unit.

In order to relate radiation dosages to typical spacecraft missions, a tabulation was made of the integrated radiation dosages expected for three missions: the proposed Grand Tour, *Mariner Venus-Mercury 1973*, and *Viking*. Allowable material radiation levels were determined from several sources including, for example, Ref. 2.

The results of the study indicated that the combined radiation dosages would not be of sufficient intensity to cause any significant degradation of the mechanical properties of the advanced composites. It was felt that surface damage, due to nonpenetrating radiation which might occur, could easily be prevented by the use of a thin sacrificial protective coating over the composite material. The data did indicate, however, that the combined effects of cryogenic temperatures and vacuum on advanced composite materials could be significant. An evaluation of these effects is in process.

In-house fabrication of advanced composite test specimens¹ is being completed. Some problems have been en-

¹Materials support provided and coordinated by Mr. W. Jensen of the JPL Materials Section.

countered in resin flow control during the cure phase of the boron-epoxy materials. Boron filament volume percentages that have been obtained were on the order of 65 to 70%. The graphite-epoxy materials, which are being investigated as possible backup composite materials, have provided more reasonable flow control and have yielded fiber volume percentages on the order of 50 to 55%. Although testing has begun, the data obtained have not yet been sufficiently reduced, analyzed, or evaluated for program usage.

Cost considerations have led to the selection of a suitable backup design fabrication method of making liners from formed and welded sheet aluminum. The designs will be based on the use of liners that are fabricated by spinning aluminum on a mandrel. The requirements relating to the need for precision holding fixtures during liner welding operations are being studied.

4. Critical Problem Areas

Critical problem areas identified to be in need of further work include:

- (1) Design criteria and allowables.
- (2) Structural attachments.
- (3) Fracture mechanics and nondestructive testing applications.
- (4) Thermal-vacuum effects.

Limited work is in progress in all of these problem areas. Full treatment of any of these areas is, unfortunately, beyond the scope of this program.

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2. Jaffe, L. D., and Rittenhouse, J. B., *Behavior of Materials in Space Environments*, Technical Report 32-150. Jet Propulsion Laboratory, Pasadena, Calif., Nov. 1, 1961.

B. Nonlinear Vibration of an Infinite Long Circular Cylinder, J. C. Chen

1. Introduction

In SPS 37-59, Vol. III, pp. 158-162, the nonlinear vibration of nonlinearly coupled structural systems was discussed where the only nonlinearity comes from the coupling units. All of the component systems are linear.

However, in other nonlinear vibration of structural systems, the nonlinearities are due to the fact that the vibrational amplitudes are large. This type of nonlinearity is called geometric nonlinearity. The governing differential equations of motion of a large amplitude vibration are nonlinear. This is sometimes due to nonnegligible nonlinear terms in strain-displacement relation or due to large angles in the motion, such as $\sin \theta$ cannot be approximated as θ , as in the case of infinitesimal motion. This article presents a discussion on the vibration of geometric nonlinearity based on an infinite long circular thin-walled cylinder. As in SPS 37-59, Vol. III, the goal of this study is to achieve a systematic approach to the problem.

The geometric nonlinear vibrations of structural systems received renewed attention in the literature for two reasons:

- (1) The growing appreciation of the importance of nonlinear effects in determining the stability and response of structural systems.
- (2) Our increased ability to handle a more complex description of a structural system via high-speed digital computers.

Many people consider that a nonlinear vibration is just an extension of linear vibration since frequently the nonlinear terms are, in fact, the smaller terms. Therefore, a numerical scheme is developed, either by the finite difference method or by the iterative method, to take care of the nonlinear terms and the solution is obtained somewhat blindly. In the following, an infinite long thin-walled circular cylinder, under periodic dynamic loading, will be analyzed to demonstrate that some nonlinear phenomena cannot be obtained by the straightforward numerical method. Although the geometric nonlinearity is small due to the thinness of the cylinder wall, some of the special phenomena that only exist in the nonlinear vibration can be obtained only by careful mathematical considerations. The method used will have the advantage of automation that can be achieved for more complicated structural systems.

2. The Equations of Motion

The equations of motion for a thin circular cylinder become nonlinear when finite displacements are considered. These equations have long been employed in nonlinear analyses of thin cylindrical shells (Refs. 1 and 2) and are simply repeated here:

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = \rho h \frac{\partial^2 u}{\partial t^2} \quad (1)$$

$$\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = \rho h \frac{\partial^2 v}{\partial t^2} \quad (2)$$

$$\begin{aligned} \frac{\partial^2 M_x}{\partial x^2} + 2 \frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} + \frac{\partial}{\partial x} \left(N_x \frac{\partial w}{\partial x} + N_{xy} \frac{\partial w}{\partial y} \right) \\ + \frac{\partial}{\partial y} \left(N_{xy} \frac{\partial w}{\partial y} + N_y \frac{\partial w}{\partial x} \right) - \frac{N_y}{R} + q(x, y, t) = \rho h \frac{\partial^2 w}{\partial t^2} \end{aligned} \quad (3)$$

where h is the shell thickness, R is the radius of midplane, and ρ is the material density (Fig. 1). The variables u , v , and w are the midplane displacements in the axial, circumferential, and radial directions, respectively. Forces and moments per unit length (N_x , N_y , M_x , etc.), defined in terms of the stresses, are

$$\left. \begin{aligned} N_x &= \int_{-h/2}^{h/2} \sigma_{xx} dz \\ N_{xy} &= \int_{-h/2}^{h/2} \sigma_{xy} dz \\ N_y &= \int_{-h/2}^{h/2} \sigma_{yy} dz \\ M_x &= \int_{-h/2}^{h/2} \sigma_{xx} z dz \\ M_{xy} &= \int_{-h/2}^{h/2} \sigma_{xy} z dz \\ M_y &= \int_{-h/2}^{h/2} \sigma_{yy} z dz \end{aligned} \right\} \quad (4)$$

The applied load $q(x, y, t)$ acts in the radial direction.

Equations (1-3) contain both forces and moments as well as displacements. To express them in a more workable form, the following procedure is used:

- (1) The strains are related to the displacements u , v , and w .

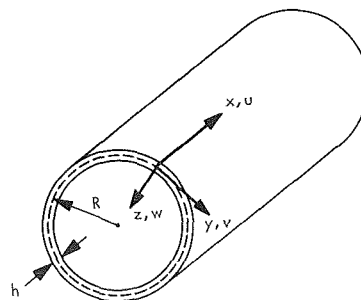


Fig. 1. Shell geometry

(2) Hooke's law is used to relate the stresses to the strains.

(3) By combining the relations from procedures (1) and (2), the stresses are written in terms of the displacements and then employed in Eqs. (1-4).

The strain-displacement relations are approximated by

$$\left. \begin{aligned} \epsilon_{xx} &= \frac{\partial u}{\partial x} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 - z \frac{\partial^2 w}{\partial x^2} \\ \epsilon_{yy} &= \frac{\partial v}{\partial y} + \frac{w}{R} + \frac{1}{2} \left(\frac{\partial w}{\partial y} \right)^2 - z \frac{\partial^2 w}{\partial y^2} \\ \epsilon_{xy} &= \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial w}{\partial x} \frac{\partial w}{\partial y} - 2z \frac{\partial^2 w}{\partial x \partial y} \end{aligned} \right\} \quad (5)$$

Transverse shear deformation is neglected, and the transverse normal stress is assumed to be negligible. These assumptions may be written as

$$\epsilon_{xz} = 0 \quad \epsilon_{yz} = 0 \quad \sigma_{zz} = 0$$

These relations are commonly employed in nonlinear analyses of thin cylinders. Terms like

$$\frac{1}{2} \left(\frac{\partial w}{\partial y} \right)^2$$

give rise to nonlinearities in the equations of motion.

Next, Hooke's law is used to relate the stresses to the strains and, with the preceding assumptions, we obtain

$$\sigma_{yy} = \frac{E}{1 - \nu^2} (\epsilon_{yy} + \nu \epsilon_{xx})$$

$$\sigma_{xx} = \frac{E}{1 - \nu^2} (\epsilon_{xx} + \nu \epsilon_{yy})$$

$$\sigma_{xy} = \frac{E}{2(1 + \nu)} \epsilon_{xy}$$

where E is Young's modulus and ν is Poisson's ratio. Employing these stresses and Eqs. (5) into Eqs. (4) and carrying out the integration through the thickness gives

the following forces and moments in terms of the displacements:

$$\left. \begin{aligned} N_y &= \frac{Eh}{1 - \nu^2} \left\{ \frac{\partial v}{\partial y} + \frac{w}{R} + \frac{1}{2} \left(\frac{\partial w}{\partial y} \right)^2 \right. \\ &\quad \left. + \nu \left[\frac{\partial u}{\partial x} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \right] \right\} \\ N_x &= \frac{Eh}{1 - \nu^2} \left\{ \frac{\partial u}{\partial x} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \right. \\ &\quad \left. + \nu \left[\frac{\partial v}{\partial y} + \frac{w}{R} + \frac{1}{2} \left(\frac{\partial w}{\partial y} \right)^2 \right] \right\} \\ N_{xy} &= \frac{Eh}{2(1 + \nu)} \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial w}{\partial x} \frac{\partial w}{\partial y} \right) \\ M_y &= -D \left(\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2} \right) \\ M_x &= -D \left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2} \right) \\ M_{xy} &= -D(1 - \nu) \frac{\partial^2 w}{\partial x \partial y} \end{aligned} \right\} \quad (6)$$

where the bending stiffness $D = Eh_3/[12(1 - \nu^2)]$.

3. Derivation of an Infinite Long Shell

To specialize the cylinder equations to an infinite long cylinder, the following assumptions are made:

- (1) The displacements w and v , as well as the radial load q , are taken to be functions of only the circumferential coordinate y and time t .
- (2) The thickness is taken to be constant and thin, so that $(h/R)^2$ is negligible in comparison with unity.
- (3) The forces N_x and N_{xy} are assumed to be zero throughout the shell.

The boundary conditions require that N_x and N_{xy} vanish at the ends of the shell; for flexural vibrations, rapid changes in the x -direction are not anticipated, and it becomes reasonable to assume that $N_x = 0$ and $N_{xy} = 0$ throughout the ring.

When these assumptions are used, Eqs. (6) reduce to

$$\left. \begin{aligned} N_y &= \frac{Eh}{1-\nu^2} \left[\frac{\partial v}{\partial y} + \frac{w}{R} + \frac{1}{2} \left(\frac{\partial w}{\partial y} \right)^2 \right] \\ N_x &= 0 \\ N_{xy} &= 0 \\ M_y &= -D \frac{\partial^2 w}{\partial y^2} \\ M_x &= -\nu D \frac{\partial^2 w}{\partial y^2} \\ M_{xy} &= 0 \end{aligned} \right\} \quad (7)$$

Substituting Eqs. (7) into Eqs. (1-3) yields the following ring equations:

$$0 = \rho h \frac{\partial^2 u}{\partial t^2} \quad (8)$$

$$\frac{\partial N_y}{\partial y} = \rho h \frac{\partial^2 v}{\partial t^2} \quad (9)$$

$$D \frac{\partial^4 w}{\partial y^4} + \frac{N_y}{R} - \frac{\partial}{\partial y} \left(N_y \frac{\partial w}{\partial y} \right) + \rho h \frac{\partial^2 w}{\partial t^2} = q(y, t) \quad (10)$$

where

$$N_y = \frac{Eh}{1-\nu^2} \left[\frac{\partial v}{\partial y} + \frac{w}{R} + \frac{1}{2} \left(\frac{\partial w}{\partial y} \right)^2 \right] \quad (11)$$

Equation (8) is unimportant for the problems dealt with here and is not considered further. Equations (9), (10), and (11) can be nondimensionalized by letting

$$\left. \begin{aligned} N_y &= \frac{Eh}{1-\nu^2} N, & y &= R\theta, & r &= \frac{1}{12^{1/2}} \frac{h}{R}, \\ \bar{q} &= \frac{R^2 q}{\Omega^2 \rho h w_m^3}, & w &= w_m W, & t &= \frac{\tau}{\omega}, \\ \Omega^2 &= \frac{E}{(1-\nu^2) \rho R^2} \end{aligned} \right\} \quad (12)$$

where

$$\begin{aligned} w_m &= \text{expected maximum deflection} \\ \omega &= \text{frequency} \end{aligned}$$

Eliminating v from Eqs. (9) and (11) gives the relation

$$\frac{\partial^2 N}{\partial \theta^2} - \frac{\omega^2}{\Omega^2} \frac{\partial^2 N}{\partial \tau^2} = -\frac{w_m}{R} \frac{\omega^2}{\Omega^2} \frac{\partial^2 W}{\partial \tau^2} - \frac{1}{2} \left(\frac{w_m}{R} \right)^2 \frac{\omega^2}{\Omega^2} \frac{\partial^2}{\partial \tau^2} \left[\left(\frac{\partial W}{\partial \theta} \right)^2 \right] \quad (13)$$

Now let

$$\frac{w_m}{R} = \epsilon \ll 1 \quad (14)$$

where ϵ is a small parameter. Physically this means that the maximum shell deflection is much smaller than the shell radius. Then

$$\frac{\partial^2 N}{\partial \theta^2} - \frac{\omega^2}{\Omega^2} \frac{\partial^2 N}{\partial \tau^2} = -\epsilon \frac{\omega^2}{\Omega^2} \frac{\partial^2 W}{\partial \tau^2} - \frac{1}{2} \epsilon^2 \frac{\omega^2}{\Omega^2} \frac{\partial^2}{\partial \tau^2} \left[\left(\frac{\partial W}{\partial \theta} \right)^2 \right] \quad (15)$$

$$\epsilon r^2 \frac{\partial^4 W}{\partial \theta^4} - \epsilon \frac{\partial}{\partial \theta} \left(N \frac{\partial W}{\partial \theta} \right) + \epsilon \frac{\omega^2}{\Omega^2} \frac{\partial^2 W}{\partial \tau^2} + N = \epsilon^3 \bar{q} \quad (16)$$

Equations (15) and (16) can be considered as the non-dimensionalized equilibrium conditions.

4. Perturbation Equations

One of the commonest methods for treating nonlinear problems in mechanics is the perturbation method, which consists of developing the desired quantities in power series of some small parameters. The coefficients of the series are then determined by solving a sequence of linear problems. The advantage of this method is that it is relatively foolproof, in the sense that it can often be applied fairly safely even in the absence of foreknowledge regarding the general character of the solution. The mathematical proof of the existence and convergence of the perturbation solution is beyond the scope of this article, (Ref. 3). We simply assume that

$$\left. \begin{aligned} N &= N_0 + \epsilon N_1 + \epsilon^2 N_2 + \dots \\ W &= W_0 + \epsilon W_1 + \epsilon^2 W_2 + \dots \end{aligned} \right\} \quad (17)$$

Also,

$$\frac{\omega^2}{\Omega^2} = \lambda = \lambda_0 + \epsilon \lambda_1 + \epsilon^2 \lambda_2 + \dots \quad (18)$$

Substituting Eqs. (17) and (18) into Eqs. (15) and (16) and equating the coefficients of like order of ϵ terms gives

$$\epsilon^0 \begin{cases} \frac{\partial^2 N_0}{\partial \theta^2} - \lambda_0 \frac{\partial^2 N_0}{\partial \tau^2} = 0 \\ N_0 = 0 \end{cases} \quad (19)$$

$$\epsilon^1 \begin{cases} \frac{\partial^2 N_1}{\partial \theta^2} - \lambda_0 \frac{\partial^2 N_1}{\partial \tau^2} - \lambda_1 \frac{\partial^2 N_0}{\partial \tau^2} = -\lambda_0 \frac{\partial^2 W_0}{\partial \tau^2} \\ r^2 \frac{\partial^4 W}{\partial \theta^4} - \frac{\partial}{\partial \theta} \left(N_0 \frac{\partial W_0}{\partial \theta} \right) + \lambda_0 \frac{\partial^2 W_0}{\partial \tau^2} + N_1 = 0 \end{cases} \quad (20)$$

$$\epsilon^2 \begin{cases} \frac{\partial^2 N_2}{\partial \theta^2} - \lambda_0 \frac{\partial^2 N_2}{\partial \tau^2} + \lambda_0 \frac{\partial^2 W_1}{\partial \tau^2} = \lambda_1 \frac{\partial^2 N_1}{\partial \tau^2} - \lambda_1 \frac{\partial^2 W_0}{\partial \tau^2} - \frac{1}{2} \lambda_0 \frac{\partial^2}{\partial \tau^2} \left[\left(\frac{\partial W_0}{\partial \theta} \right)^2 \right] \\ r^2 \frac{\partial^4 W_1}{\partial \theta^4} - \frac{\partial}{\partial \theta} \left(N_1 \frac{\partial W_0}{\partial \theta} \right) + \lambda_0 \frac{\partial^2 W_1}{\partial \tau^2} + \lambda_1 \frac{\partial^2 W_0}{\partial \tau^2} + N_2 = 0 \end{cases} \quad (21)$$

$$\epsilon^3 \begin{cases} \frac{\partial^2 N_3}{\partial \theta^2} - \lambda_0 \frac{\partial^2 N_3}{\partial \tau^2} + \lambda_0 \frac{\partial^2 W_2}{\partial \tau^2} = \lambda_1 \frac{\partial^2 N_2}{\partial \tau^2} + \lambda_2 \frac{\partial^2 N_1}{\partial \tau^2} - \lambda_1 \frac{\partial^2 W_1}{\partial \tau^2} - \lambda_2 \frac{\partial^2 W_0}{\partial \tau^2} - \lambda_0 \frac{\partial^2}{\partial \tau^2} \left(\frac{\partial W_0}{\partial \theta} \frac{\partial W_1}{\partial \theta} \right) - \frac{1}{2} \lambda_1 \frac{\partial^2}{\partial \tau^2} \left[\left(\frac{\partial W_0}{\partial \theta} \right)^2 \right] \\ r^2 \frac{\partial^4 W_2}{\partial \theta^4} - \frac{\partial}{\partial \theta} \left(N_1 \frac{\partial W_1}{\partial \theta} + N_2 \frac{\partial W_0}{\partial \theta} \right) + \lambda_0 \frac{\partial^2 W_2}{\partial \tau^2} + \lambda_1 \frac{\partial^2 W_1}{\partial \tau^2} + \lambda_2 \frac{\partial^2 W_0}{\partial \tau^2} + N_3 = \bar{q} \end{cases} \quad (22)$$

We are looking for the steady-state solution which must satisfy two conditions, namely, the periodicity conditions in θ and τ . Mathematically, it means for instance that the solutions must satisfy

$$\begin{aligned} W(\theta, \tau) &= W(\theta, \tau + 2\pi) \\ W(\theta, \tau) &= W(2\pi + \theta, \tau) \end{aligned}$$

Equations (25) and (26) satisfy the continuity requirements indicated previously and n is the circumferential wave number.

Without losing great accuracy, Eqs. (25) and (26) can be approximated as

Let the loading function $\bar{q}$ to be of the form

$$\bar{q} = F \cos \tau \cos n\theta \quad (23)$$

$$N_1 = a \cos \tau \cos n\theta + b \sin \tau \sin n\theta \quad (27)$$

$$W_0 = \bar{a} \cos \tau \cos n\theta + \bar{b} \sin \tau \sin n\theta \quad (28)$$

Equation (19) gives a trivial solution

$$N_0 = 0 \quad (24)$$

Equations (20) are a system of linear partial differential equations whose solutions, by considering the loading function in Eq. (23), will be of the form

$$N_1 = \sum_{n=0}^{\infty} [a_n \cos \tau \cos n\theta + b_n \sin \tau \sin n\theta] \quad (25)$$

$$W_0 = \sum_{n=0}^{\infty} [\bar{a}_n \cos \tau \cos n\theta + \bar{b}_n \sin \tau \sin n\theta] \quad (26)$$

The terms with $\cos \tau \cos n\theta$ are called driven mode since they are similar to the mode of forcing function, and the terms with $\sin \tau \sin n\theta$ are called companion modes since they are being excited through nonlinear coupling. Figure 2 shows the form of W_0 .

Substituting Eqs. (27) and (28) into Eqs. (20), one obtains the characteristic equation for λ_0 :

$$\lambda_0^2 - (n^2 + n^4 r^2 + 1) \lambda_0 + n^6 r^2 = 0 \quad (29)$$

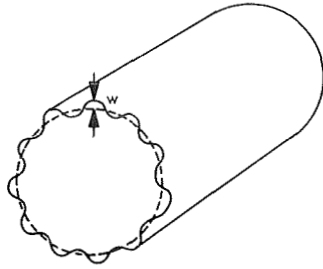


Fig. 2. Deformation pattern

and the relation

$$\left. \begin{aligned} \bar{a} &= \left(1 - \frac{n^2}{\lambda_0}\right) a \\ \bar{b} &= \left(1 - \frac{n^2}{\lambda_0}\right) b \end{aligned} \right\} \quad (30)$$

Since Eq. (29) is of quadratic form, there are two solutions for λ_0 . Figure 3 is the plot for λ_0 as a function of circumferential wave number n for

$$r = \frac{1}{400}$$

If one assumes that the circumferential wave number is such that $n^4 r^2 < 1$, then the two solutions can be approximated as

$$\left. \begin{aligned} \lambda_{01} &= n^2 + 1 \\ \lambda_{02} &= \frac{n^6 r^2}{n^2 + 1} \end{aligned} \right\} \quad (31)$$

The first expression in Eq. (31) is the upper curve and the second is the lower curve in Fig. 3. Since r represents the nondimensional bending rigidity, the upper curve in Fig. 3 is the membrane frequency and the lower curve is the bending frequency. Throughout this investigation, only the bending vibration will be considered because of the extremely high frequency of the membrane vibration.

Knowing N_1 , W_0 , and λ_0 , Eq. (21) can be readily solved in which terms involving N_1 and W_0 can be considered as forcing functions. The periodicity condition indicated previously requires that all of the coefficients of $\cos \tau \cos n\theta$ and $\sin \tau \sin n\theta$ in the forcing function be eliminated, since otherwise terms as $\tau \cos \tau$ and $\tau \sin \tau$ would arise in the general solution. These are called secular terms. After solving Eq. (21), Eq. (22) can be solved in a similar fashion.

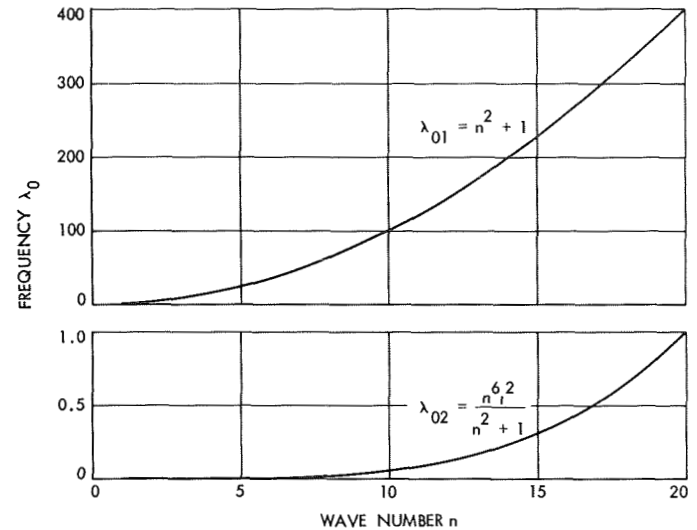


Fig. 3. Frequency as a function of wave number

5. Conclusions

The nonlinear equations of motions of a vibrating infinite long circular cylinder have been reduced to a sequence of linear equation. From the mathematical theory, we are assured that the solutions of these linearized equations will converge to the true solution and it is not anticipated that there will be any difficulties in obtaining the solution.

In the linear vibration, it has been mathematically proved that the solution is unique, which means that given an initial condition, there is only one response under certain loading conditions. However, this unique theorem cannot be applied to the nonlinear problem, therefore we will encounter multiple responses at certain loading conditions. Physically, this cannot happen, which means some of the solutions obtained are not stable. Thus, a stability analysis is required which will enable us to select a correct response among many responses. In subsequent investigations, the solutions, as well as the stability analysis, will be obtained and the results will be compared with the existing experimental results.

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C. Vehicle Mobility Tests: Soft Soil Slopes, I. Kloc²

1. Introduction

In order to provide soil support and thrust for vehicle mobility, an effective lunar and planetary program for roving vehicles requires some advanced information about the behavior characteristics of the various terrains that may be encountered. Current testing concepts of vehicle mobility generally refer to horizontal terrains. Information about soft, steep-sloped surfaces for estimating vehicle performance is limited because of the more complex problems involved.

An exploratory testing program is outlined for evaluation of vehicle mobility performance on soft, level, and sloping terrains. Specific information is given about vehicle characteristics, soil material, equipment, instrumentation, and an analysis of the test results.

The basic objectives of these tests are to: (1) obtain vehicle performance data on soft steep slopes for potential lunar applications considering low soil-wheel contact pressure of less than 0.75 psi and small sinkages, (2) establish criteria for planning representative single wheel tests on slopes, and (3) guide the formulation of a mathematical soil-wheel interaction model for horizontal and sloping terrains.

The soils considered were an air-dried cohesionless sand and a wet sand with mixed frictional and cohesive properties. Slopes were prepared artificially and varied up to near the angle of repose.

2. Mobility on Steep Slopes

Soft soil slope climbing immobilization is due to either one or the combination of the following factors: (1) lack of soil-wheel interface friction which would enable the soil to transfer the thrust for mobility purposes, and (2) exceeding the slope bearing capacity for the combined loads of vehicle weight and thrust required for mobility. In both cases, with an increasing slope, wheel rotation produces a down-the-slope soil transport accompanied by larger wheel sinkages under continuously degrading conditions. A lunar roving vehicle with limited maneuvering capability may be unable to extricate itself from the inside

of a crater if any of the above limiting conditions is reached.

In principle, the study of soil-wheel interaction points to two basic aspects. One corresponds to the operational conditions of a vehicle whereby mobility is always guaranteed, particularly for low contact pressures and reduced sinkage. In these cases, the design objective for mobility is concerned mainly with an optimization problem. The other aspect relates to the limiting operational conditions of a vehicle on steep slopes where the slope reaches its limiting equilibrium state due to self weight. In this case, safety rather than efficiency constitutes the constraint factor.

Conceptually, the problem of soft soil-wheel interaction covers the normal vehicle operation up to and including immobilization. Each case is intimately connected with a specific soil failure condition. Thus, the problem of soil-wheel interaction pertains to the study of soil mechanics and the theory of plasticity.

Some understanding of vehicle mobility was gained by performing the following experiments.

3. Test Description and Discussion

Test measurements consisted of graphical and/or magnetic recordings of the vehicle drawbar pull, vehicle velocity, velocity of each wheel, wheel motor currents, and battery terminal voltage. All measurements were time-related by suitable indications on the instrumentation records. The test program is shown in Table 1.

Figure 4 shows the sloped soil testing facility in side elevation. The test track is constructed of plywood walls and is 6.5 ft wide. The soil track permits mobility for two vehicle lengths, or 12 ft, on both the horizontal and sloped portions.

Two lateral, slope-adjustable, pipe rails serve as a means of guiding and supporting a soil hopper for spreading sand to form a 15-in. minimum height soil bed. These rails also support a counterweighting arrangement for lightening the load applied by the vehicle to the sand bed

Table 1. SLRV mobility slope tests

Vehicle weight, lb	Slope, deg				
	Dry sand			Wet sand	
158	0	20	25	30	35
40	—	—	25	—	35

²The author gratefully acknowledges the assistance of M. Baligh and T. D. Lu in analyzing vehicle test results and in performing the laboratory soil tests at the California Institute of Technology Soil Mechanics Laboratory, and R. E. Imus, J. K. Hawkes, and H. C. Primus for their participation in the detailed design of the testing facility, instrumentation, and vehicle control, respectively.

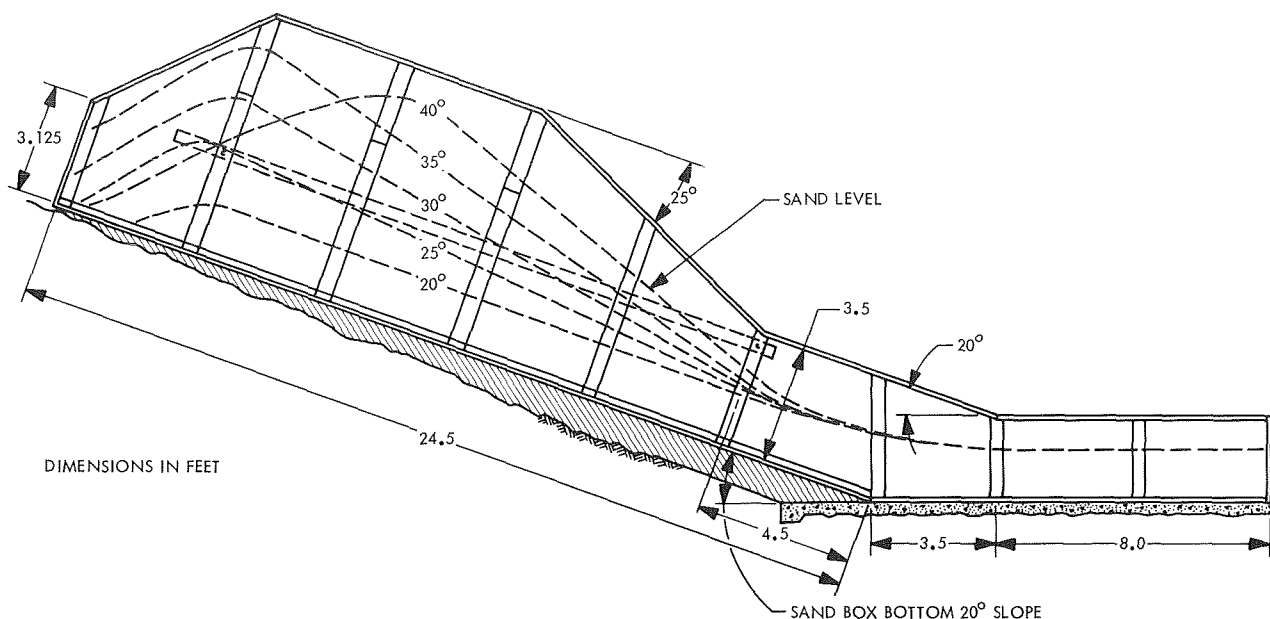


Fig. 4. Sloped soil testing facility

for tests involving less than full vehicle weight. The soil hopper is supported within the carriage frame on end pivots, permitting it to swivel freely as the carriage displaces back and forth along the pipe rails.

Typically, the soil bed consists of medium to fine white silica sand poured from the hopper at a constant rate from a variable height of 6 to 24 in. through an adjustable opening held at a constant 0.10-in. width. This produces a soil bed with the density increasing linearly with depth. The average sand density is 101 lb/ft³.

Some of the tests were conducted on slightly cohesive soil produced by wetting the sand. This was accomplished by spraying water through a set of jets mounted across the carriage frame. Soil strength properties are given in Fig. 5.

The test vehicle is the General Motors surveyor lunar roving vehicle (SLRV), which is 2 ft wide by 6 ft long. It has six wheels with smooth tires of the pneumatic wire-reinforced type, rubber-covered, and operated at a typical tire pressure equal or less than 0.5 psi. The tire dimensions are 18 in. in diameter and 8 in. in width. The vehicle has no velocity control and is operated only on a "stop-go" basis by remote radio control. Wheel rotation on the order of 15 rev/min provides a movement of 70 ft/min over horizontal terrain. The vehicle drawbar pull was measured parallel to the ground support by means of a load cell connected at the rear end of the vehicle. Drag was applied

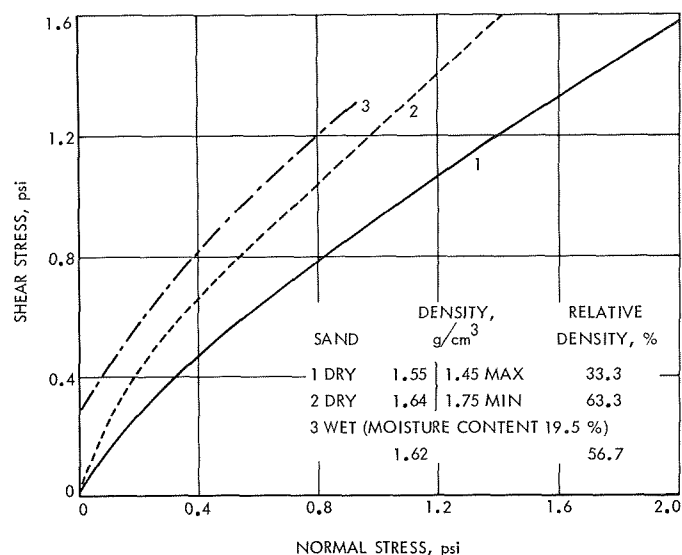


Fig. 5. Direct shear tests for dry and wet, medium to fine silica sand

by means of a braking arrangement consisting of a cable reel-up drum upon which the winding of the cable, attached to the vehicle through the load cell, could be controlled. Drag was exerted on the cable as it was unwound by movement of the vehicle using a brake shoe arrangement. Drag was maintained parallel to the track surface by means of two rollers which could be adjusted to match the soil slope angle.

Some of the tests were made at full vehicle weight and others at a reduced one-quarter vehicle weight in order to obtain approximate soil-wheel contact stress levels which may apply to lunar gravity conditions. Load lightening was effected by using counterbalancing weights applied to each wheel or by using Negator springs³ for the same purpose (Fig. 6).

In order to obtain more accurate data with respect to vehicle performance, it was necessary to eliminate the effect of friction in the brake drag system, to balance out the effect of the downward component of the weight of

the carriage frame, and to match the velocity of the carriage frame to that of the SLRV vehicle when the tests involving one-quarter vehicle weight were run. This was accomplished by using a counterweighting arrangement shown in Fig. 7. Brake drag and the frictional effect on the drag cable, produced as it passed under the rollers, was balanced out by using a counterweighted cable which passed over pulley B and which was attached to the front end of the vehicle. Initial adjustment was made without the vehicle. The counterweight was adjusted until the cable could be pushed in either direction by application of a very small force. This arrangement, when attached to the vehicle, was all that was needed for tests conducted at full vehicle weight.

³Provided by Ametek-Hunter

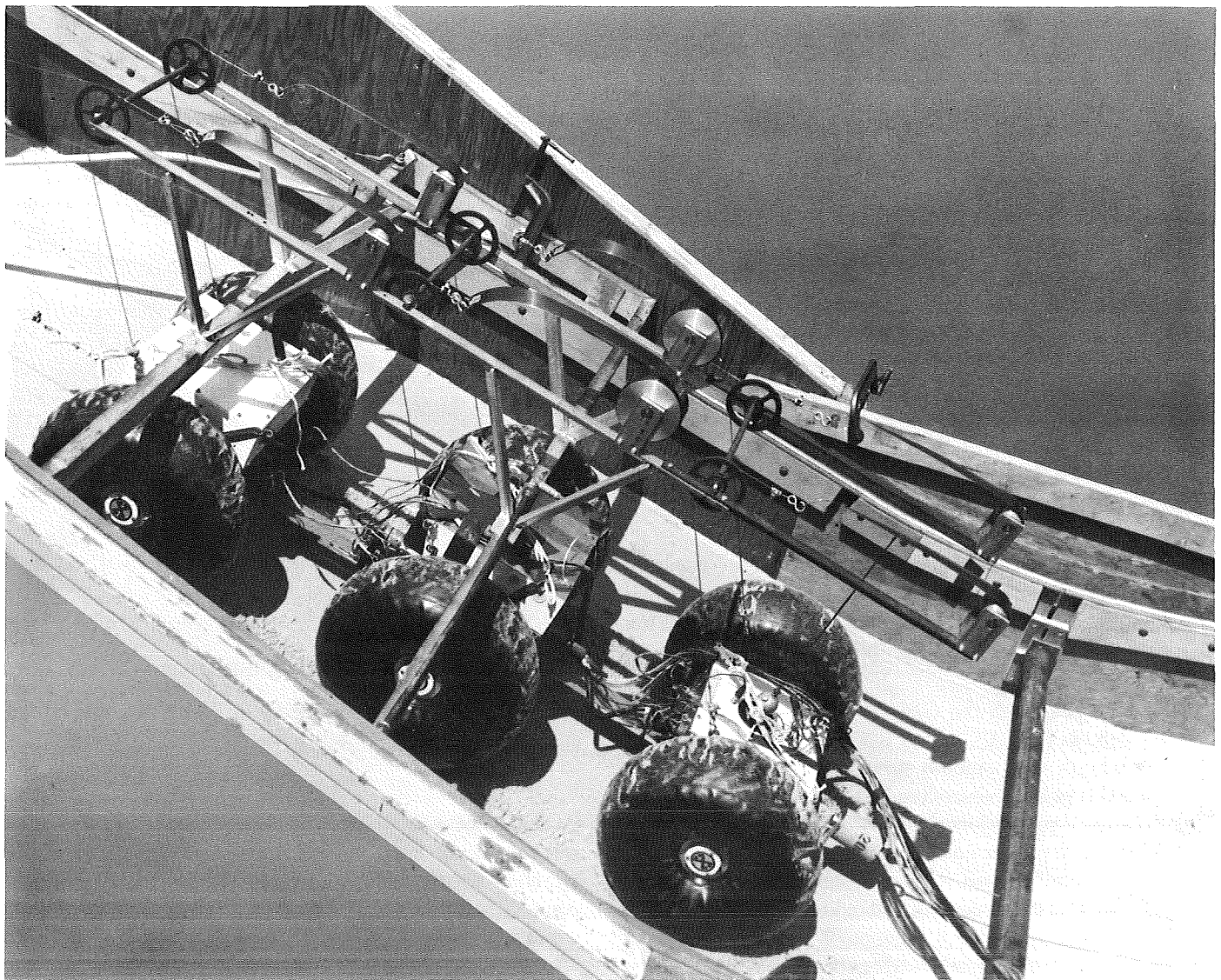


Fig. 6. Vehicle carriage and weight-lightening system

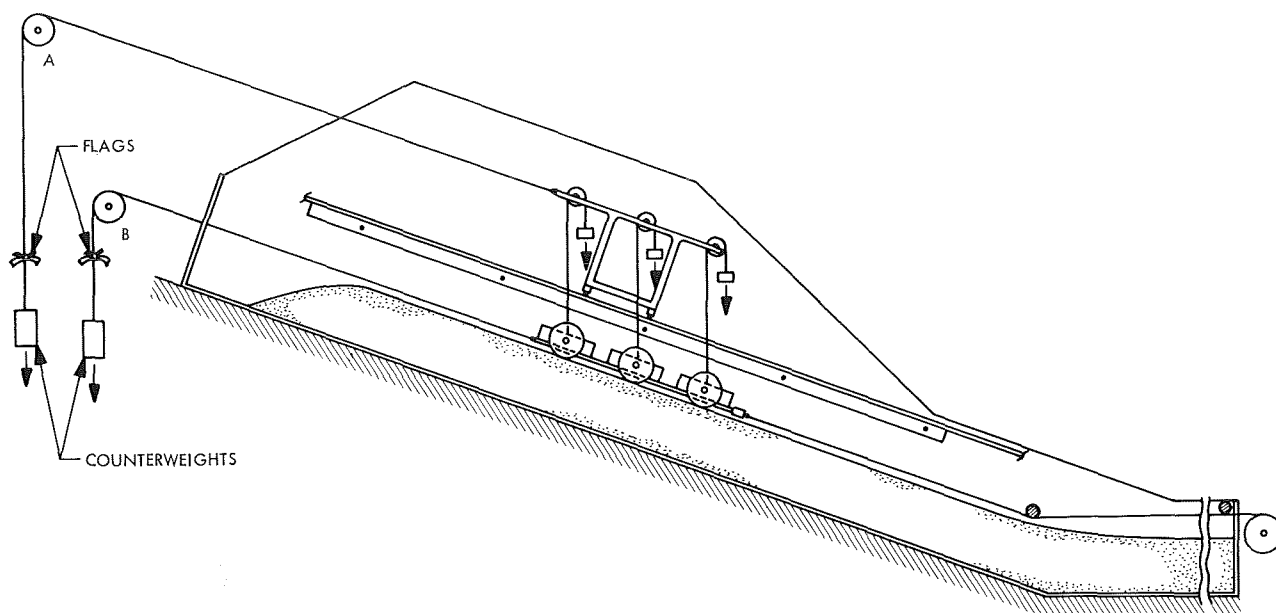


Fig. 7. Vehicle-lightening counterweight arrangement

For the one-quarter weight tests, a second counterweighting arrangement was used to offset the downward slope component of the weight of the carriage frame. Another counterweighted cable, passing over pulley A, was attached to the forward end of the carriage frame. The counterweight was adjusted until the frame could be moved in either direction when a small force was applied.

In order to maintain equal velocities for the SLRV vehicle and the carriage frame, marks or flags were placed on both cables, as indicated in Fig. 7, and registered. The flags were monitored and the carriage frame movement

was adjusted as necessary to maintain flag registration. Although the arrangement did not make it possible to maintain equal velocities on an absolute basis, it was possible to maintain equivalent velocities within acceptable limits.

4. Analysis of Test Results

A summary of the performance of vehicle mobility on dry level soil and on dry and wet sloping terrain is given in Table 2.

Table 2. SLRV test results

Slope, deg	Vehicle weight, lb	Nominal tire inflation pressure, psi	Wheel slip, ^a %	Soil thrust to normal load ratio ^b	Maximum pull force, lb	Maximum wheel axle torque, in.-lb		
						Front	Middle	Rear
25	158	0.5	90-95	0.465-0.548	12.0	82	43	188
	158	0.25	83-94	0.465-0.52	8.5	98	53	192
	40	0.25	95-96	0.455-0.565	4.0	— ^c	— ^c	— ^c
20	158	0.5	60-97	0.412-0.62	38	101	70	205
0	158	0.5	— ^c -80	0-0.511	81	141	90	241
	40	0.25	14-— ^c	0-0.70	28	34	24	58
30 ^d	158	0.25	32-94	0.576-0.715	19	86	74	229

^aSlip percentage $S = (V_{10} - V) 100/V_{10}$, where V_{10} = wheel peripheral velocity and V = wheel axle translational velocity.
^bSoil thrust = load cell pull force + vehicle weight component parallel to slope.
^cData reduction not available at this time.
^dWet sand.

The following are the most important vehicle mobility facts considering low soil-wheel contact pressures.

- (1) For horizontal and sloping terrains, far removed from the soil's natural angle of repose, the lower the soil-wheel contact pressure the more significant the improvement of the thrust performance. This occurs at comparatively reduced slip values accompanied also by proportionally reduced wheel torques.
- (2) The beneficial effect of low contact pressure degrades continuously with increase in the slope. When the slope approaches its natural angle of repose, there is no significant mobility performance gain, either in torque or thrust, regardless of the level of the soil-wheel low contact pressure.
- (3) On a stable cohesive steep soil slope the overall mobility performance is improved as compared with a cohesionless soil. Viewed as a soil bearing capacity problem, the slope may be capable of supporting the vehicle load but may be unable to provide the required soil-wheel interface thrust friction to develop vehicle motion. For this test the fact that on a 30-deg wet cohesive slope the SLRV could climb and develop additional pull and become immobilized on a 35-deg slope indicates that the soil-wheel interface friction lies between 0.63 and 0.68.
- (4) For the same normal load and soil characteristics, the higher the slope the larger the slip percentage required to develop the same thrust force.

D. Outer-Planet Infrared Radiometer Cooling,

L. D. Stimpson

1. Introduction

An infrared radiometer experiment is being considered in the Thermoelectric Outer-Planet Spacecraft (TOPS) study project. Present detectors require cryogenic cooling. Order-of-magnitude type estimates have been made of the temperature control requirements for several different design concepts. These estimates show that this is a com-

pletely new development to achieve, and they point out some of the problem areas needing early attention in order to demonstrate feasibility of this type instrument for this application.

The estimated cooling requirements for the TOPS infrared bolometer detector are given in Table 3 and range from 80°K at Jupiter to 20°K at Neptune. Also shown in Table 3 are planet encounter constraints for the 9.2- and 11.5-yr Grand Tour missions from Ref. 1. The latter indicates typical encounter science planet and solar view angle requirements.

2. Approach

The approach taken for a first estimate of the heat leaks and required cooling was to thermally isolate the cooled infrared unit from the rest of the spacecraft and the sun using some advanced structural-thermal concepts. The cooled unit (IR) would be mounted on an encounter science scan platform together with two encounter TV cameras (wide angle TV-A and narrow angle TV-B), a UV photometer (UVP), and a near-infrared radiometer (NIR).

The temperature requirements and power dissipations for the encounter science are given in Table 4. The distinctly different requirement for the cooled IR detector is evident. The 3-mW power requirement assumes three fixed detectors; a much larger power requirement (~0.1 W) exists if a filter wheel is used. The similarity in temperature (and view) requirements for the other instruments allows them to be packaged together. However, the IR instrument should be thermally isolated from the rest.

A recommended arrangement of the encounter science is shown in Fig. 8. Two shields are incorporated: one to radiatively isolate the IR instrument from the other instruments, and the other to isolate it from the sun. The radiators will be viewing out of the ecliptic. Figure 9 shows a typical encounter sequence where the sun can illuminate the science over a view angle greater than 180 deg.

Table 3. Detector temperature and encounter constraints

Planet	Detector temperature requirement, °K	Planet radius, km	Encounter altitude, km		Planet-probe-sun minimum angle, deg		Estimated duration of sun occultation, h	
			9.2-yr mission	11.5-yr mission	9.2-yr mission	11.5-yr mission	9.2-yr mission	11.5-yr mission
Jupiter	80	71,200	217,900	582,100	4	0	1.9	4.0
Saturn	50	59,600	6,400	75,200	30	0	1.0	1.4
Uranus	30	25,700	21,300	88,200	4	2	0.9	1.3
Neptune	20	24,900	19,000	85,000	20	0	0.7	1.2

Table 4. Encounter science temperature requirements

Instrument	Operating units, °C/°F		Electrical heat (I ² R), W	Heat required at Neptune, W	Overall dimensions, in.
	Lower	Upper			
TV-A	-40/-40	-15/+5	7	0	4 × 5 × 12
TV-B	-40/-40	-15/+5	7	0	6 × 6 × 20
UVP	-40/-40	+25/+77	1.8	2	5 × 5 × 12
NIR	-100/-148	+25/+77	1.8	2	5 × 5 × 20
IR	-273/-460	-253/-424	0.003	-0.008	4 × 4 × 16

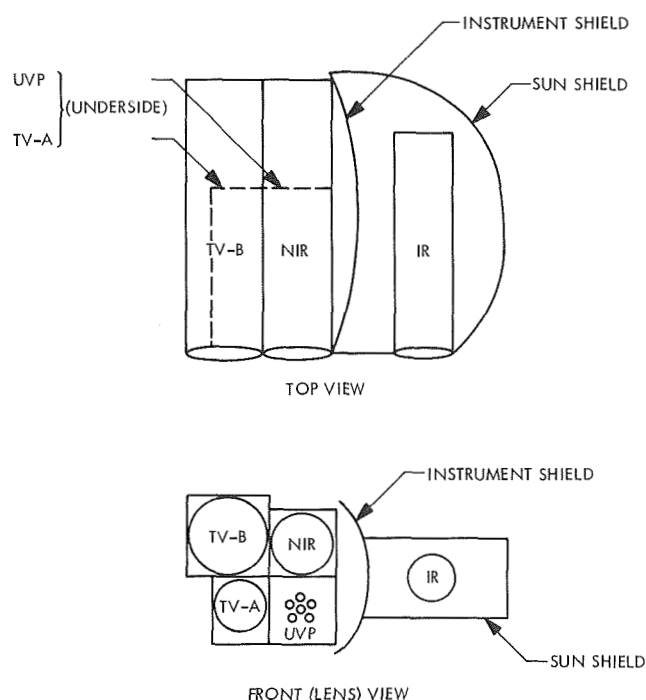


Fig. 8. Encounter science configuration

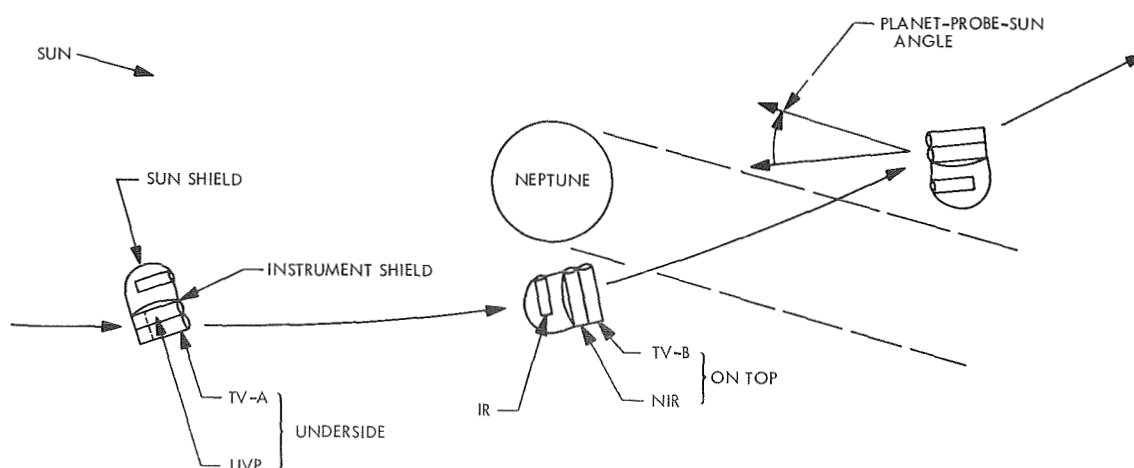


Fig. 9. Encounter science view angles

Several different instrument design concepts are under consideration for measuring the planetary infrared radiation. The NIR would scan the frequency band I in Fig. 10, using stepped or rotary filters. The IR instrument, using a cryogenically cooled bolometer, would sample three infrared bands as shown for Jupiter and Neptune. These three band measurements (2a, 2b, and 2c) can be made with either a single detector and a stepped filter or three separate detectors, each with their own filter.

Several different optical arrangements are shown conceptually in Fig. 11. One consists of a reflecting mirror and lenses for focusing the irradiated energy through a filter wheel onto a detector. Another also has the reflecting mirror but has a second mirror mounted on a tuning fork for scanning three separate detectors. A third arrangement consists of direct focusing of the irradiated energy through lenses (no mirror). We will treat the first optical arrangement and make comparisons with the other options.

3. Heat Sources

In Fig. 11a, the sensor package requiring cooling consists of the detector and a preamplifier. The sources of heat into the sensor package include the electrical power into the preamplifier and filter wheel stepper motor, heat radiating from the planet and reflected by the mirror through the lens, heat radiating from the warm mirror and the interior of the tube, and heat conducted in through the structural supports and the electrical cabling. Although the detector is the only element required to meet the above cooling requirement, the preamplifier and filter are so closely associated with the detector that they will also require cooling. Thus, in this preliminary estimate, the

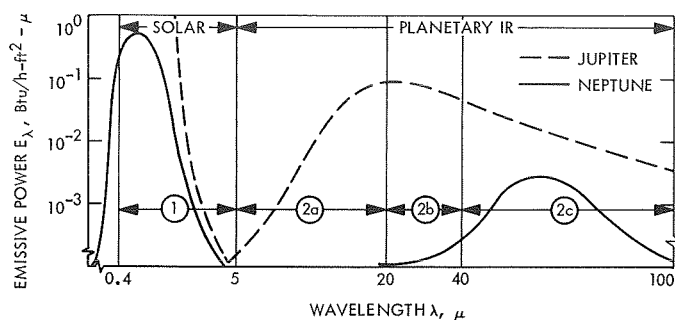


Fig. 10. Detector channel wavelength characteristics

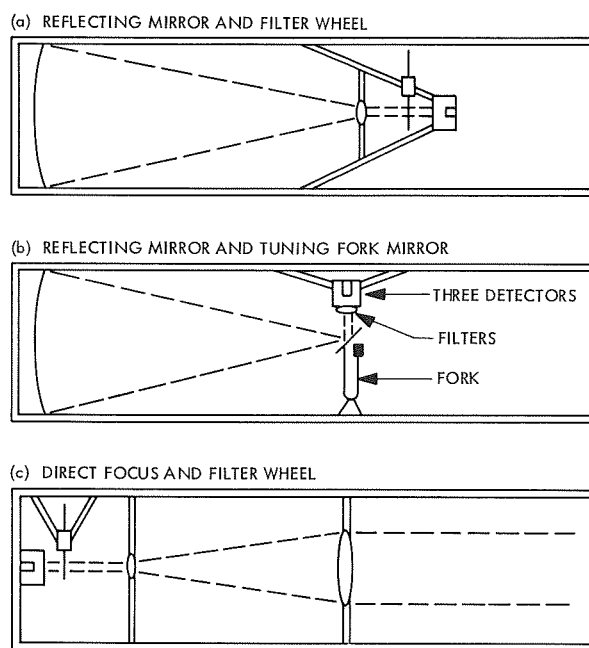


Fig. 11. Optical configurations

sensor package to be cooled will consist of the detector, preamplifier, and filter. The total heat received by this package will need to be rejected through a radiator.

The heat from the electrical resistance (I^2R) into the preamplifier for one detector is 1 mW. The pulsating load into the filter stepper motor averaged during encounters, where it will be operating almost continuously, is estimated at 100 mW.

It is of interest now to note the passive radiation capability at the different planets as shown in Table 5. One square foot of radiator ($\epsilon = 0.88$) can dissipate 190 mW at Jupiter but can only dissipate 0.75 mW at Neptune because of the lower radiator temperature. It is quite obvious that the I^2R heating from the filter stepper motor cannot

Table 5. Passive radiator capability and structural temperatures

Planet	Average structural temperature, °R	Radiator temperature, °R	Radiator capability, mW/ft²
Jupiter	242	144	190.0
Saturn	179	90	29.0
Uranus	127	54	3.8
Neptune	102	36	0.75

be dissipated passively. Consequently, an active cryogenic cooling system will be required, or the motor might be located external to the tube or even eliminated (Fig. 11b).

The energy radiating from a planet has been estimated, assuming a gray-body sphere in temperature equilibrium with the sun. The resulting planet temperatures are listed in Table 6. Some recent information does indicate that the planets are warmer, which might be a result of some internal energy generation or possibly from the greenhouse effect. Using the temperatures listed in Table 6 and the encounter geometry given in Table 3, the planet infrared radiation on the cooled sensor has been estimated. (Saturn infrared radiation is high because of a close flyby distance.) These estimates and heat inputs from other sources also are found in Table 6.

The encounter science configuration shown in Fig. 8 will result in the mirror and tube having temperatures intermediate to the radiator and structure temperatures given in Table 5. Their precise levels will depend upon heat conduction from the warmer structure and irradiation from the shields. Structural-thermal isolation techniques and thermal modeling need to be investigated further to obtain these precise levels. However, for the present estimate of heat leaks into the sensor package, it is assumed the mirror and tube will be midway between the values given in Table 5 for radiator and structure. The mirror and tube interior, having gold finishes, will irradiate 40 mW onto the sensor package at Jupiter and 0.7 mW at Neptune.

Eight electrical wires into the sensor package act as conductive heat leaks from the nearby warmer structure. These wires include three signal, one power, and one shield to the preamplifier, plus two power and one common to the filter. Assuming the latter is remote, then there are only five wires to consider. These internal instrumentation wires are assumed to be 32 to 36 gage (one-fourth of the flight cable wire diameter). Taking higher conductivity at low temperatures into account and using well-isolated supports (1-ft distance) result in 1–2.5 mW

Table 6. Heat inputs to sensor from various sources

Planet	Planet temperature, °R	Heat input, mW					
		Planet infrared	Mirror and tube infrared	Electrical I ² R	Electrical cabling	Structural support	Total ^a
Jupiter	220	8	40	1 ^a	1.2	0.8	51.0
Saturn	163	24	10	1 ^a	2.0	0.7	38.0
Uranus	131	3.5	2	1 ^a	2.5	0.6	9.6
Neptune	92	1.5	0.7	1 ^a	2.5	0.5	6.2

^aOne detector, filter wheel not included.

(Table 6). The increase with time is due to the higher conductivity having a dominant effect over the decreasing temperature difference.

The sensor package support is assumed to have four members attached to the tube, and the radiator interconnect provides some additional support. The members would be a composite material with low thermal conductivity but high strength such as a boron fiber or glass-filled polyimide. The radiator interconnect would have a high thermal conductivity but would need to be thermally isolated along with the radiator from the warmer tube.

Using the critical buckling load for an individual member as the strength criteria (both ends rigid), and assuming the equivalent of 1 lb supported under a 100-g load result in structural member dimensions of $0.03 \times 0.15 \times 4.5$ in. The heat leak through the four composite members would be less than 1 mW.

4. Development Required

Comparing the total estimated heat leaking into the sensor package shown in Table 6 with the passive radiator capability given in Table 5 indicates the need for active cryogenic cooling for the outermost planets (8 ft² is required for a passive radiator at Neptune). The totals in Table 6 assume the filter stepper motor in the configuration of Fig. 11a to be completely removed and isolated. If considered at all, it also will still leak heat conductively into the sensor package. The configuration of Fig. 11b adds 2 mW for the two extra detectors while the configuration of Fig. 11c still has the stepper motor.

A number of developments are required to make this instrument practical. These include the filter and/or tuning fork design, thermal isolation approach using high-strength and low thermal conductive materials, associated structural static and dynamic considerations, ac-

tive thermal control approaches, and cryogenic materials compatibility.

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E. Tensile Strength of Fiber-Reinforced Composite Materials, C. Zweben

1. Introduction

The recent development of high-strength, high-modulus fibers, such as boron and graphite, presents an opportunity for significant improvements in rocket motor case and pressure vessel design. Preliminary studies indicate that not only are significant weight savings possible, but the use of filament-wound pressure vessels permits a greater freedom in the choice of fuel systems since they permit the use of thin, nonstructural liners of materials that are compatible with the fluids to be contained. However, the efficient use of composite materials for such applications requires a good understanding of the fracture behavior of these materials as well as reliable design data for ultimate strength. Despite the widespread use of these materials throughout the aerospace industry, the state of knowledge relating to the strength and fracture behavior of these materials is at a low level.

Therefore, a program has been initiated to develop reliable design data for the strength of advanced composite materials and to provide an understanding of fracture behavior sufficient to insure the reliability of filament-wound pressure vessels. The effort to understand the factors that affect composite strength is further motivated by the significant disparity between the strength of small laboratory tensile coupons and that observed in tests on filament-wound pressure vessels (Refs. 1-3).

2. Method of Analysis

The main difficulty in determining the tensile strength of composite materials is that the fibers do not have a unique failure strength as do the more familiar materials, such as aluminum and steel. The fibers in use, such as boron, graphite, and glass, exhibit a significant scatter in strength because they are brittle and sensitive to the inevitable presence of flaws. Coefficients of variation as high as 25% for ultimate stress values are not uncommon for these fibers. The sensitivity to flaws also results in a reduction in average strength with increasing fiber length because of the greater probability of finding flaws of a given severity.

The scatter in fiber strength results in a dispersion in the strength of composite materials containing them and, apparently, in a decrease in strength with increasing volume of loaded material (Refs. 1-3). However, there are also indications that for materials with a constant length and width the tensile strength increases with the number of layers, leveling off at 6 to 10 layers so that there appear to be several different phenomena that affect the strength of composite materials.

Since a typical filament-wound structure contains a volume of material that is several orders of magnitude larger than a tensile coupon, the applicability of strength data obtained from the latter for design of large structures is in question.

Ideally, one would like to have a theory that would predict the strength of a given composite structure from the properties of the constituents, that is, the fibers and matrix. Such a theory does not exist. However, there has been some work done that attempts to predict the strength behavior of composite materials which provides useful information (Refs. 4-8). In order to verify these analyses, a large body of experimental data has been collected and is compared with the theoretical approach which is summarized in Ref. 7.

The approach to composite tensile strength developed by Zweben and Rosen (Ref. 6) is based on the concept of lower and upper bounds. Each bound is associated with a particular mode of failure.

The weakest link mode of failure is associated with the first fiber break. The expected stress, σ_1 , at which this event will occur is generally too conservative, since failure, for most composites, is associated with an accumula-

tion of fiber breaks. Therefore, as a rule, a better lower bound is obtained by using σ_2 , the stress at which the first overstressed fiber, adjacent to one of the scattered fiber breaks, will fracture. This stress is a measure of the tendency for fiber breaks to propagate. An upper bound, σ_u , is provided by the cumulative weakening theory (Ref. 4). This theory, which considers a composite to be a chain whose links are bundles of fibers, neglects the effect of load concentrations on composite strength. Since load concentrations lower the strength of a composite, σ_u can be expected to be greater than the observed failure loads.

3. Comparison With Experimental Data

In order to test the general applicability of the bounding approach, comparison was made with as many fiber/matrix combinations for which data could be obtained. In order to compare the theoretical predictions with experimental data, it is necessary to know the fiber strength distribution, matrix properties, and specimen geometry. Generally, all of this information is not determined or reported by investigators. Therefore, it is difficult to get good data with which to compare theoretical predictions. Furthermore, the reliability of reported data is difficult to assess. Therefore, an attempt was made to obtain as much experimental data as possible with the hope that, in the aggregate, trends could be discerned. Table 7 shows that, generally, the experimental data does fall within the expected bounds.

It is interesting to compare the boron/aluminum composites reported by Shimizu and Kreider/Marciano (Ref. 13); both use the same types of fiber (manufactured by United Aircraft) and matrix (6061). However, the composites in the latter reference are fabricated by plasma spraying which produces a better fiber-matrix bond than the hot pressing technique used to produce the composites of Shimizu. This better bonding appears to result in a weakest link mode of failure and hence a lower failure strength. However, one would expect the transverse strength properties to be enhanced by the better bond.

Figure 12 shows the effect of specimen size on strength using the data of Gaylord. The bounds σ_2 and σ_u are shown for comparison.

The results of this study tend to verify the approach to composite strength described in Refs. 4-8. Because of this, the analyses will be extended in order to provide a more definitive prediction of composite strength from constituent properties.

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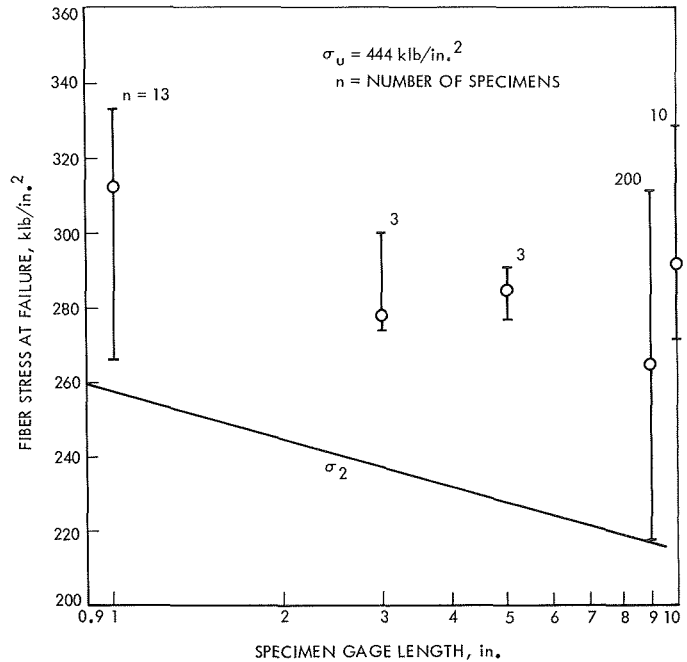


Fig. 12. Effect of specimen gage length on strength of graphite/epoxy composites

Table 7. Comparison of bounds with experimental failure stresses for fiber-reinforced composites

Fiber/Matrix	σ_2 , klb/in. ²	σ_u , klb/in. ²	Experimental fiber stress at failure, klb/in. ²	Source of data	Comments
Glass/epoxy	142	186	150-168	Ref. 8	Monolayer
	130	167	143-163	Ref. 8	Monolayer
	88	142	92, 97	Ref. 9	Monolayer
	480	761	473-541	Ref. 10	
	93	187	150, 166	Ref. 11	
	181	246	200-352	Ref. 12	σ_u probably not applicable because of the small number (24) of filaments in each specimen. The three sets of data represent tests at different strain rates (monolayer).
	172	243	177-352	Ref. 12	
	158	237	232-302	Ref. 12	
Graphite/epoxy	217	547	217-312	Gaylord, W. ^a	One specimen failed at 208 klb/in. ² , the remaining 119 specimens failed within the range shown.
Boron/aluminum (6061)	254	736	264-412	Sumner, E. ^a	
	354	565	379-466	Shimizu, H. ^a	Apparent weakest link mode of failure, $\sigma_1 = 241$ klb/in. ²
	396	575	254-374	Ref. 13	
Boron/epoxy	333	498	390-435	Ref. 14	
	317	497	307-348	Jensen, W. M. ^a	

^aPrivate communication.

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F. Thermal Joint Conductance, J. A. Hultberg

1. Introduction

Thermal joint conductance is of interest to temperature-control engineers because the majority of heat which is dissipated in a spacecraft must be transferred to a space radiator through a metal-to-metal bolted joint. In *Mariner*-type spacecraft, the shear plates serve as space radiators and electronic modules are mounted to these shear plates.

The investigation of thermal joint conductance is not unique to space vacuum applications. Aircraft designers needed this information for the design of aircraft which operate at high altitudes. For this reason, literature on thermal joint conductance dates back to before the space effort. So much literature has been published that it should suffice here to mention two of the literature surveys on the subject (Refs. 1 and 2).

2. Types of Measurements

The investigations in the literature generally fall into two types. The first type is a reporting of the total joint conductance of an actual joint. The joint conductance in this case is calculated from the known energy transferred per unit time and a representative temperature difference across the joint. Usually the experiments include different types of bolted joints, different materials, and different bolt torques. The purpose of this type of investigation was to provide data for actual applications. Attempts of investigation to perform a meaningful correlation of such data have proved inconclusive, because a complete characterization of the joint has not been reported. Considering the difficulty of correlating thermal conductivity measurements from several investigations, it is not surprising that due to the addition of a physical discontinuity that thermal joint conductance correlations are, indeed, difficult.

The second type of joint conductance measurement is one in which only the resistance due to the joint is found.

The experiment is performed with rods whose length is long in comparison to their cross-sectional diameter. The ends of the rods are pressed together to form the joint. The end of one rod is heated while the end of the other rod is cooled. The temperature gradient thus produced in the rods is measured, plotted, and extrapolated to the joint discontinuity. The temperature across the joint thus appears as a discontinuity. The joint conductance is then calculated by dividing the total heat flux by the apparent temperature discontinuity across the joint. For a particular joint, the test is repeated for several pressure levels. The heat input is usually electrical so that the total heat input may be easily determined. Thermal conductivity can be a byproduct of a joint conductance experiment of this type. A heat balance on both sides of the joint must verify that the heat flowing into the joint equals the heat leaving the joint when steady-state conditions are met. In actual practice, the heat balance is considered to be satisfied when the heat flow in equals the heat flow out to within 1 or 2%.

3. Measurements at JPL

The joint conductance work at JPL has been both in-house effort and contract monitoring of two continuing university investigations. At the Massachusetts Institute of Technology a complete investigation was made of a surface which was both rough and wavy, concentrating on the microscopic joint conductance effects. The University of Illinois investigation was concentrated on macroscopic joint conductance. Both university investigations included analytic predictions which correlated with their experimental work.

The purpose of the in-house work is to bridge the gap between the university research and the actual spacecraft hardware. The analysis and integration of the conductance versus pressure data for long rods into an actual bolted joint was begun by T. J. Lardner (SPS 37-19, Vol. IV, pp. 83-85) and continued by J. A. Hultberg (SPS 37-38, Vol. IV, pp. 61-63 and SPS 37-39, Vol. IV, pp. 53-55). The work by Lardner concentrated on the calculation of pressure versus axial distance for two plates pressed together. The work by Hultberg demonstrated the method of calculation of total joint conductance for several configurations given the conductance versus axial distance. Both investigations neglected the bolt itself and considered only the loading effects of the bolt. For the pressure calculations, this infers that the bolt applies a uniform pressure distribution under the entire bolt head, and for the total conductance calculations the heat transfer through the bolt itself is neglected.

For the experimental portion of the in-house effort, a joint conductance facility was constructed. The apparatus is typical of joint conductance apparatus used to measure the contact conductance of long rods whose ends are pressed together. The apparatus is equipped with a dead-weight loading mechanism capable of 10,000 lb. The vacuum chamber has a diameter large enough to accommodate flat plates 12 in. in diameter. A general view of the apparatus and vacuum pumping station is shown in Fig. 13. The apparatus may accommodate joint conductance tests with long rods or simulated bolted joints with large diameter flat plates. University research on long rods was exhaustive, and no further research was necessary. Flat plate tests, where the heat flows from the periphery of one plate to the periphery of the other plate, were performed. Figure 14 shows the installation of the plates in the apparatus. The AZ31B magnesium plates are 12 in. in diameter and 0.25 in. thick. The bolted joint is simulated by application of a load with the deadweight loading apparatus through a 1-in.-diam alumina spacer. The alumina spacers, which are each 0.5 in. thick, are used to reduce heat flow to the loading rods. Heat is supplied through resistors attached to the upper plate. Figure 15 shows the plates separated with a heater wire instead of the resistors on the heater plate. This method

of heating the plate was found to be unsatisfactory. The load delivered by the dead weight loading mechanism was measured with a load cell. The load cell calibration was verified at the beginning and at the end of the tests. Throughout the tests, the vacuum chamber surrounding the plates was held at a pressure lower than 5×10^{-6} torr. The edge temperature of the heated plate was held constant with a proportional controller controlling the power to the heaters. The edge temperature of the cooled

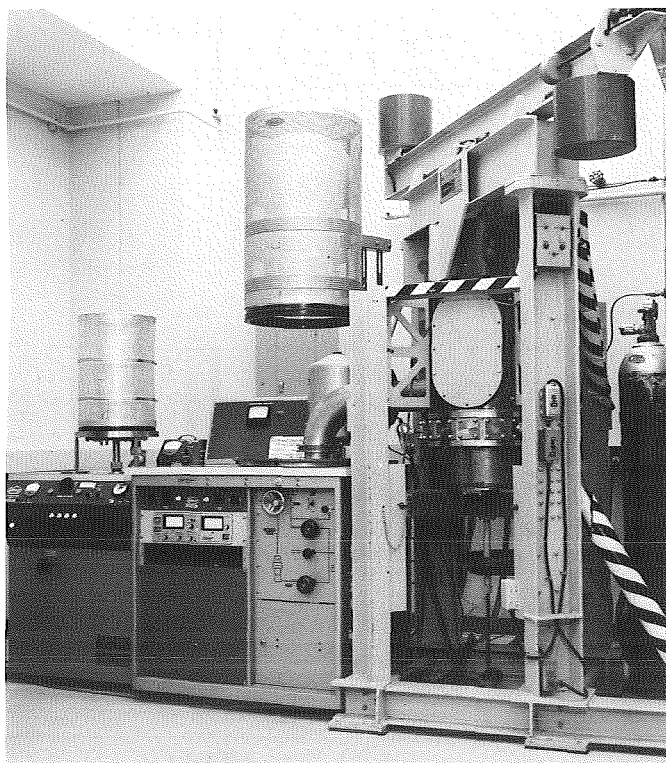


Fig. 13. Joint conductance facility

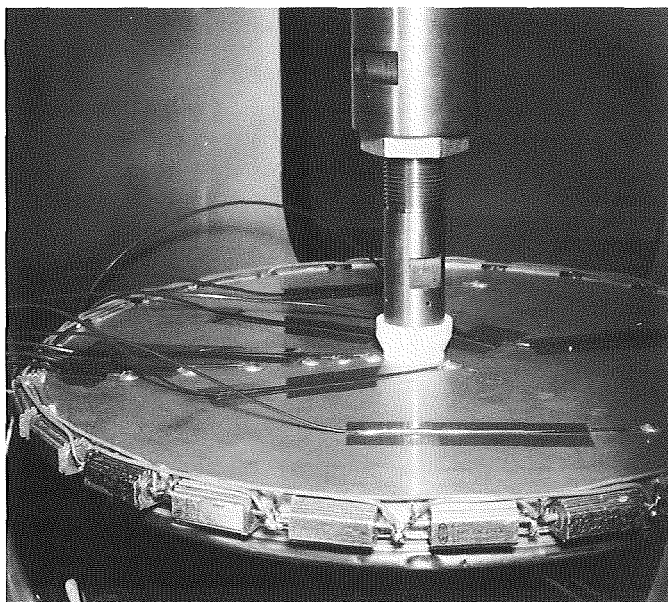


Fig. 14. Flat plates in test position in joint conductance facility

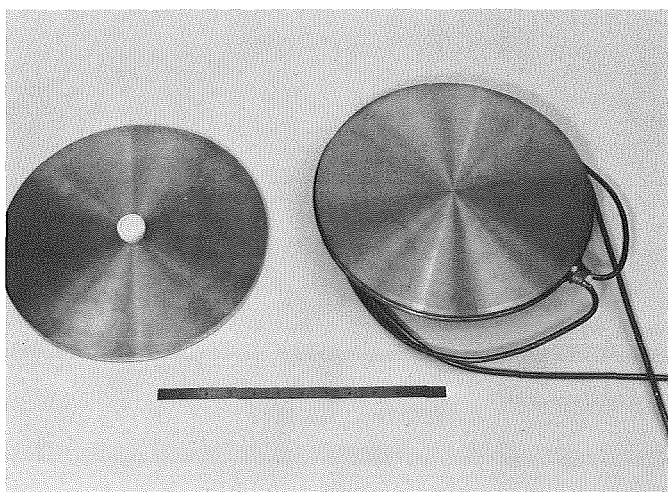


Fig. 15. Separated flat plates showing top of heated plate with alumina spacer and mating surface of cooled plate with cooling coil attached

plate was held constant by circulating liquid from a temperature-controlled liquid chiller through a copper tube attached to its edge.

Four tests were performed with the plates. The first test was with flat as-machined plates. Since the purpose of this investigation was to integrate the conductance versus pressure data into the total conductance for a simulated bolted joint, it was necessary to develop this pressure versus conductance data. This could be developed from

analytic methods developed in the university research, or it could be measured in an actual test using long rods with a surface finish the same as the flat plates. In order to eliminate uncertainties from these methods, one of the plates was machined, leaving a boss a few thousandths of an inch high and 1 in. in diameter. In this way, the effects of a radial pressure distribution are eliminated, since the pressure may be assumed to be uniform over the 1-in. diameter. Also the surface finish is the same as in the test without the boss; so this variable is eliminated. For all of the tests, the load was varied from 2000 to 10,000 lb and back to 2000 lb in 2000-lb increments. The load was then reduced in smaller increments until the plates would separate. Next, the plates were ground and lapped flat. The first test was then repeated. This test was then repeated for different edge temperature levels. One plate was then machined to leave a boss as for the as-machined tests. This test data will be available⁴.

One of the first checks of the data is a heat balance to verify the heat flow into the edge of the heated plate equals the heat flow out of the edge of the cooled plate. Although a plot of the data (Fig. 16) seems reasonable,

⁴Hultberg, J., *Thermal Joint Conductance* (JPL internal document), in process.

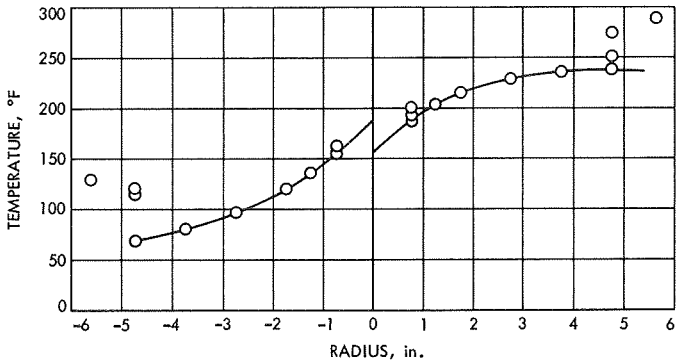


Fig. 16. Typical plot of data flat plates, ground and lapped (Run 3.2B)

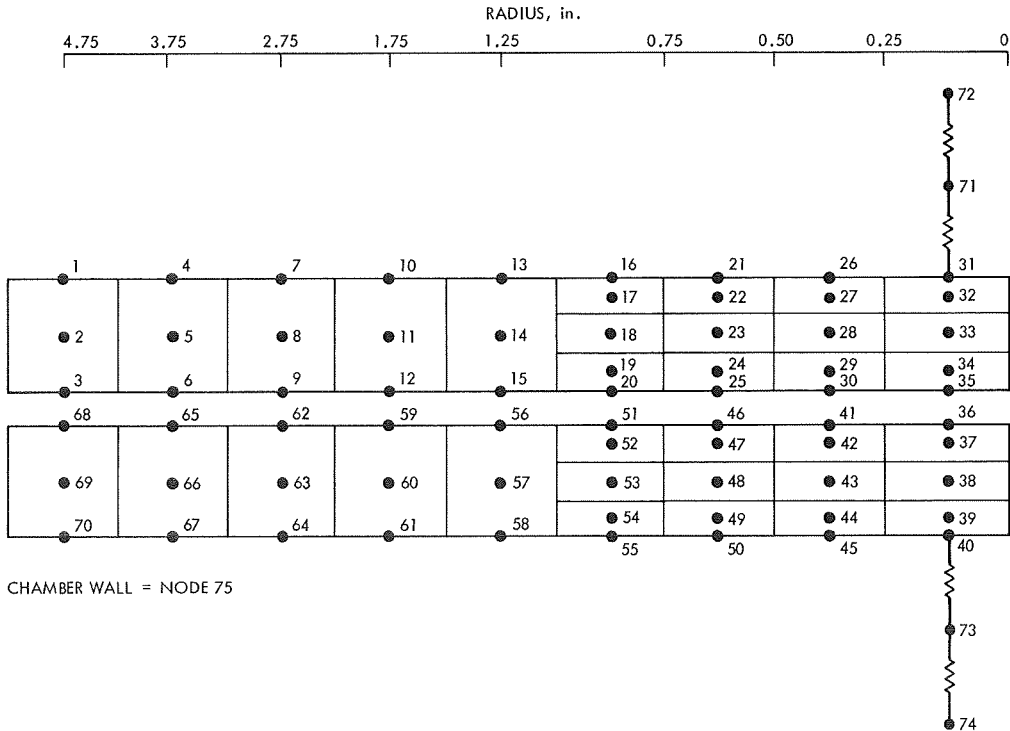


Fig. 17. Nodal model used to analyze flat plate data

a closer inspection of the data shows that, based on conduction only, the heat balance is not satisfied. A nodal thermal model was constructed to account for both conduction and radiation. This nodal model is shown in Fig. 17. The model was constructed so that the locations of the nodes would correspond to the location of the measuring thermocouples. In the model, the edge temperatures of the hot and cold plates were specified as boundary nodes. The heat loss through the rods used to transmit the load was also accounted for. However, even this model was unable to account for the obvious discrepancy in the data of both the flat plates and the flat plates with the 1-in.-diam boss. The experimental data is available so that other investigators may try correlations which may seem more appropriate. Table 8 shows the comparison of temperatures between test 2.7 and the thermal model. It is interesting to note that magnitude of the temperature differences between adjacent radii on one plate are not the same as the temperature differences between corresponding radii on the other plate. For instance, compare the 3.75- and 2.75-in. radii on the top plate where the temperature difference is from 271.8 to 261.4°F, or 10.4°F,

Table 8. Comparison of analytic model and experimental data for Run 2.7

Radius, in.	Run 2.7 temperature, °F	Analytic model temperature, °F
Top		
4.75	283.4	283.7 (A boundary node)
4.75	283.7	
4.75	284.5	
3.75	273.5	271.8
3.75	274.2	
3.75	274.7	
2.75	261.9	261.4
1.75	243.3	246.7
1.25	228.6	235.7
0.75	211.5	219.3
0.75	211.8	
0.75	209.8	
Bottom		
0.75	127.0	97.1
0.75	128.0	
0.75	128.6	
1.25	105.4	83.9
1.75	90.5	74.8
2.75	70.5	62.8
3.75	56.4	54.4
3.75	59.8	
3.75	61.7	
4.75	44.9	44.9 (A boundary node)

with the bottom plate at the same radii where the temperature difference is from 54.4 to 62.8°F, or -8.4°F. This demonstrates that even the computer model shows that a heat balance based on conduction only is not sufficient for analysis of the experimental data. The majority of this

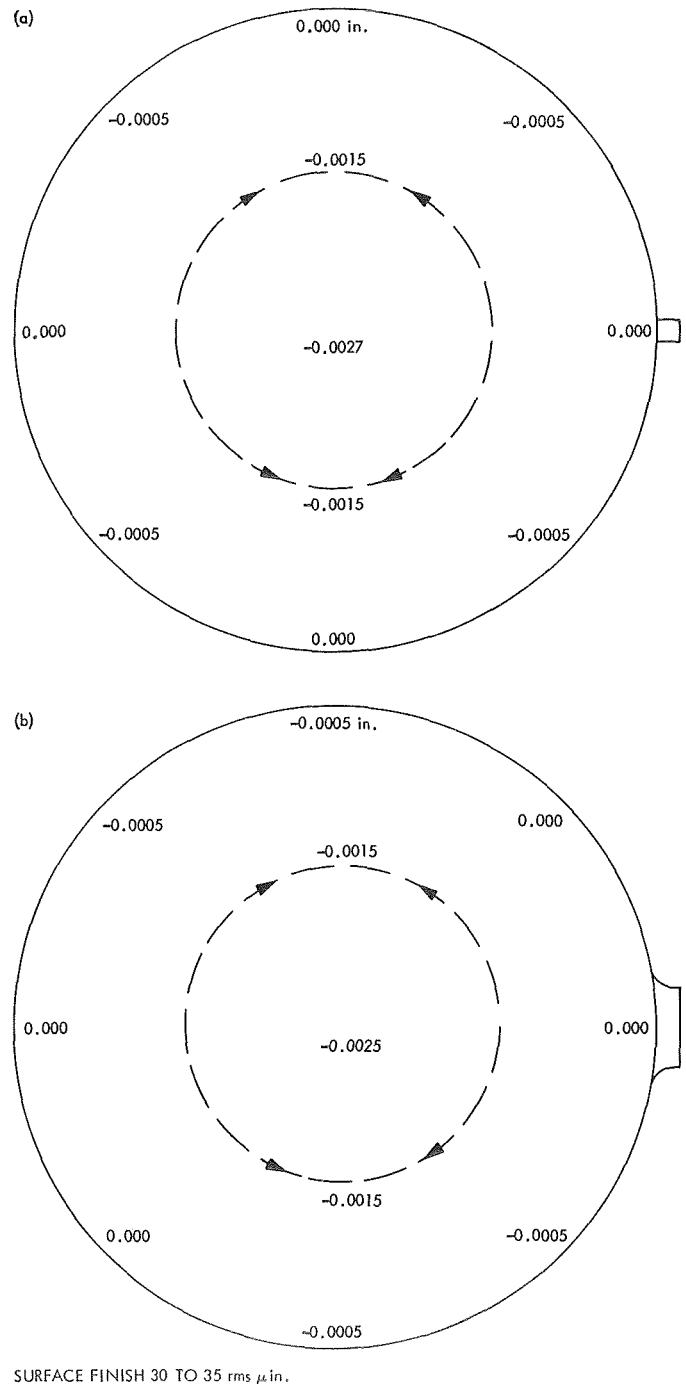


Fig. 18. Flatness deviation of: (a) heated plate, (b) cooled plate

difference is caused by the heat flow through the aluminum insulators and loading rods. The remainder is caused by radiation loss from the plates and radiation between the plates.

The work by Lardner (SPS 37-38, Vol. IV, pp. 83-85) indicates that for flat plates loaded as in the experimental configuration, the contact pressure goes to zero at a radius of 1 in. from the center line. This infers that beyond this radius, contact pressures are very low (or zero) so that heat transfer will be by radiation only. The problem of implementing the experiment so that this sort of loading data is valid, centers around the ability to make the plates flat. Considerable time and effort was expended toward this end with not very satisfying results. The plates were carefully machined to achieve surfaces as flat as possible with this process. The plates were then ground and lapped. Care was taken during the lapping process to avoid the build-up of heat in the plates. Inspection results of the plates are shown in Figs. 18a and 18b. These figures show that the plates are certainly not flat, and at least some of the discrepancies in the correlation of the experimental data and the analytical model can be attributed to this. Another problem area was realized after this test: the relatively large heat loss through the alumina insulators and loading rods. This heat loss is accounted for in the

analytic model, but no measurements were taken to verify the results. This heat loss does not represent an actual bolted joint; therefore it should have been eliminated, or at least carefully accounted for. The heat loss could have been greatly reduced or even eliminated by guard heating each loading rod to the same temperature as the plate under the alumina insulator. It seems unnecessary to repeat the tests with guarded loading rods until a method of producing a sufficiently flat plate can be found to make the conclusion with the analytic model meaningful. An alternate method would be to calculate pressure distributions for a non-flat plate, but this aspect was not pursued in this investigation. It is interesting to note that the work of Professors Chao and Clausing began with an attempt to produce flat surfaces on the ends of rods. This more or less inadvertently led them to the investigation of the macroscopic joint conductance where the mating surfaces of the rods were slightly rounded.

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XV. Data Handling Systems

ASTRIONICS DIVISION

A. Quasi-Linearity of Digital Record/Reproduce Process on Magnetic Tape Recorders,

J. C. Ashlock

1. Introduction

It has been reported previously (SPS 37-49, Vol. III, pp. 361-363) that a study of the record/reproduce process of digital magnetic tape recorders has been undertaken with the aim of generating a model of this process. The model resulting from this study and limited experimental data concerning its performance was described in detail in the SPS article cited above.

2. Quasi-Linearity

In the course of obtaining comprehensive experimental verification of the range of controlling parameters over which the model is valid, it was noted that the linear sum of the appropriate series of output voltage pulses corresponding to isolated transitions of record current could be used to obtain a multi-bit output voltage waveform. This concept is illustrated in Fig. 1.

It has been known for many years that this "quasi-linearity" can result in good approximations to multi-bit output waveforms under certain conditions. For example, consider a section of a recording layer as shown

in Fig. 2. Three magnetized regions are shown; one corresponds to positive remanent magnetization, a second corresponds to negative remanent magnetization, and a third represents the transition region between the other two levels. Figure 3 represents the remanent magnetization averaged through the thickness of the recording layer and plotted as a function of distance on the tape. It had previously been observed that as long as these magnetization transitions as written on tape do not overlap, quasi-linearity could be used to generate the resulting multi-bit output waveform. However, as the density is increased and the transition regions between bits on tape move closer together, the point will be reached where these regions will interfere with each other during the writing process. Since the writing process is inherently non-linear, this would establish an upper limit on density beyond which quasi-linearity would not apply. This limit appeared to be about 2500 bits/in. for standard $\gamma\text{-Fe}_2\text{O}_3$ tape. What was new in the observation made during the validation effort was the fact that the multi-bit output waveform generated by the model discussed in SPS 37-49, Vol. III, could also be arrived at using the appropriate linear sum of isolated transition playback waveforms generated by the same model. Furthermore, this waveform corresponded closely to what was observed experimentally for the same parameters. This appeared to be true for all densities to at least 20,000 flux reversals/in.

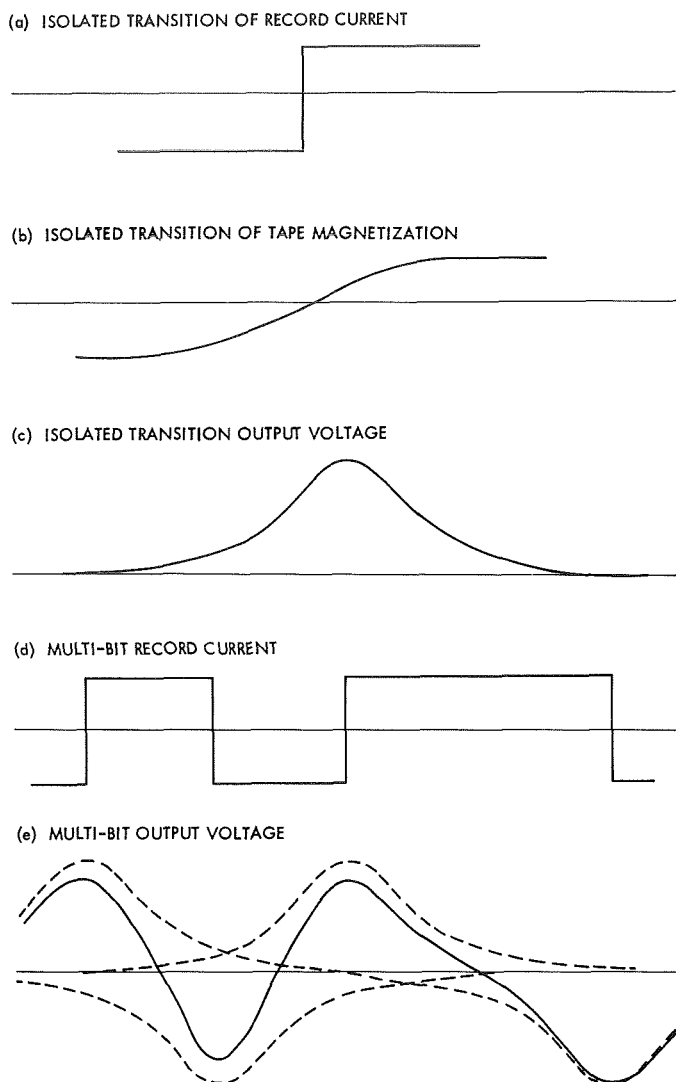


Fig. 1. Quasi-linearity principle

The underlying reasons for this quasi-linearity have been pursued. Details of this investigation are beyond the scope of this article, but it has been found (Ref. 1) to be valid at all densities provided two conditions hold. First, the record head fringing field must show a negligible rise time compared to the bit duration; the significant factor is the *field* rise time—not the input current rise time. It is possible that frequency response limitations of the write head could preclude quasi-linearity even when ideal step functions of input current are used.

The second condition is that the Preisach diagram for the tape being used must be of such a nature that the intersection points of the lines corresponding to head field extrema fall outside the Preisach function (Ref. 2). Because of the difficulty of measuring a Preisach diagram,

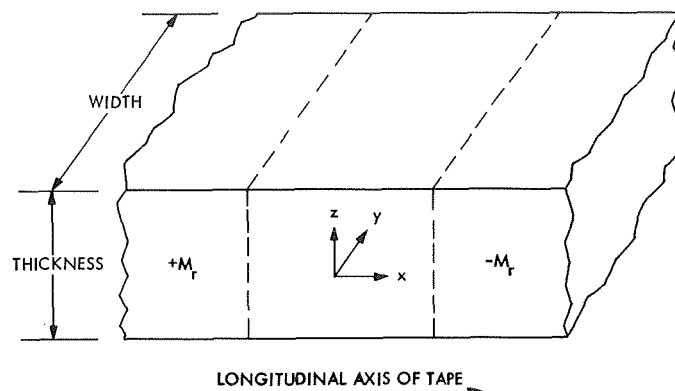


Fig. 2. Longitudinal section of recording layer

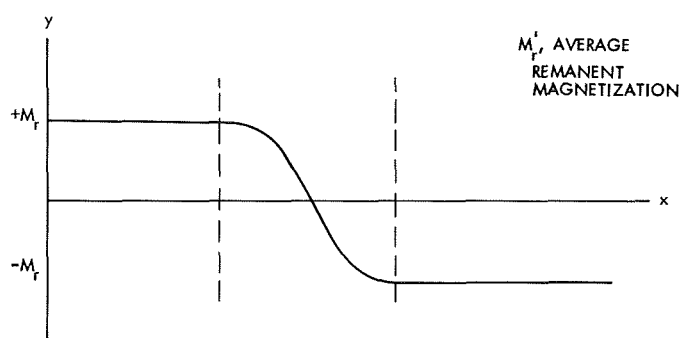


Fig. 3. Remanent magnetization averaged across the thicknesses of the recording layer

a simple test for this condition cannot be given. It appears likely, however, that the record head field gradient is usually high enough at small head-to-tape spacings to satisfy the second condition for most commercial recording media even at very high bit densities since the critical longitudinal field gradient is very large close to the surface. However, if the head-to-tape spacing were to be increased during the writing process (e.g., flying heads), at some point quasi-linearity will cease to be valid. The reproduce head-to-tape spacing used has, of course, no effect on quasi-linearity.

The validity of quasi-linearity in no way assumes or requires that the record process be linear. In fact, it is recognized that in order for the record process to exhibit memory, it must be non-linear. It is correct to say, however, that under the conditions of quasi-linearity, the essential non-linearity of the record process is sufficiently characterized in each isolated transition output pulse.

It has been demonstrated that the existence of quasi-linearity is not an idiosyncrasy of the model discussed

in SPS 37-49, Vol. III, or of the particular test configuration used in the experimental validation of that model. It is, rather, an inherent characteristic of the record process under certain conditions, and *one of the primary values of the investigation of quasi-linearity is in defining the conditions under which quasi-linearity will exist.*

3. Application to Bit Detection

One significance of the quasi-linearity concept in tape recording is that because the output of the playback heads can be generated from a single pulse that is characteristic of the system by using a linear technique, even when these pulses overlap, the techniques of linear filter and detection theory can be brought to bear on the problem of reconstructing (i.e., detecting) the recorded bit stream. Thus, the bit detection problem can be taken out of the realm of non-linear problems, even though the basic record process is still recognized as (necessarily) highly non-linear; in at least one sense, the digital tape recorder can be modeled as a linear communication channel with intersymbol interference.

Ignoring for the moment irregularities in the playback signal due to flutter and dropouts, and assuming timing is known, the detection problem can be stated as one of finding the optimum receiver structure for the case of

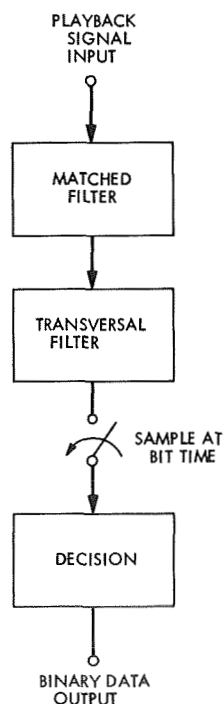


Fig. 4. Suboptimum equalizer structure

synchronous transmission of binary data over fixed, known, dispersive channels. This problem has been solved by the use of a decision-theory approach (Ref. 3); this approach leads to the equation that specifies the operations that the receiver must perform on its input to minimize the probability of error. The interpretation of these equations results in the optimum receiver structure—a non-linear device that, in this case, is unfortunately too complex and impractical to construct. Nonetheless, it does offer insight into attractive, practical, sub-optimum structures. One attractive receiver structure that can be developed from such insight is the equalizer shown in Fig. 4. This is a cascade of a filter matched to the signal, followed by a transversal filter. As a matter of interest, the structure is similar to that employed by the telephone industry to compensate for the distortion of pulse-amplitude-modulated signals in their transmission channels (Ref. 4), and it has been the subject of considerable analysis. Grossly, this structure may be thought of as a matched filter to reduce the effects of noise, followed by a transversal filter to reduce the effects of intersymbol interference.

The transversal filter is simple in principle, as can be seen from Fig. 5. A distorted signal enters a tapped delay line and is picked off at various taps (in this case, equally spaced at intervals of one bit length) on the delay line, delayed in time but unchanged in wave shape. The signal from each tap is passed through the associated attenuator (which may have either a plus or minus sign associated with it), and all the attenuator output signals are then summed. If we represent the pulse applied to the transversal filter by $h(t)$ and the response of the filter as $g(t)$, then we adjust the tap gains c_i so that $g(t_0) > g(t)$ for all $t \neq t_0$, and $g(t_0 + kT) = 0$ for all $k \neq 0$, where t_0 is time of detection, k is an integer, and T is the duration of a bit. Then the effect of intersymbol interference is

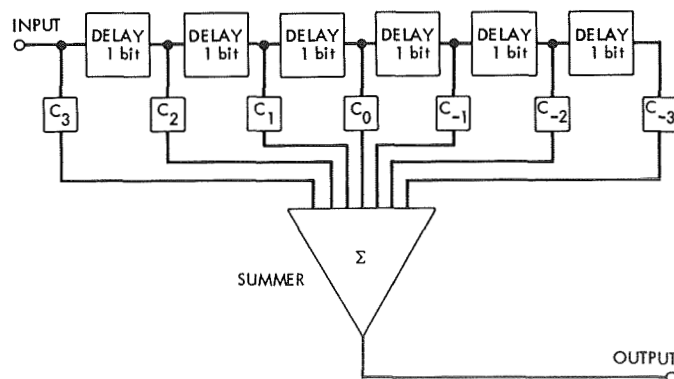


Fig. 5. Transversal filter schematic

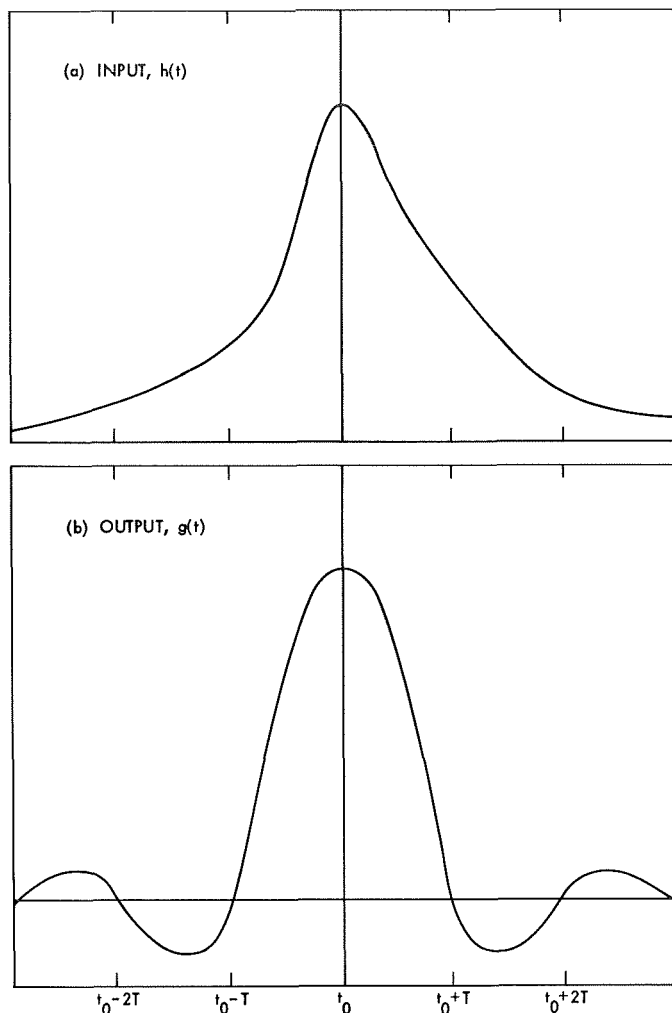


Fig. 6. Ideal response of transversal filter to characteristic function

eliminated at the times of detection, $t_0 \pm kT$ (Fig. 6). Of course, since linear superposition of characteristic pulses is valid prior to the matched and/or transversal filters, and since both the matched and transversal filters are linear, linear superposition of the output $g(t)$ illustrated in Fig. 6, to generate the signal at that point, is also valid. The requirement that $g(t_0 + kT) = 0$ for all $k \neq 0$ implies a delay line of infinite length. This is not feasible in practice, of course, so various measures of distortion have been defined. The tap gain coefficients are then adjusted to minimize the distortion criteria of concern. (See Ref. 4 for a comprehensive account of these distortion measures and minimization procedures.)

4. Current Work

The approach currently being pursued is to employ a transversal filter similar to that indicated in Fig. 5 to

reduce the effect of intersymbol interference. The lab work is being complemented by computer simulations of such filters, using the model discussed in SPS 37-49, Vol. III, coupled with the quasi-linearity phenomenon. Considerable effort has been directed toward fabricating a breadboard detector to operate at 10,000 bits/in. Initial indications of testing on this breadboard at 10,000 bits/in. are promising.

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B. A Highly Reliable Interface Circuit for Driving FET Switching Trees,

R. Easton and E. Hilbert

1. Introduction

This article describes a driver circuit for interfacing between transistor-transistor logic (TTL) and field effect transistor (FET) switching trees. It was designed for use in switching analog signals in the measurement processor of the thermoelectric outer planet spacecraft (TOPS) data handling subsystem (SPS 37-57, Vol. III, p. 41; SPS 37-59, Vol. III, p. 65). The principal objectives of the design are to minimize power dissipation and to maximize reliability. This driver can drive several hundred FETs at a time. It requires only 6 mW of power in its basic non-redundant version and 20 in its redundant version at 100% duty cycle. In normal operation, the duty cycle would be much less than this. The basic non-redundant driver has a probability of operation after 12 yr of 0.97. The redundant version has a probability of operation of 0.999, assuming the failure rates shown in Table I. Both versions can switch at rates of well over 100 kHz.

2. Basic Strobed Driver

A basic strobed driver was designed as shown in Fig. 7. Q_A and $\bar{Q}_A$ are one of the TTL flip-flop's outputs

from the sensor address register. In the absence of a strobe pulse, the entire circuit is off and only leakage current power dissipation occurs (approximately $10 \mu\text{W}$). When the driver is strobed, one side of the driver switches to 0 V and the other side to -15 V , depending on the states of Q_A and $\bar{Q}_A$. Power dissipation while on is about 6 mW . The switching rise and fall times are approximately $1 \mu\text{s}$, even for loads of several thousand picofarads. The driver outputs remain in the selected states as long as the strobe pulse is high. When the strobe pulse ends, the high output falls to -15 V with a time constant determined by the capacitive load and $100 \text{ k}\Omega$. Thus, in the absence of a strobe pulse, all tree FETs are off.

The use of a strobed driver circuit has the advantage that since the tree will not switch until the strobe pulse is high, it can be easily inhibited from switching while the new sensor address is being shifted into the sensor address register. This eliminates unnecessary power dissipation in the capacitive load of the FET. For a 512-sensor tree being switched at 10,000 samples/s, the power dissipated in the internal capacitance of the FET would be about 50 mW . In addition, the use of the strobe pulse effectively lowers the power duty cycle of the

Table 1. Assumed component failure rates

Component	Number of failures/h
Resistors	0.001×10^{-6}
Capacitors	0.005×10^{-6}
Diodes	0.01×10^{-6}
Transistors	0.02×10^{-6}
FETs	0.02×10^{-6}
Monolithic integrated circuits	0.20×10^{-6}

driver itself, making the tree and driver's power dissipation approximately equal to that of the tree alone.

3. Reliability Improvement of Basic Driver

Figure 8 shows a binary tree, including the register and drivers. A 512-sensor tree would require nine flip-flops and nine drivers. Trees can be made highly reliable through the use of redundancy. The first important step to increase driver reliability is to isolate each FET from the driver line with about $20 \text{ k}\Omega$. This prevents FET failures involving a short from gate to source or drain from affecting the driver control on the rest of the FETs.

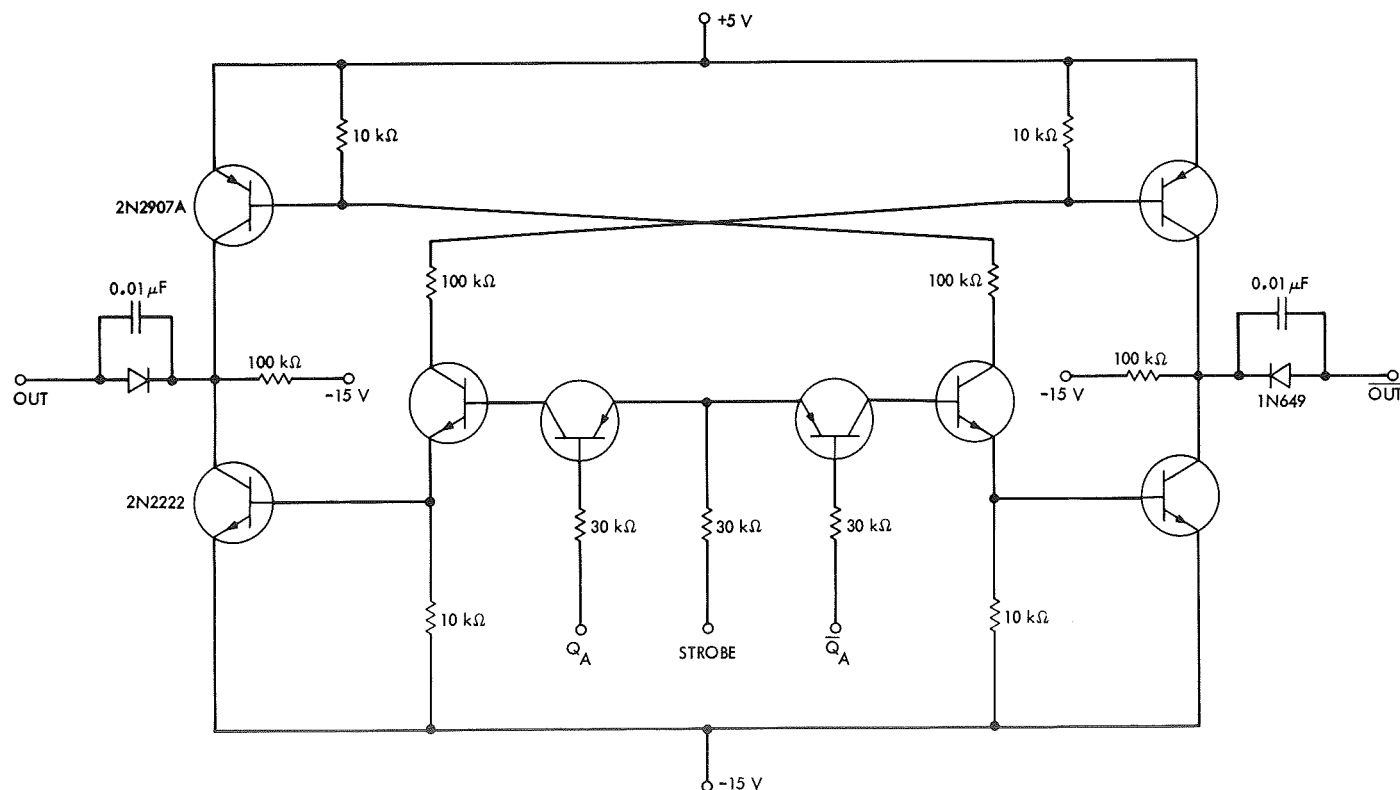


Fig. 7. Basic bipolar driver

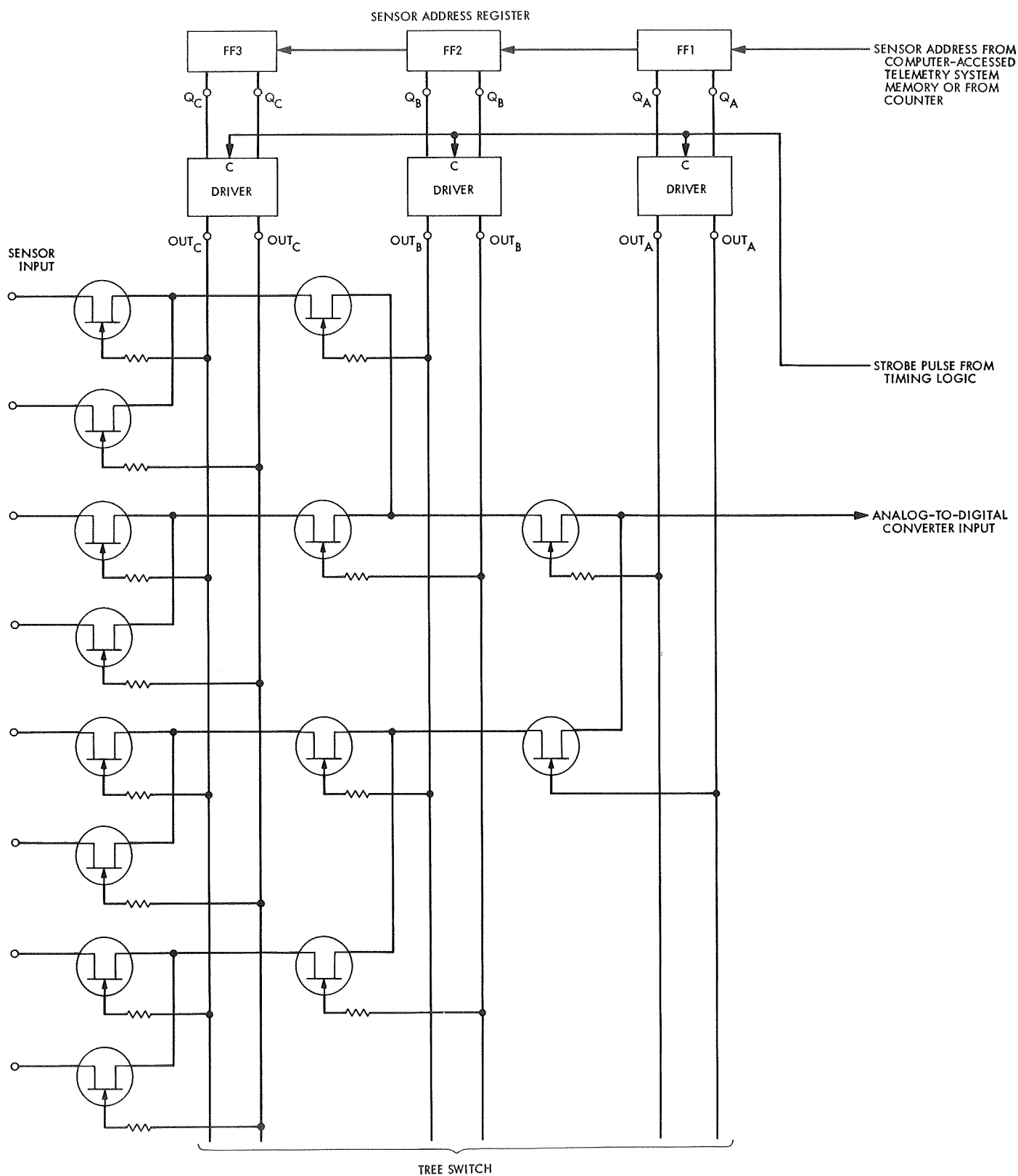


Fig. 8. Driver as used in binary tree

Using the parts reliability numbers indicated in Table 1, a single hybrid-flip-flop/driver combination has a probability of success in 12 yr of 0.97. For a nine driver combination, the probability of success is about 0.77. To improve this reliability, the basic driver circuit can be modified as shown in Fig. 9. This circuit is more complex and requires four parallel redundant sensor address flip-flops (Q_{A1} , Q_{A2} , Q_{A3} , Q_{A4}) per driver and is, therefore, more likely to have a single failure. However, no short or open of any one of the components between any of its terminals can cause loss of operation (i.e., power dissipation will increase slightly and driver waveforms will change, but no degradation will be noted when looking at the tree switches signal output). In addition, the loss of any one flip-flop and most multiple failures will not affect its operation. The probability of no failure after 12 yr of any component within one redundant flip-flop/driver combination is about 0.89 and the probability of no component failures in any of nine redundant units is only 0.33. However, the probability of a single redundant flip-flop/driver combination maintaining operation for 12 yr is approximately 0.999 and the probability of all nine redundant units working for 12 yr is about 0.99.

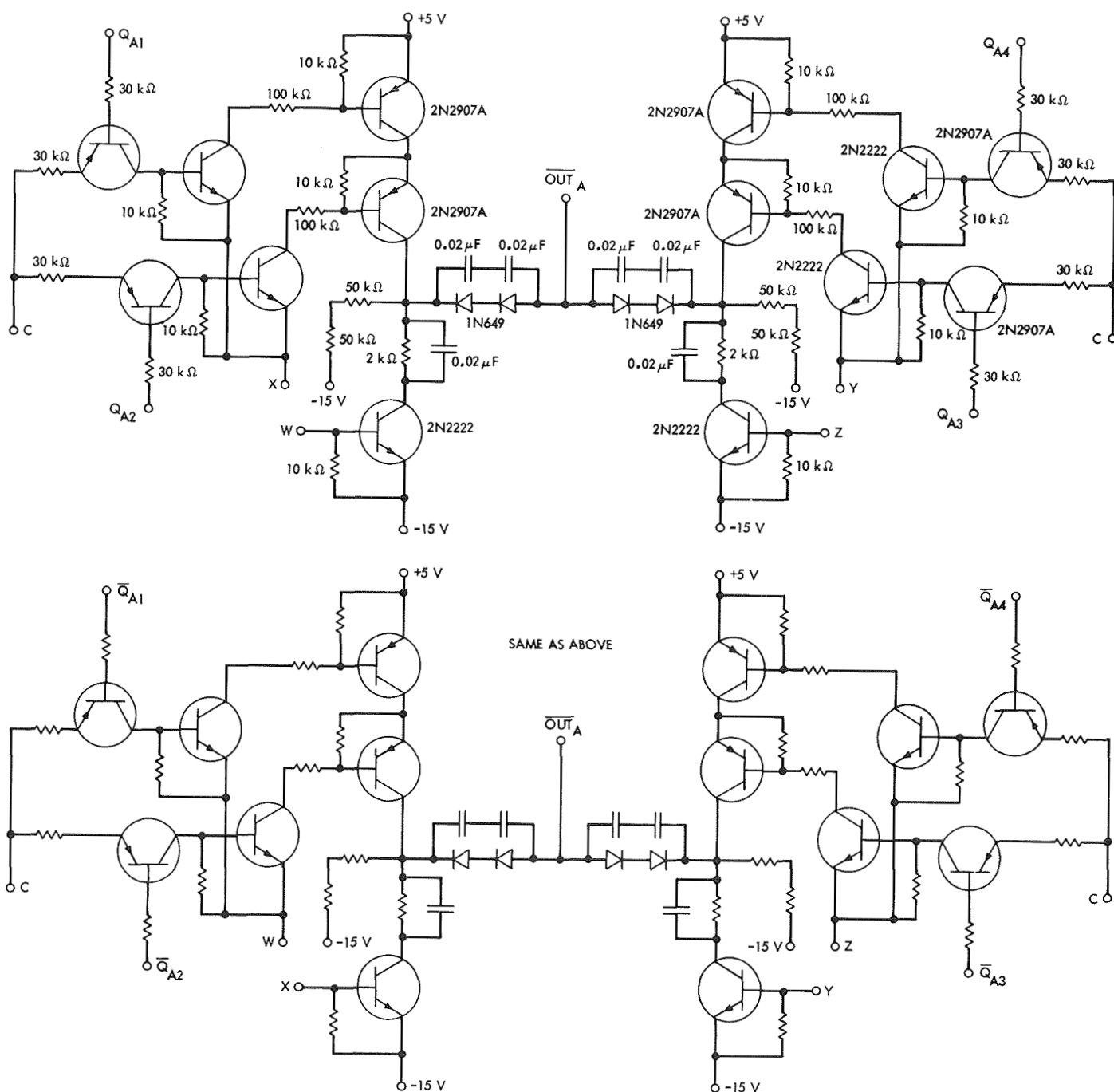
The above reliability values assume the use of discrete component failure rates except for assumed monolithic sensor address register flip-flops. No combination of commercially available monolithic driver circuits were found that could be interconnected to give similar reliability. Due to the resistor and capacitor requirements of this driver circuit, it appears to be best suited to construction using discrete or hybrid technology.

4. Power Dissipation of Redundant Driver

Since only n drivers are required for 2^n sensors, one can afford to sacrifice a slight amount of weight and volume for this added reliability. In addition, there is little power penalty for the added redundancy because of the original power efficiency of the basic circuit. A redundant complementary driver was breadboarded and tested. Its performance was verified under various failure conditions. A plot of unloaded driver power for one redundant driver unit (not including control flip-flops) is given in Fig. 10. Also included in Fig. 10 is the typical average power consumption of the lowest-power, commercially available driver and the estimated average power consumed in the total capacitive load of a 512-sensor binary tree. Total power consumed (excluding flip-flops) by the 512-sensor multiplexer is the tree power plus nine times the driver power.

5. Conclusions

It can be concluded that it is possible to build an FET tree driver that requires much less power and provides much higher reliability than commercially available monolithic units or the *Mariner* 1969 discrete component unit. Both the basic circuit and the redundant circuit discussed have been breadboarded and tested with discrete components. The redundant driver has also been fabricated on a 1- × 1-in. substrate that utilized hybrid chip capacitors, LID-type (leadless inverted device) semiconductors, and thick-film resistors and wiring. A full-sized tree system for 384 sensors incorporating nine of these hybrid redundant driver units is being constructed.



TERMINALS WITH THE SAME NAME ARE TIED TOGETHER

Fig. 9. Redundant complementary driver

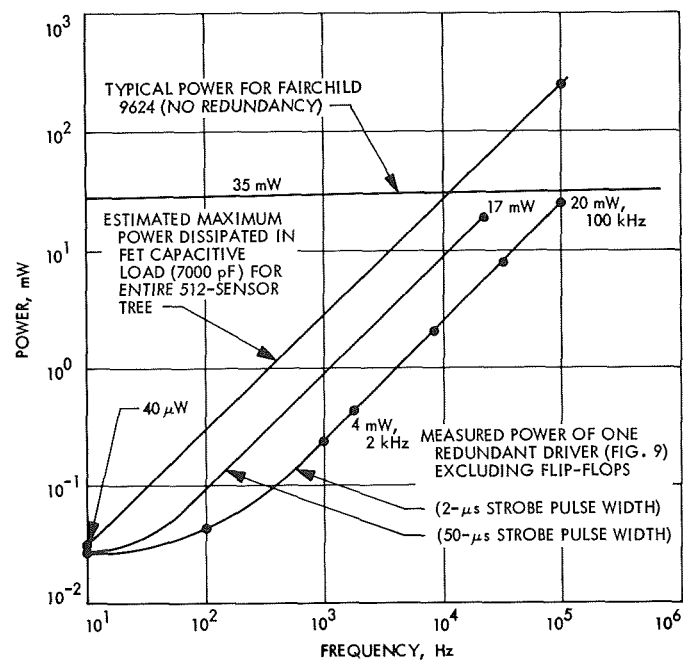


Fig. 10. Power comparisons

XVI. Astrionics Research

ASTRIONICS DIVISION

A. Superconducting Josephson Junctions Fabricated on Anodized Tantalum

Substrates, *P. V. Mason and H. B. Thacker*

1. Introduction

Thin-film Josephson junctions offer the possibility of nanosecond-switching-time computer elements that can be fabricated at high densities with microcircuit techniques (SPS 37-51, Vol. III, pp. 72-75). Major problems that must be overcome are:

- (1) Present fabrication techniques do not have high yield.
- (2) Junction characteristics are not reproducible from sample to sample.
- (3) Junctions deteriorate when stored at room temperature.

This article discusses preliminary results on a junction formed by anodically oxidizing tantalum, then overlaying the oxide with a superconducting thin film.

2. Functional Description

A Josephson junction consists of two superconductors separated by a few atomic layers of insulating barrier. Since the Josephson effect depends on tunneling of super-electrons through the barrier, the maximum Josephson current $I_{J\max}$ (which is the critical design parameter) is strongly dependent on film thickness. Additionally, dif-

fusion of metal can reduce the barrier, or diffusion of oxygen atoms can increase it, leading to changes with time, especially at room temperature. Certain fabrication processes (SPS 37-51, Vol. III, and Ref. 1) overcome one or the other of these problems, but none seem entirely suitable.

Anodic oxidation (application of positive voltage to tantalum in a water solution containing an oxygen-rich compound such as sodium sulfate) has been extensively studied by Young (Ref. 2) and others. It is known to yield a dense, uniform, pinhole-free oxide whose thickness can be closely controlled by choosing the correct voltage, temperature, and terminating current densities.

3. Fabrication Techniques

Fabrication consists of the following steps:

- (1) Electropolishing of a tantalum sheet to remove microscopic roughness by Tegert's method (Ref. 3).
- (2) Removal of the resulting insulating film by hydrogen reduction.
- (3) Evaporation of a silicon monoxide mask over all except a 0.5-mm strip to define the active area.
- (4) Anodization at 0.5 V for 30 s in a solution of 100 g sodium sulfate in 1 l of water.
- (5) Vacuum evaporation of a tin cross strip to form the second electrode of the junction.

The current-voltage characteristic at 3.0°K of the resulting film is shown in Fig. 1 ($I_{J\max}$ was 84 μA).

The theoretical tunneling curve was also calculated and is shown in Fig. 1. As a result of the different energy gaps of tin and tantalum, the theory predicts a peak in current at the voltage corresponding to the difference; however, this was not observed.

4. Conclusion

The junction of Fig. 1 was stored for 6 mo at 77°K. When the characteristics were remeasured, no significant change had taken place. The film was accidentally destroyed before the effects of cycling to room temperature could be studied.

These results show promise as a useful fabrication method. Further work must be done to determine effects of room-temperature storage, to increase $I_{J\max}$, and to determine yield and reproducibility.

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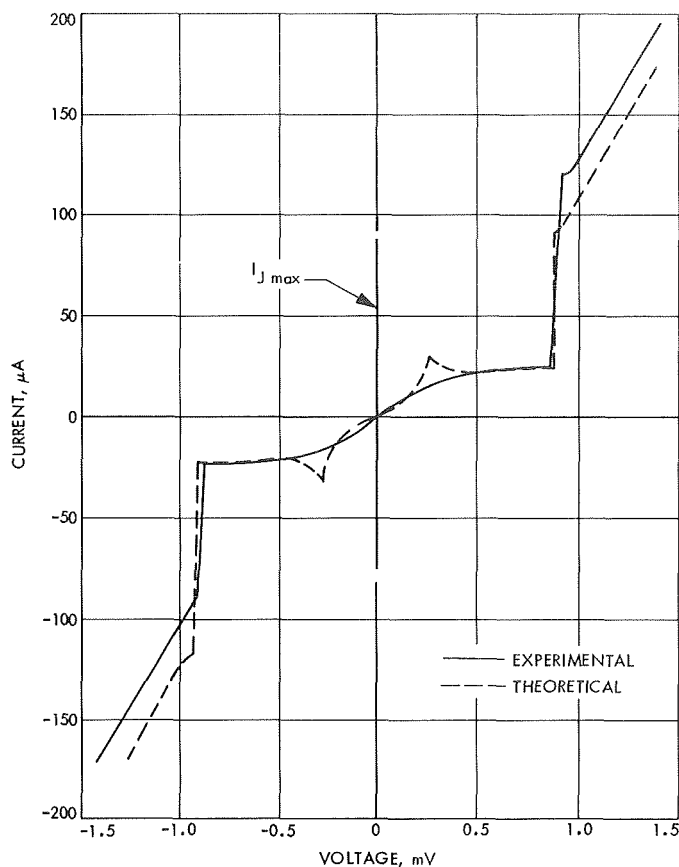


Fig. 1. Current-voltage characteristics of tantalum/tantalum oxide/tin Josephson junction

XVII. Aerophysics Facilities

ENVIRONMENTAL SCIENCES DIVISION

A. Thrust and Plume Measurements Taken on Miniature Nozzles in a Vacuum Chamber, *G. Herrera*

1. Introduction

The nozzle flow and vacuum exhaust technology program¹ is composed of an analytical and experimental effort. For the experimental portion of the program, the 25-ft Space Simulator (Ref. 1) and Molsink (Ref. 2) in the Environmental Sciences Division were used. Technical personnel from the Aerodynamic Facilities Section participated in planning the tests and were responsible for providing hardware, equipment, and running the tests.

The purpose of the test performed in the Space Simulator was to measure the thrust and the pressure distribution inside the nozzle and at the exit. Several conical nozzles were evaluated at the pressures listed in Table 1 (Fig. 1).

2. Test Procedure

Thrust measurements were taken with a wind tunnel strain gage balance that was adapted for this application.

¹For a discussion of the objectives, overall plan, and analytical efforts of this task, see Chapter XXII-B.

The static pressure distribution on the nozzle surface was determined by means of three orifices, each connected to a pressure transducer. Pitot pressure surveys were taken at the nozzle exit with a small probe mounted on a remotely actuated traverse.

The general test procedure consisted of evacuating the simulator to approximately 10–20 torr. Once steady-state conditions had been achieved, gaseous nitrogen at 70°F was introduced into the plenum chamber and subsequently to the nozzle. In order to prevent taking data with a separated boundary layer condition, data were taken before the ambient pressure exceeded the nozzle exit pressure.

Table 1. Nozzle specifications

Nozzle	Area ratio	Throat diam, in.	Exit diam, in.	Plenum pressure, psia
IA	60:1	0.100	0.775	5–25
IB	60:1	0.100	0.775	5–25
IC	60:1	0.100	0.775	5–25
IIIA	240:1	0.050	0.775	5–25
IIIB	15:1	0.200	0.775	5–25

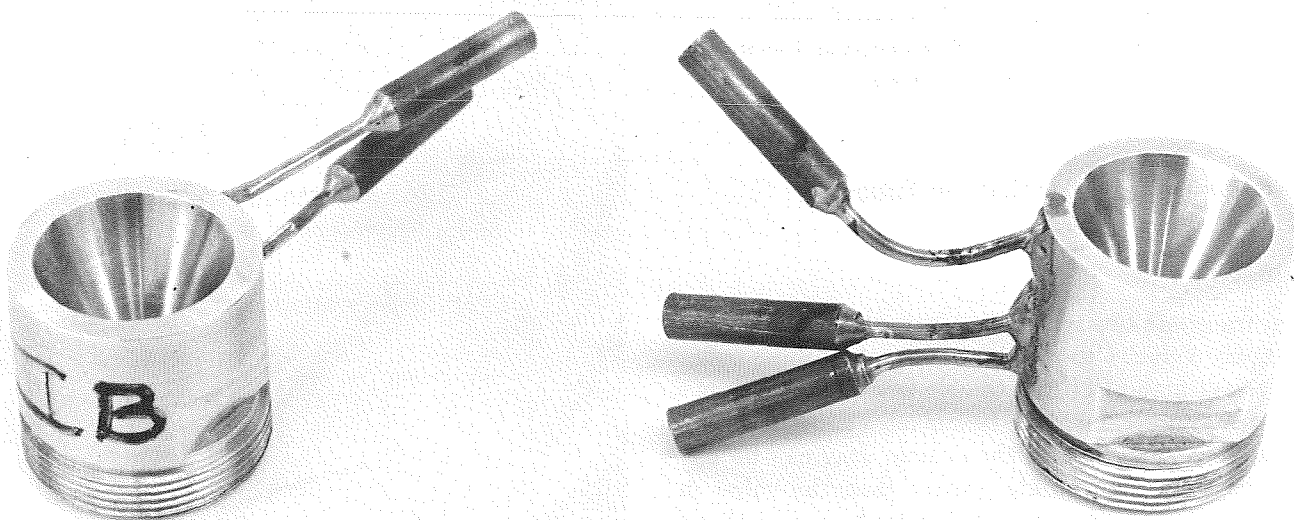


Fig. 1. Conical nozzles evaluated in 25-ft Space Simulator and Molsink

The main objectives of the test performed in the Molsink were:

- (1) Checkout and evaluation of all experimental hardware and subsystems to be used in the facility.
- (2) Measurement of the static pressure variation on the nozzle surface.
- (3) Determination of the density distribution in the nozzle plume with an electron beam/photomultiplier system.
- (4) Measurement of the nitrogen mass-flow per unit area at the crystal by means of quartz crystals mounted on the Molsink walls. These quartz crystals are frequency sensitive and can detect changes in mass.

Nozzle IIIA, which was tested in the simulator, was run in the Molsink in a plenum pressure range of 21–26 psia. Because of the excellent pumping capability of the Molsink, it was possible to maintain it in the 10^{-5} – 10^{-6} torr range for several hours with nitrogen gas flowing at 1–2 g/s. The test procedure consisted of evacuating the Molsink to about 10^{-6} torr and subsequently introducing the gaseous nitrogen. During the runs data were taken and, by means of computer time sharing, were recorded and printed on paper.

Pressure and temperature data were taken for both the plenum and nozzle. Because of a malfunctioning transducer, the nozzle exit pressure data was considered unreliable. The transducer connected to this particular orifice did not respond properly and indicated an extremely low pressure. A plume density data measurement taken at the nozzle exit was limited primarily because of a high-voltage short in the electron beam system. The data taken can only be used qualitatively but was of benefit in evaluating the electron beam/photomultiplier system. Information from the 12 quartz crystals installed on the Molsink walls was obtained for all the runs.

3. Concluding Remarks

Because of the complexity of the electromechanical systems associated with this test, a substantial amount of time was devoted to the checkout and evaluation phases. This time was necessary in order to fully check, fix, and find solutions for the hardware. Now that the systems have been exposed to the Molsink environment, those that did not operate properly can be fixed and improved. Following the conclusion of the Molsink test in December 1969, the test hardware was removed, inspected, and plans are underway to repair and make improvements prior to the next test.

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B. Aerodynamics of Vehicles in Tubes,

B. Walker and B. Dayman, Jr.

1. Introduction

Many transportation systems are currently being studied that require vehicles to operate in tunnels (tubes) under conditions of high blockage (vehicle frontal area occupying a significant portion of the tunnel cross-sectional area). When this condition exists, the vehicle drag can be one to two orders of magnitude greater than the value in free air. In addition, the air velocities in the tunnel ahead of and behind the vehicle can be appreciable relative to the vehicle velocity.

A proper understanding of these aerodynamic characteristics is necessary to efficiently design systems that utilize vehicles in tunnels, which in turn will make it possible to choose an optimum transportation system. In conjunction with theoretical studies by the Aeronautics Department of the California Institute of Technology, an experimental program was initiated at JPL on the aerodynamics of vehicles traveling in tubes.

2. Description

Currently operational is a 70-ft long, 2-in.-diam, vertically oriented aluminum tube that utilizes gravity to propel the models being tested. The tube (with two of the models used) is shown in Fig. 2. (The tube on the right is a utility tube.) Figure 3 is a schematic of the tube.

This 70-ft tube replaces an original 32-ft long, 2-in.-diam, vertically oriented Plexiglas tube that was used to develop the necessary testing techniques for the more advanced, high-precision facility now in operation. The 70-ft tube is instrumented along its length with magnetic-coil pickups to indicate model position along the tube as a function of time. This configuration yields data from which model velocity, acceleration, and drag can be determined. Also, wall pressure ports are provided for recording the pressure history along the tube (Fig. 4). The tube is aligned to within ± 0.015 in. over the 70-ft length.

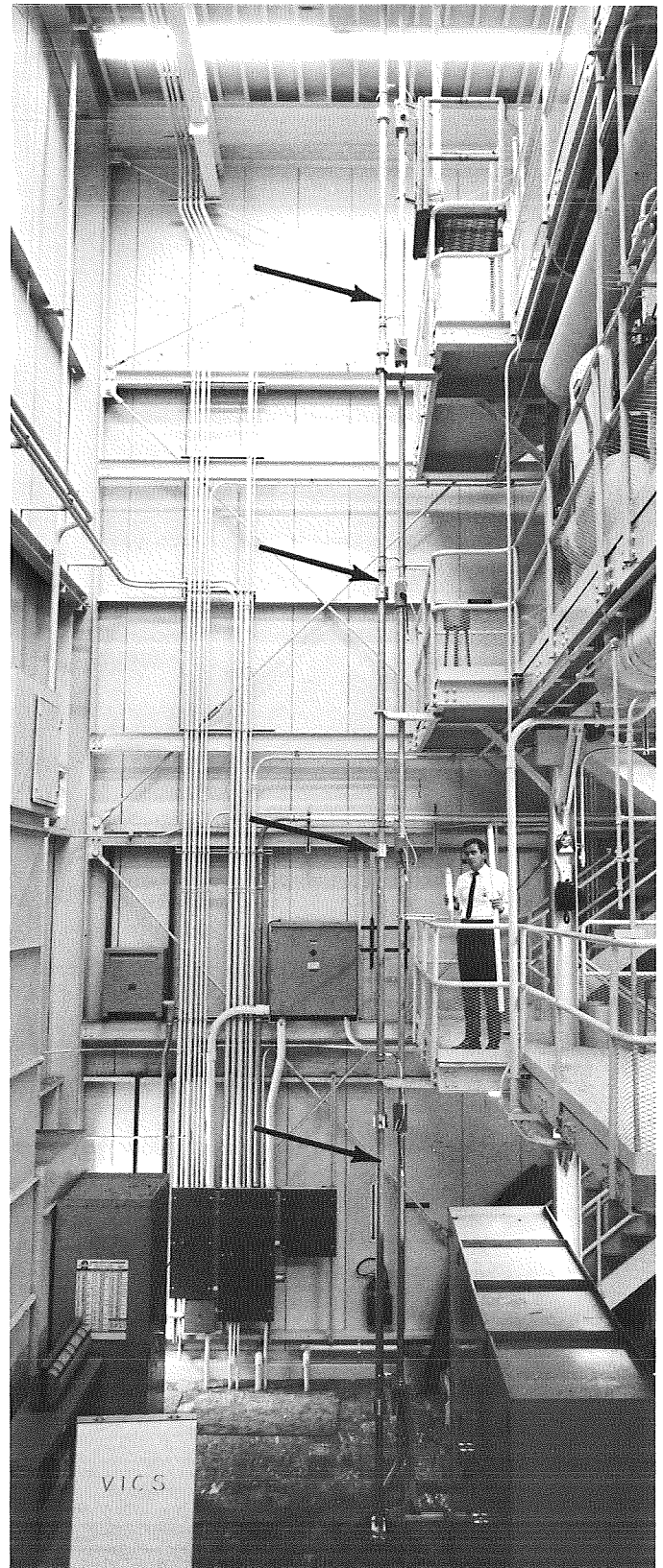


Fig. 2. JPL 70-ft drop tube

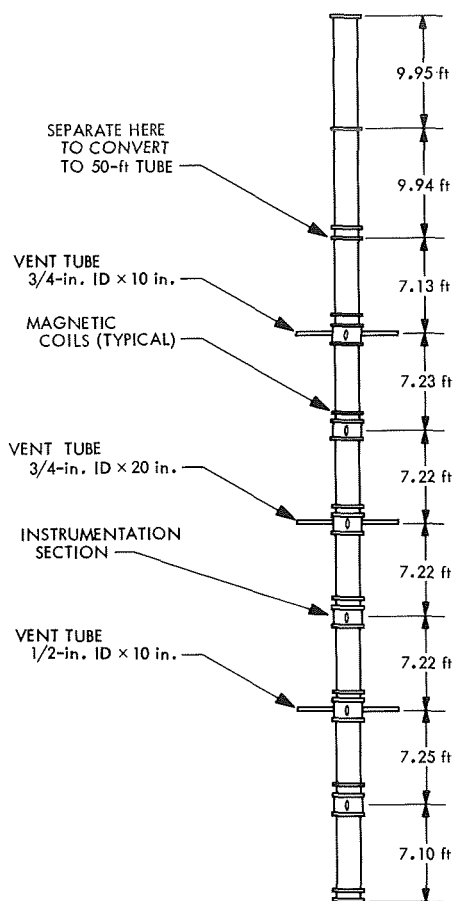


Fig. 3. Schematic of 70-ft drop tube

3. Discussion

The present facility is a very versatile one; several geometric characteristics of the 70-ft tube are readily alterable. The tube length can be decreased to 50 ft or

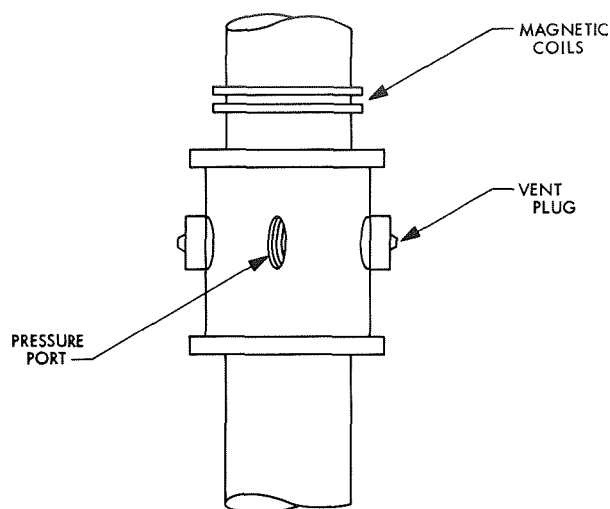


Fig. 4. Typical instrumentation section

effectively increased up to an infinite length. Also, venting ports are provided along the entire length of the tube. The interior walls of the tube are aerodynamically smooth but controlled roughness of the walls can be introduced. It can be pressurized up to 300 psi and can use different fluids to obtain data through a wide range of Reynolds numbers. A model launcher can be attached to vary the initial velocity of the test models. The effects of model eccentricity in the tube and several models traveling simultaneously in the tube can be investigated.

To date, model vehicles of various geometries have been constructed and some results have been obtained at model velocities from 15 to 45 ft/s under both unsteady (model accelerating) and steady (model at constant velocity) conditions. Figure 5 shows an example of the model velocity data that can be obtained. Model

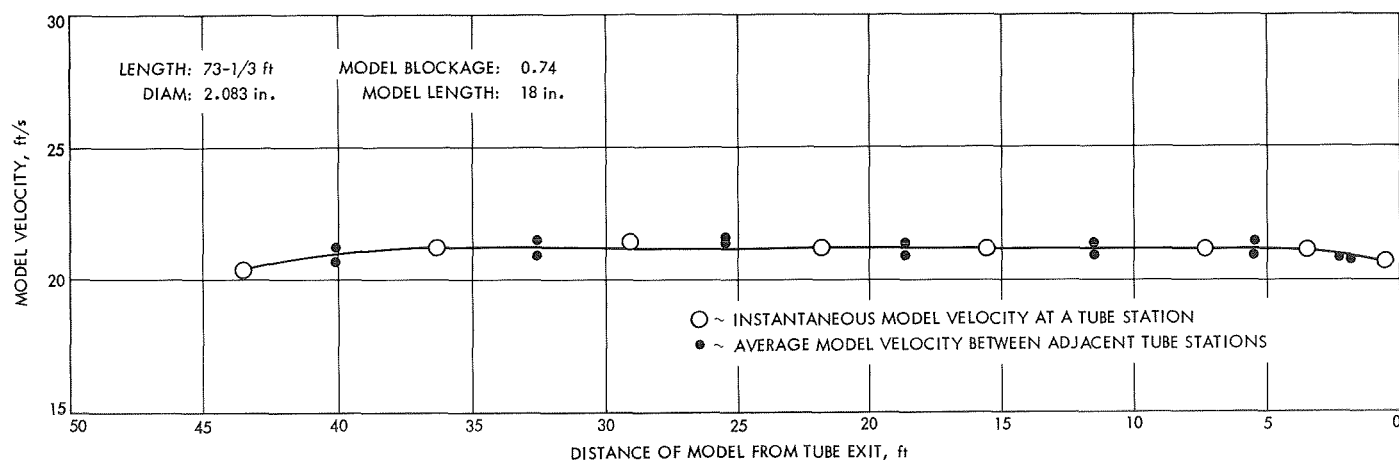


Fig. 5. Sample of velocity history data

blockages (ratio of cross-sectional area of model to cross-sectional area of tube) of 0.45, 0.74, and 0.90 and model lengths of 1.5 and 5 ft ($l/d = 8.6$ and 29) with various nose and base shapes have been tested. The data are now in the process of being analyzed in conjunction with the development of analytical models formulated for the understanding of the aerodynamic characteristics of vehicles traveling in tubes. The comparison of the experimental data with both the steady and unsteady analytical models is already quite good.

C. Planetary Entry Flight Dynamic Research,

P. Jaffe

1. Introduction

The key to accurately predicting the flight performance of a passive (one requiring no attitude control) planetary entry probe is the proper determination of the dynamic stability coefficient for all regimes of its trajectory. This is difficult for a blunt body, because the influence of this coefficient, in a free-flight test, is relatively small making its determination through testing difficult.

In the past five years, there has been considerable progress in obtaining this kind of information, at least for planar motion. However, many researchers now believe that there can be a substantial difference between the dynamic stability in the planar and nonplanar modes. Anticipating the need, a program was started about four years ago to extend the wind tunnel free-flight testing technique into the nonplanar regime (Ref. 1). During the

last year, this capability has been refined to a high level of precision permitting the acquisition of nonplanar data at accuracies not before possible.

2. Discussion

Briefly, the technique is as follows: Models are launched upstream into the oncoming flow at a prescribed angle-of-attack and roll rate; as the model leaves the launcher, it is given a transverse angular kick. The proper combination of these three initial conditions will produce any free-flight motion desired. The free-flight motion is recorded with a high-speed 35-mm movie camera operating at about 2000 frames/s. The camera is synchronized to a strobe light that effectively decreases the exposure time to about $4 \mu\text{s}$ and greatly increases the image resolution. Two views of the model in flight (60-deg apart) are obtained by means of the mirror system described in Fig. 6. Both views are recorded together on each frame of data. The film is then manually read on a film reader; the two model orientation angles are measured on each frame, the model's translational position measured, and the time frame relationship determined. In addition to the angular and translational information, an accurate record of the model's spin in flight is required. Two methods of recording this spin are available: (1) for slender configurations, a spiral stripe painted on the model is sufficient to determine spin to about 1%; (2) for blunt configurations, a fiber optics system mounted above the launch gun is used to view a stripe painted on the base of the model and transmit the information to a camera outside the wind tunnel—the resultant accuracy is also about 1%.

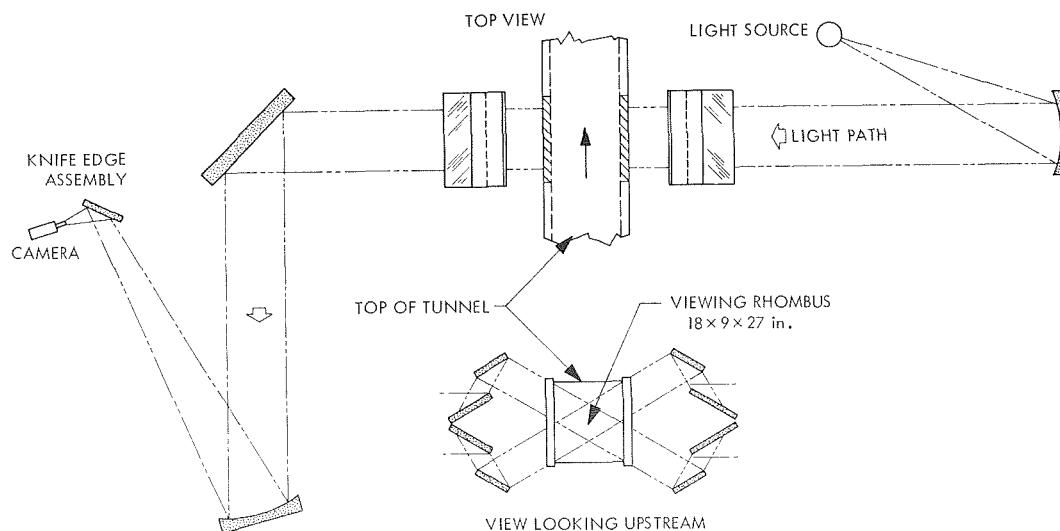


Fig. 6. Schematic of bi-planar optical system

The data reduction process from this point on is done on a computer: each pair of orientation angles is transformed to a fixed-plane alpha and beta pair (almost an orthogonal vertical and horizontal pair). This, along with the corresponding relative position between the model and free stream (obtained from the translational history) is then input to a tricyclic differential correction program in exactly the same manner as is done in a ballistic range. From the translational history the drag coefficient is determined; from the tricyclic fit the static and dynamic stability coefficients are determined.

The first test programs that used the improved system were performed on two widely different configurations, a blunted 60-deg (half-angle) cone and a sharp 10-deg (half-angle) cone. The blunt-body test consisted of 21 shots, 19 of which were reducible; 18 were conducted at Mach 3.97 and the remaining 3 were shot at Mach 1.65. Thirteen shots, all at Mach 4.56, were made during the 10-deg cone test, 7 of which were reducible. The diameters of the blunt models and of the sharp cones were 1.5 and 1.0 in., respectively. Both were constructed with a dense core and a light plastic exterior shell.

A typical alpha-beta plot from each test is shown in Figs. 7 and 8; the stars are the raw data points and the smooth curve is from the fit equation. The virtual absence of raw data scatter should be noted. It is almost impossible to hand draw fair curves through the raw data points that have a mean deviation greater than 0.1 deg for the 10-deg cones and 0.5 deg for the 60-deg cones.

Although the curve fits were good, there appeared to be a difference in the phasing between the computed alpha-beta modes and the raw data, particularly for the

blunt bodies (Fig. 8). Investigation brought out two problem areas that are generally neglected in ballistic range analysis. First, in the derivation of the tricyclic equation it is assumed that the roll rate to velocity ratio is constant; this produces an error in the vector rotation rates. For the case shown in Fig. 8, the velocity changed 7% during the flight and the vector rates changed 1.7% and 0.6% (the total effect is additive). Second, the non-rolling-axis system used in the derivation differs by a small amount from the fixed-plane axes (Ref. 2). The major part of the error resulting from these two components can be eliminated iteratively by using the information from the first pass through the tricyclic program and inputting it along with the original alpha-beta data in a modified tricyclic program. The corrected alpha-beta curve is given in Fig. 9. It is obvious that the fit has been improved but not to the extent desired. The exact reason why the improved fit was not better is currently under investigation.

3. Concluding Remarks

To date all of the reducible data from the 10-deg sharp cone test (7 flights) and 10 runs from the 60-deg blunt cone test have been read and put through the reduction as outlined. Release of this data is being held up until resolution of the data reduction problem is completed.

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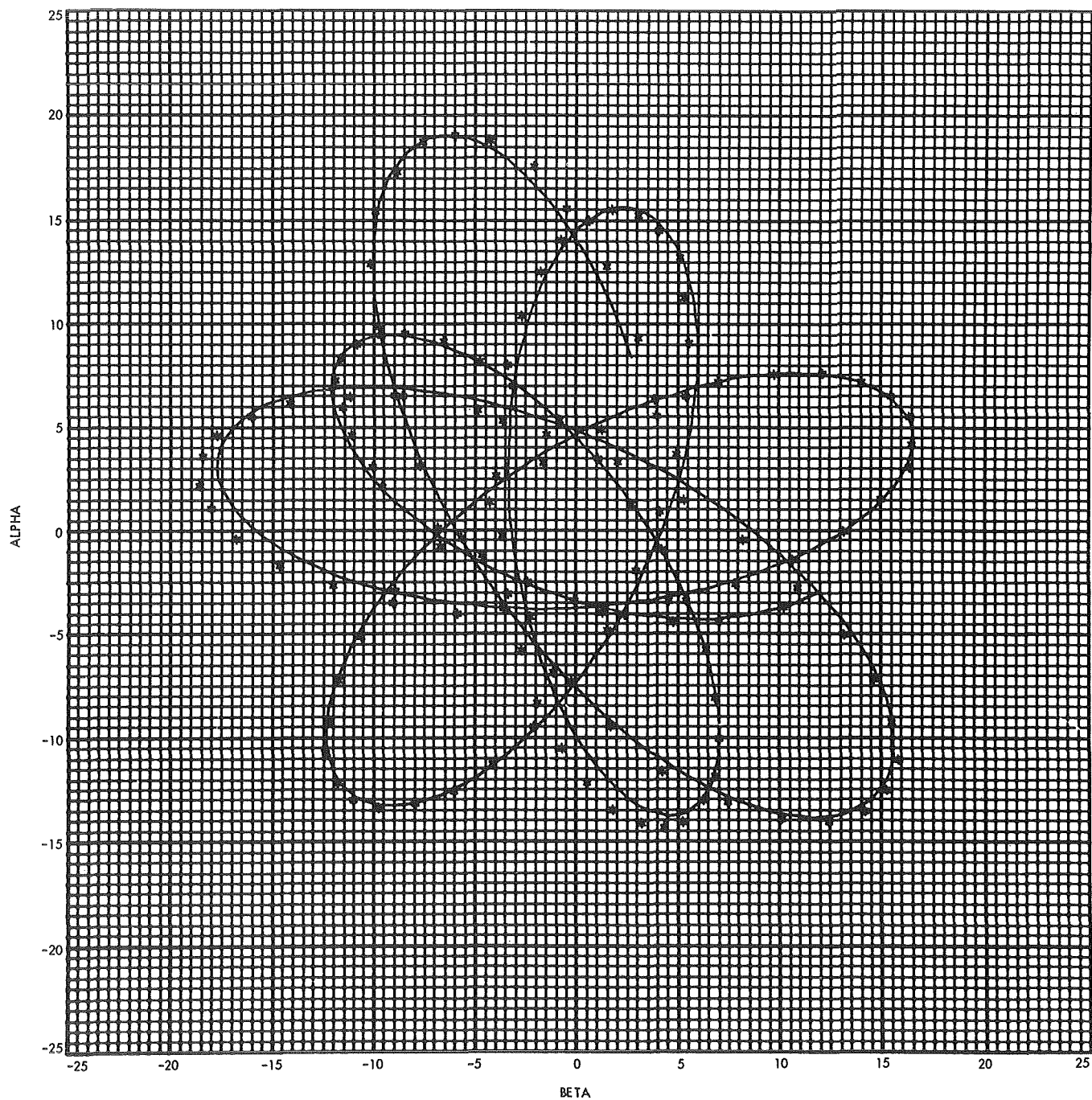


Fig. 7. Typical alpha-beta plot of a 10-deg cone

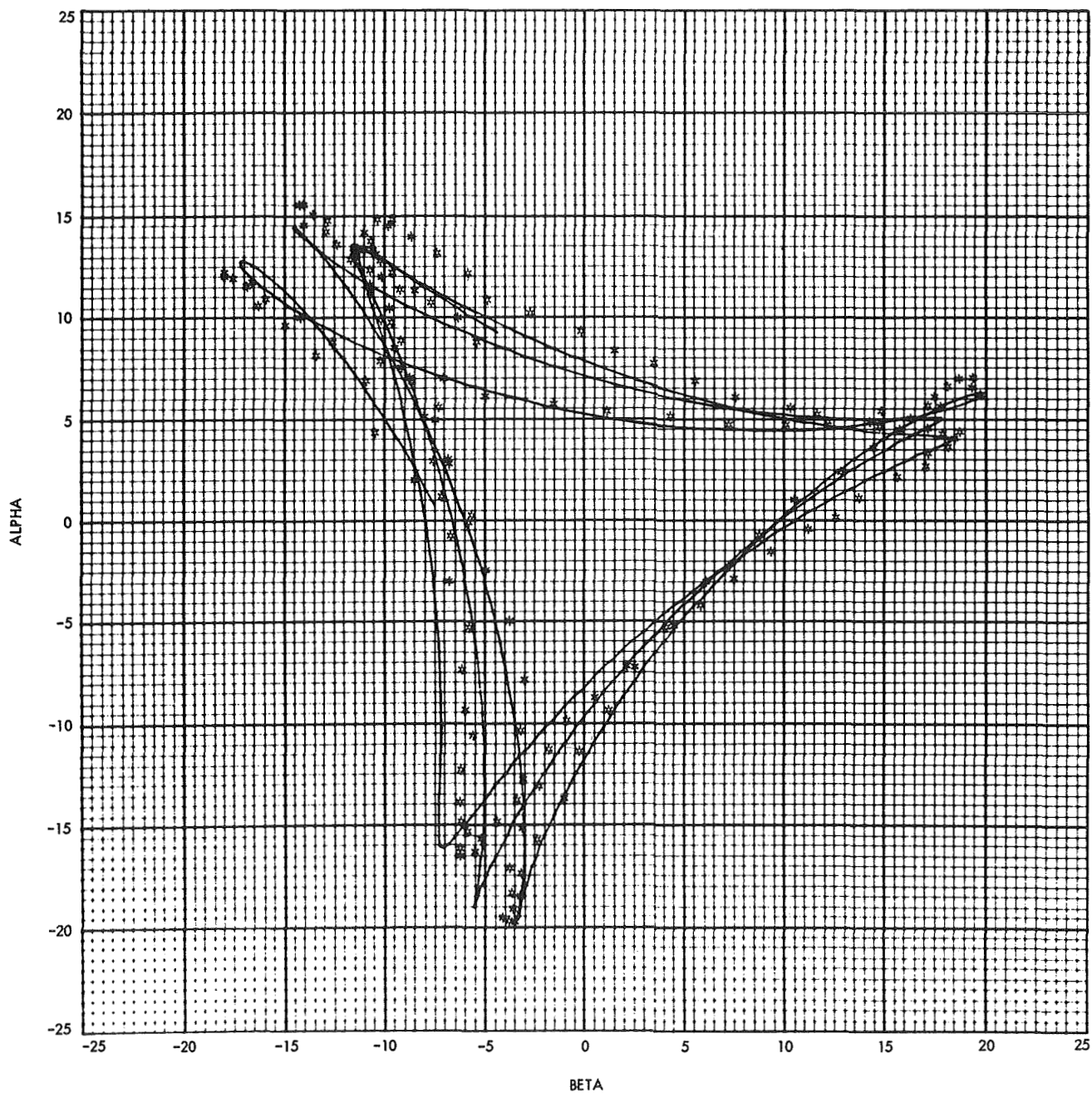


Fig. 8. Typical alpha-beta plot of a blunt 60-deg cone

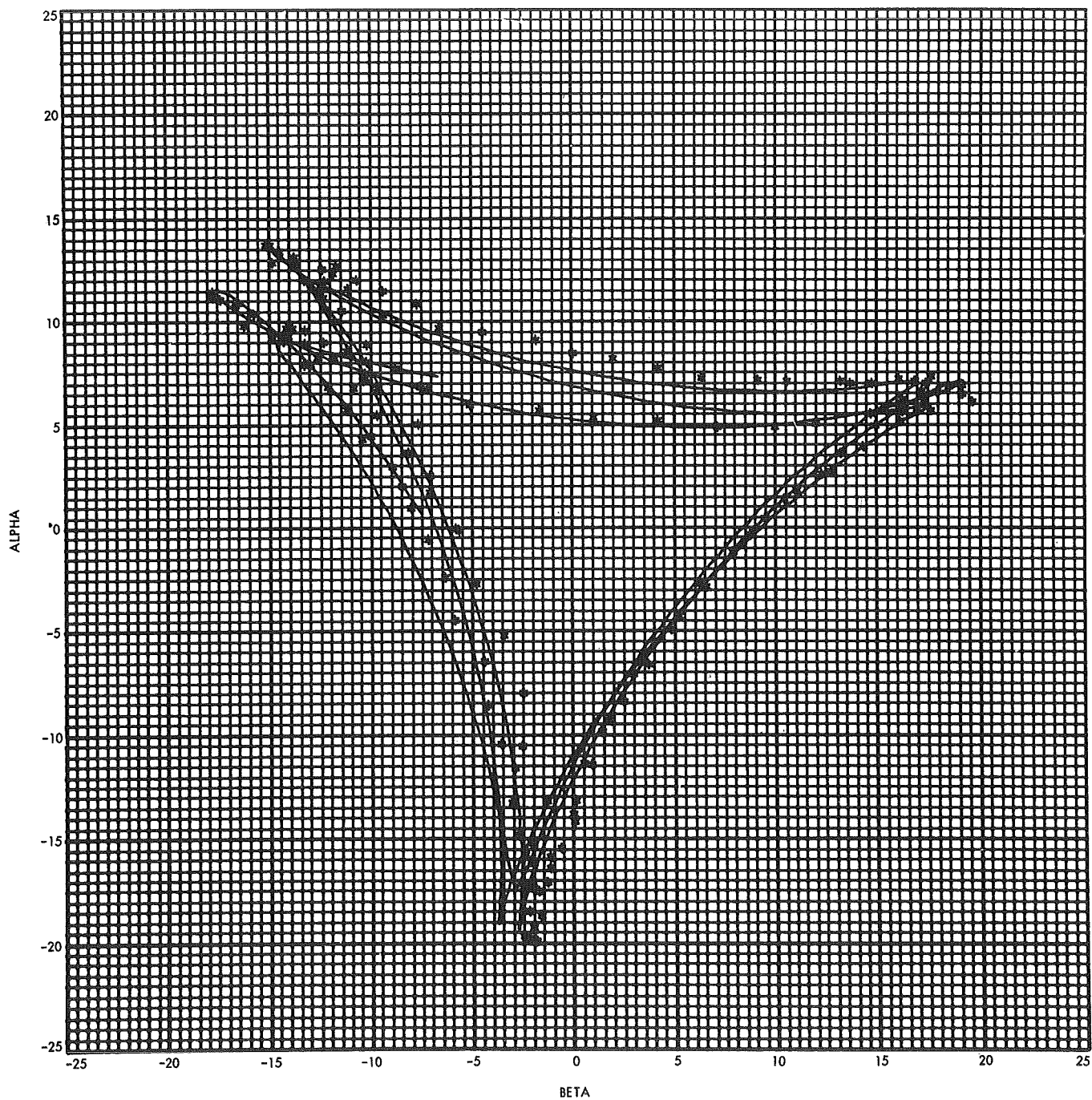


Fig. 9. Corrected alpha-beta plot of Fig. 8

XVIII. Space Simulators and Facility Engineering

ENVIRONMENTAL SCIENCES DIVISION

A. Design Considerations for Accelerated Radiation Tests, C. G. Miller and R. H. Parker

1. Introduction

Outer planet missions require electronics and science instruments that have a useful lifetime of 10 yr. During this time they are continuously within the gamma and neutron fields of the radioisotope thermoelectric generator (RTG), which provides electrical power to the spacecraft. Additionally, these electronics and science instruments are subjected to a number of other radiation stresses: a single exposure to Jovian radiation belts, solar flares, solar wind, and cosmic radiation. One means of verifying the adequacy of components and of design for the 10-yr useful lifetime is by accelerated testing in an adequately simulated environment. The intent is to simulate, in a short testing period, the effects to be expected in the real spacecraft mission, taking into account all of the stresses involved in radiation-cryogenic-vacuum related environments.

2. Jovian Radiation Belts

Jovian radiation belt tests need not be accelerated. This is because the belts are localized over a relatively small volume that is quickly traversed. Also, the significant exposure volume within the belts is smaller still. Although the uncertainty in the actual significant time is considerable, it may be taken (for illustrative purposes) to be less than 8 h. Since these times are reasonable for

total dose tests, they need not be accelerated. In addition, because the Jovian rates are high (compared to the solar wind rates), they should not be accelerated. Other radiation belts are considered to be significantly less damaging, and separate tests need not be considered.

3. Solar Flares

Solar flare rates will be lower than the Jovian radiation belt rates, and total doses can almost certainly be obtained by allowing 0.5 h/flare (acceleration by a factor of around $100\times$). A 12-h test will conservatively simulate a 10-yr mission.

4. Solar Wind

The solar wind, whose effects are manifested as surface problems, has not interfered with past missions. The average flux of solar wind particles over a 12-yr period, with an assumed average energy of 3 keV, is 10^7 protons/cm²-s, and at 1 AU the flux is 10^8 protons/cm²-s. The $1/r^2$ decrease means that at Jovian distances the flux is reduced by a factor of 1/25. Since these protons have a range of 10^{-5} g/cm², if we assume a target with a thickness equal to the proton range, the delivered power density is 4.8×10^{-8} W/cm² at 1 AU, and is 1.8×10^{-9} W/cm² at Jupiter orbit. For those components that rely on radiative cooling for their heat loss, typically unsupported thin films, too great an acceleration of the above natural solar wind rates may lead to annealing

effects in the test, which are not present in the real space irradiation.

The power radiated from a blackbody (in watts per square centimeter) is equal to $(T/645)^4$ where T is the temperature in degrees Kelvin. Table 1 gives the expected equilibrium temperature of a radiatively cooled thin film, subjected to various accelerated dose rates of solar wind, calculated for various initial component temperature and power densities. In programming a test, larger accelerations (that lead to larger temperature rises) can be tolerated for systems that operate at low temperatures than for systems that operate at high temperatures, be-

Table 1. Equilibrium temperature of a radiatively cooled thin film, subjected to various doses of (accelerated rate) solar wind

Acceleration, times nominal	Duration of test	Power density, W/cm ²	Initial component temperature, °K	Equilibrium temperature, °K
Earth orbit				
1	3650 days	4.8×10^{-8}	300 (earth assumed)	300
10 ²	36.5 days	4.8×10^{-6}	300 (earth assumed)	300
10 ⁴	0.36 day	4.8×10^{-4}	300 (earth assumed)	300.5
10 ⁵	5.2 min	4.8×10^{-3}	300 (earth assumed)	308
10 ⁶	0.52 min	4.8×10^{-2}	300 (earth assumed)	358
Mission average				
1	3650 days	4.8×10^{-9}	300 (assumed)	300
10 ²	36.5 days	4.8×10^{-7}	300 (assumed)	300
10 ⁴	0.36 day	4.8×10^{-5}	300 (assumed)	300.1
10 ⁵	5.2 min	4.8×10^{-4}	300 (assumed)	300.5
10 ⁶	0.52 min	4.8×10^{-3}	300 (assumed)	308
Mission average				
1	3650 days	4.8×10^{-9}	150 (assumed)	150
10 ²	36.5 days	4.8×10^{-7}	150 (assumed)	150
10 ⁴	0.36 day	4.8×10^{-5}	150 (assumed)	150.6
10 ⁵	5.2 min	4.8×10^{-4}	150 (assumed)	155
10 ⁶	0.52 min	4.8×10^{-3}	150 (assumed)	192
Jupiter orbit				
1	3650 days	1.78×10^{-9}	150 (assumed)	150
10 ²	36.5 days	1.78×10^{-7}	150 (assumed)	150
10 ⁴	0.36 day	1.78×10^{-5}	150 (assumed)	150.1
10 ⁵	5.2 min	1.78×10^{-4}	150 (assumed)	152
10 ⁶	0.52 min	1.78×10^{-3}	150 (assumed)	169

cause the annealing temperature is related to the Debye temperature for the material under test. These restrictions are further relaxed when more efficient means of cooling may be employed (e.g., conductive cooling to the spacecraft for non-thin-film components).

5. Cosmic Radiation

Cosmic radiation is not expected to be a serious damage problem, and no additional tests are recommended.

6. RTG Radiation

a. Gamma radiation. The spectral distribution and intensity of the gamma field from the RTG, although varying with time and dependent on the initial fuel composition and construction of the RTG capsule, has been extensively studied at JPL.

The design considerations given in this article indicate the practical limits of acceleration of testing, and take into consideration costs, radiation safety needs, and requirements for spatial uniformity of accelerated test fields.

The analysis is based on a generalized outer planet spacecraft configuration in which the RTG-science-instrument spacing is taken as 180 in., and the RTG electronics spacing is taken as 72 in. Actual design geometry and environmental conditions will be determined by the Project Engineering Division as the program proceeds. The analysis is based on the simulation of the radiation of an 8000 W(thermal) source, as presently envisioned for the TOPS configuration 12j. Configuration 12k, if adopted, will have somewhat higher radiation levels, but such changes will not change the conclusions of this study.

Acceleration of delivery of total dose is accomplished in increasing the dose rates delivered to the test object by either of two methods, or by the use of both simultaneously.

Loading. Increase the gamma flux by using larger curiages of nuclear fuel or satisfactory nuclear species that give the same gamma spectrum. The apparent limitations are cost and safety precautions necessary for large sources. A practical acceleration of $25\times$ [over the output of an 8000 W(thermal) source] can be had for a moderate cost. The $25\times$ accelerated source could be used in the present 25-ft solar simulation facility.

An alternate loading acceleration has been designed that would use a fast decaying but intense source of

sodium-24 as the principal nuclide. This design allows a loading acceleration of $250\times$ at slightly more than twice the cost of the $25\times$ source. However, this source could not be ready for use in less than approximately 8 mo.

Geometrical. An increase in the gamma flux can be had by bringing the source closer to the test objects than the design value of 15 ft for the science instruments, and 6 ft for the electronics. When the source (real or simulated) is placed closer than its normal separation distance from the test object, the field strength is increased by the ratio $15^2/d^2$ or $6^2/d^2$ for science instruments or electronics, respectively. The uniformity of the exposure from the near-face to the far-face of the test object, however, decreases as the distance d from the source to the test object decreases. Practical geometrical increases are from 10 to $100\times$, depending on the size of the test object and the uniformity of flux required.

Because of the deep penetration of the gammas, the heat input per unit volume of the test object is low, and annealing is not expected to be a problem—even for a $2500\times$ acceleration.

Figure 1 shows the uniformity of dose (in terms of tolerance from the centerline dose) for varying distances up to 200 in. from the source and for test objects whose size (extent in the beam direction), is 2, 4, 8, or 16 in. The figure shows that a geometrical acceleration of $10\times$ may be had for a 4-in.-thick object with a uniformity of $\pm 6\%$ of centerline intensity, while a geometric acceleration of $100\times$ yields a variation of $\pm 20\%$. Similarly, Fig. 2 shows the uniformity of dose (from centerline dose) and geometrical acceleration possibilities for electronic bay equipment, which is normally 6 ft from the source.

b. Neutron radiation. It does not seem practical, at present, to make large loading accelerations for the neutron flux components of the RTG radiation. Therefore, any acceleration using isotopic sources will be dependent on increases via geometric means. Studies are underway to determine if larger geometrical accelerations can be tolerated for neutrons than for gammas, because of the different nature of the scattering and interaction mechanisms involved.

An alternate possibility exists for accelerated neutron testing in the use of a machine accelerator as a neutron source. While this has some limitation on the size of component accepted, the energy spectrum can be grossly simulated and fluxes as high as 10^{11} n/cm²-s can be attained in small areas.

7. Concluding Remarks

Simultaneous application of other space stresses (space vacuum, solar and ultraviolet irradiation, and controlled temperatures down to those of liquid nitrogen) will also be available in the 25-ft solar simulation facility.

Table 2 shows a selected set of operating conditions that are available that use a combination of loading and geometrical accelerations. In each case, it is possible to increase the irradiation uniformity (decrease the variation) from that presented in Figs. 1 and 2 and in Table 2, to a small fraction of the given values by mounting the test objects on a rotating table, or a table that oscillates through 180 deg, so that the two faces of test objects alternate in their exposure to the source.

The authors are indebted to J. B. Barengoltz (Environmental Requirements Section) for a critical reading of this material and for valuable suggestions.

Table 2. Operating conditions for accelerated testing (10-yr mission length)

Acceleration, times nominal			Required total test duration, days	Remarks
Loading	Geometrical	Total		
10	10	100	36	$\pm 12\%$ uniformity for 8-in.-thick test object
25	10	250	14	$\pm 12\%$ uniformity for 8-in.-thick test object
10	100	1000	3.6	$\pm 10\%$ uniformity for 2-in.-thick test object
25	100	2500	1.5	$\pm 10\%$ uniformity for 2-in.-thick test object
250	10	2500	1.5	
250	30	7500	0.5	

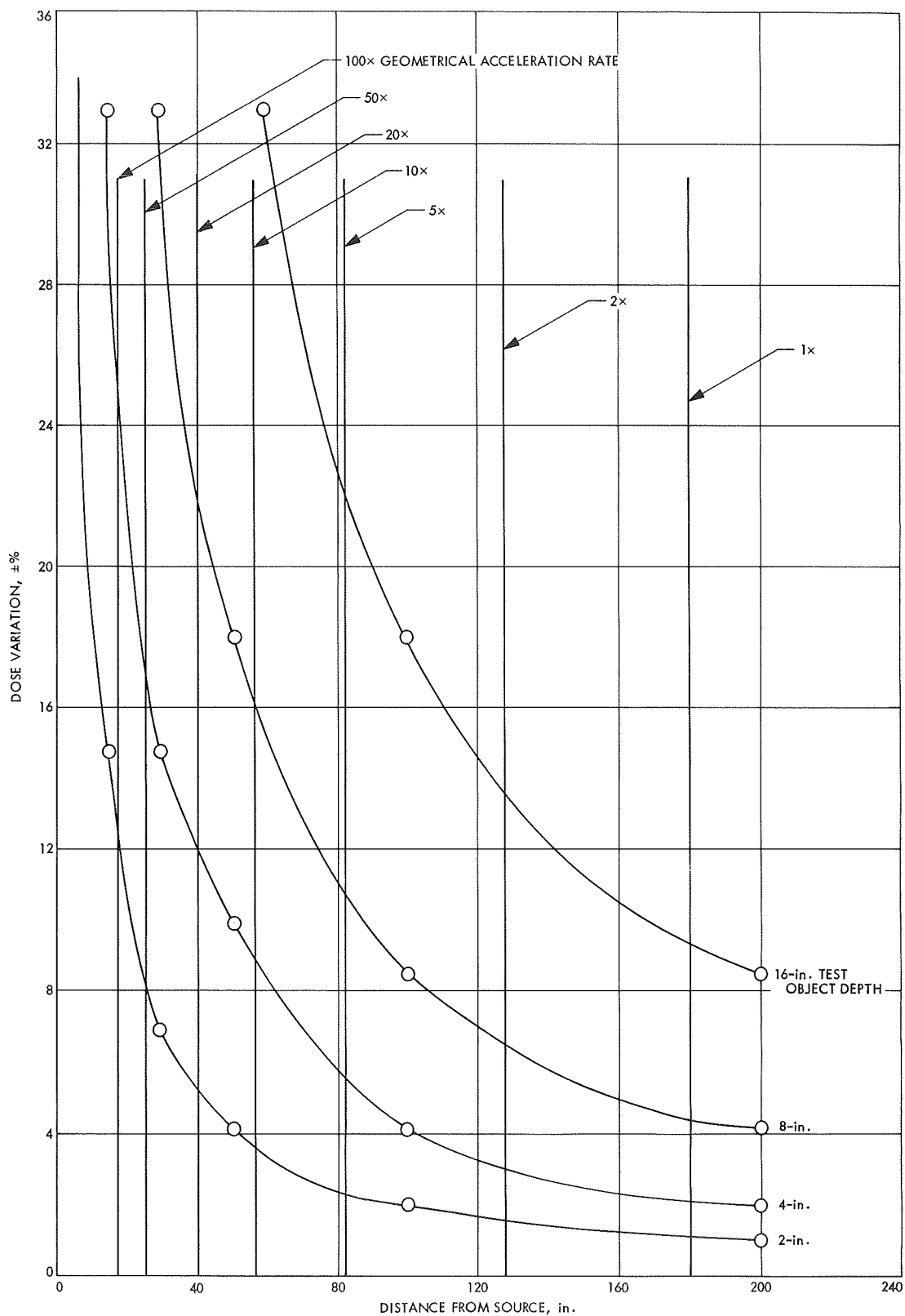


Fig. 1. Uniformity of dose as a function of distance from the source for various test objects

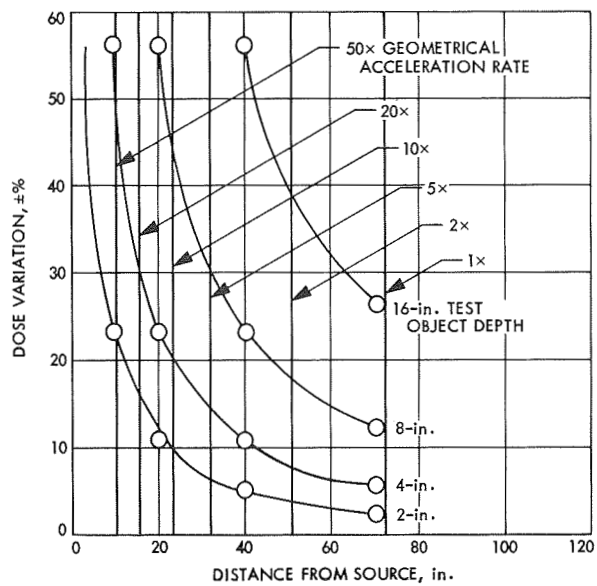


Fig. 2. Uniformity of dose as a function of distance from the source for electronic bay equipment

XIX. Solid Propellant Engineering

PROPULSION DIVISION

A. Nondestructive Testing of 1-W, 1-A Electro-Explosive Devices, V. J. Menichelli

1. Introduction

Effective nondestructive test (NDT) techniques that measure the quality and expected performance of an electro-explosive device (EED), are being investigated. The test must yield meaningful data and not cause degradation as a result of the measurement. Considerable work in the area of nondestructive testing has been performed. Instrumentation for several techniques was developed that gave meaningful data about conventional EEDs in a nondestructive manner. The following discussion describes the continuing effort being expended toward the conversion of existing instrumentation and techniques to:

- (1) Accommodate 1-A, 1-W, no-fire devices.
- (2) Simplification of test procedures.
- (3) Determination of the causes for observed abnormalities.

2. Test Technique

The theory and approach to NDT of EEDs is based upon heat transfer from the bridgewire to its surrounding environment. A lumped thermal differential equation (Refs. 1, 2, and 3) describing the temperature rise θ is:

$$c_p \frac{d\theta}{dt} + \gamma\theta = P(t) \quad (1)$$

where:

c_p = heat capacity of the system (power time/
°temperature change)

γ = heat loss factor (power/°temperature change)

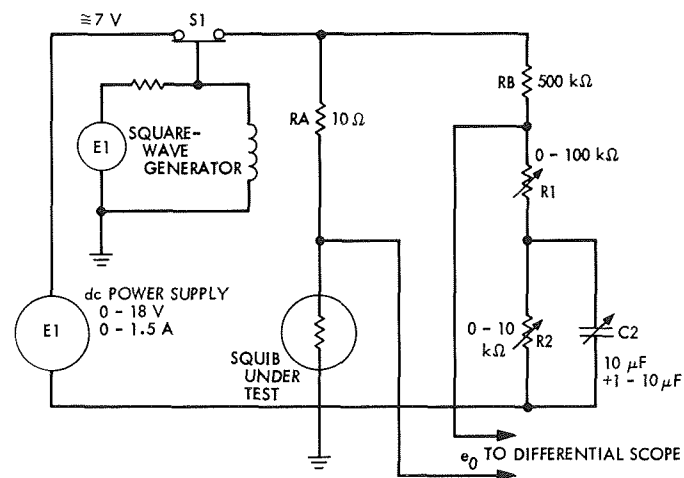
θ = temperature rise above ambient

The utilization of this description has evolved three test techniques and appropriate instrumentation to accommodate 1-A, 1-W, no-fire devices. These devices are simple to operate and yield meaningful data.

a. **Transient pulse testing.** This is a technique that applies a square-wave current pulse to the bridge circuit (Fig. 1, Ref. 2). As the bridgewire heats, the bridge circuit unbalances and provides a heating curve. This heating curve is the integral of the cooling curve response and contains the same information. A few simple manipulations of the bridge circuit provide the following output information as displayed on an oscilloscope:

- (1) The cold resistance R_0 .
- (2) The thermal time constant τ .
- (3) The heat loss factor γ [or γ/α if α is unknown; α is the temperature coefficient of resistance (TCR)].

A 7-V regulated source is periodically applied to the bridge (Wheatstone) circuit via a fast-acting mercury switch. Switch S1, operating at 10 Hz, closes for 50 ms and opens (to allow for cooling) for 50 ms. A nominal test current of 0.64 A flows through the EED squib under test. A differential scope detector at e_0 provides a detailed picture of the heating cycle. During the cooling cycle there is no signal output since the bridge circuit is passive.



$$\tau = R_2 C_2 \left(\frac{R_1}{R_1 + R_2} \right)$$

$$R_0 = R_1 \left(\frac{R_A}{R_B} \right)$$

$$I = \frac{V}{R_0} + R_A$$

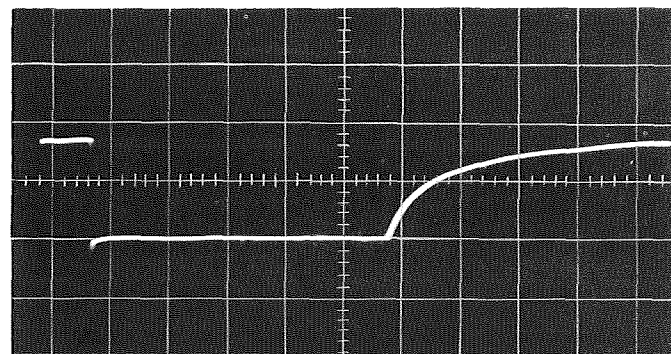
$$\gamma = I^2 R_0 \alpha \left(1 + \frac{R_1}{R_2} \right)$$

Fig. 1. Transient pulse circuitry

With R2 and C2 set at zero, R1 is adjusted so that the heating exponential starts at the baseline. This arrangement provides a complete heating display that can be used for incoming inspection by superimposing an ideal display on the scope graticule. Heating is indicated by a millivolt signal that is readily interpreted as a percent increase in resistance.

For quantitative detail, R2 is adjusted to cancel out the rise or, more properly, drop it below the baseline axis. Now instead of a $(1 - e^{-at})$ display we obtain a $(-e^{-at})$ display. The value of R2 is directly related to the error voltage, which is in turn related to γ , the heat loss factor. An increase in C2 from zero provides a time constant cancellation that yields the thermal time constant when the "best straight line" is obtained. The cancellation can be slightly ambiguous because of the presence of a residual fast (initial) heating time constant but it is quite simple to obtain the dominant thermal time constant. Figure 2 (a) illustrates a normal heating cycle while Fig. 2 (b) is indicative of certain important

(a) NORMAL DISPLAY
5 mV/cm
10 ms/cm



(b) ABNORMAL DISPLAY
5 mV/cm
10 ms/cm

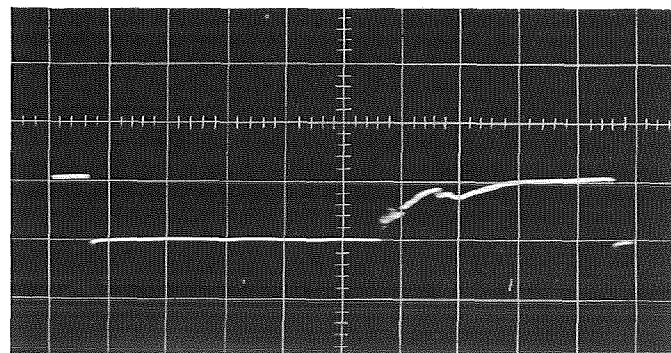


Fig. 2. Typical heating cycle displays for normal and abnormal displays

nonlinearities that may be due to poor bridgewire weld connections, current crowding, or low explosive compaction pressure. These abnormalities obscure the normal heating cycle and provide very erratic signals. Although the details of the fault or abnormality may not be fully understood, these odd responses are indicative of possible incipient failure. Deviations from the ideal response should be treated with suspicion in any high-reliability quality control operation.

b. Self-balancing bridge measurement testing. This technique yields a parameter γ/α —a watt sensitivity unit indicative of the thermal coupling of the bridgewire to the explosive. These measurements compare favorably with values based on transient pulse testing.

A self-balancing bridge is an electronic circuit arranged to maintain a Wheatstone bridge in a condition of near balance. If one element of the bridge can heat up (the RB squib) and bring the bridge to a condition of near balance, a state of sustained oscillation can be maintained. Total balance can never be reached because the feedback would then be zero. The operating frequency of the circuit is established by the selective amplifier

(i.e., 1000 Hz). The circuit diagram of the self-balancing bridge employed is shown in Fig. 3. An operational amplifier A1 is converted into a selective amplifier by means of a twin-T network operating in a degenerative mode. The bandwidth of this amplifier was measured at about 2 Hz at a center frequency of 1080 Hz; high selectivity is desirable in any self-balancing system. An emitter follower stage Q1 is employed as a current driver for a conventional push-pull power amplifier. This push-pull stage Q2-Q3 is capable of a 10-W output, but if the supply voltage is reduced, sharp clipping takes place at any desired power level; this is a means of limiting the power output to under 1 W. A power level of about 0.6 W (0.8 V rms) was the maximum level required as measured by the voltmeter V1 across the device. Resistor R9 adjusts the class B initial bias level and R10 provides for transistor (Q2-Q3) balancing. Transformers T1 (input) and T2 (output) are typical commercial units. A shorting switch S1 provides additional safety by shorting out the bridge voltage during the insertion of the squib under test. Because the measured gain from the output (across a 10- Ω resistor) to the input was 23,000, it is reduced by the attenuator network R5-R4 by a factor of $1000/8500 = 0.118$. Therefore, the loop gain is 2700

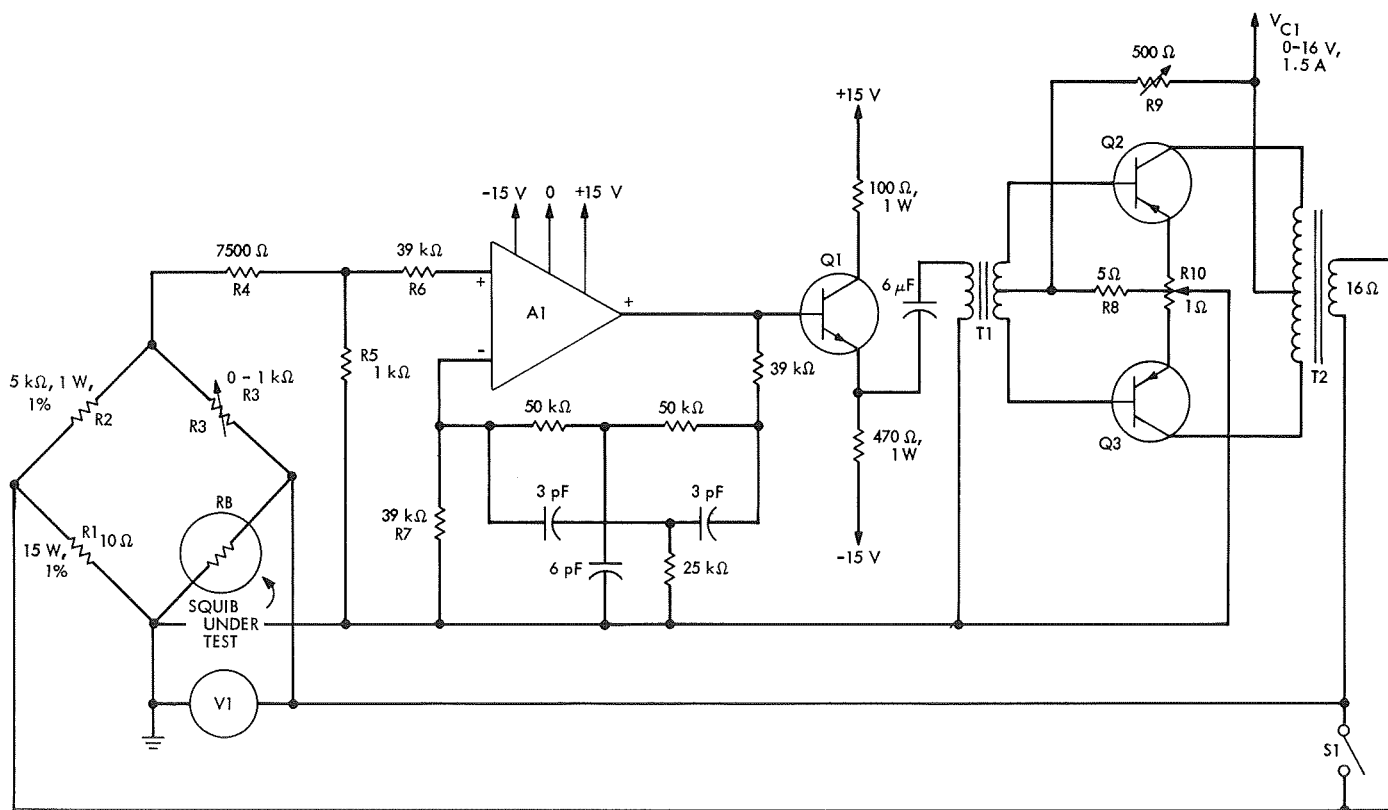


Fig. 3. Circuitry for self-balancing bridge

for a balance condition of 0.4%. If the gain is excessive in a self-balancing system, slight reactive unbalances have to be neutralized. The bridge resistor R3 will nominally operate at about 500 Ω for satisfactory resolution.

If a thermally insensitive resistor (1 Ω) is substituted as RB then a resistance change of 0.004 Ω will drive the circuit from an off state to a saturated or clipped state. This corresponds to the 0.4% balance condition based on calculations. In use, the resistance variation experienced is 2% or less. Based on an α (TCR) of 100×10^{-6} , this temperature excursion would be 200°C above ambient. The self-balancing action is very reliable and control is smooth. In reading R3 and V1, power-vs-resistance data can be obtained.

Two techniques are possible in the application of the self-balancing bridge to electro-thermal parameter measurement for squibs.

Direct thermal conductance measurement. This measurement can be made by obtaining data of power dissipation (V_B^2/R_B) VS, called for, or set resistance R_B . The power put into a squib based on a thermal model is at steady state related to temperature rise θ as

$$\gamma\theta = P \quad (2)$$

The factor γ is thermal conductance in watts per degree centigrade and is a fundamental, sought-after parameter. It describes the heat transfer of the squib bridgewire to its environment. The temperature rise θ is related to resistance change (the operating mechanism of the bridge) according to

$$R_\theta = R_0(1 + \alpha\theta) \quad (3)$$

where $R = R_B$, the operating resistance, R_0 is the cold resistance, and α is the TCR. Solving for θ yields:

$$\theta = \frac{R_\theta - R}{R_0\alpha} = \frac{\Delta R}{R_0\alpha} \quad (4)$$

Solution of Eqs. (2) and (4) yields:

$$\frac{\gamma}{\alpha} = \frac{P}{\Delta R R_0^{-1}} \quad (5)$$

The right side describes power sensitivity in terms of watts per unit change in resistance. This is simply a

slope of a curve obtained from the self-balancing bridge operation. The left side of Eq. (5) is the equivalent in terms of fundamental device parameters. No individual measurement of α is required but an erratic behavior in apparent α will upset the bridge balance behavior. Poor weld connections and wire instabilities will reflect in an erratic $P(\Delta R R_0^{-1})^{-1}$ plot.

Shunt method. This technique provides a quick answer, avoids plots, and is therefore probably most practical for inspection testing.

The squib under test is set to operate at some convenient power level. For example, a nominal 1- Ω unit will be operated at 0.5 V corresponding to a power dissipation of $\frac{1}{4}$ W. A resistor, fixed or variable, is placed across the unit. Since the self-balancing bridge attempts to keep the total arm resistance constant, the heating power increases to some higher level. The power sensitivity can then be calculated as

$$\frac{\gamma}{\alpha} = \frac{V_2^2 - V_1^2}{R_1} \left(\frac{R_1}{R_B} \right)^{-1} \quad (6)$$

The cold resistance R_0 is always obtained at the break-in point for the self-balancing bridge. Any inability to accommodate some value of R_B is indicative of a defective unit.

The self-balancing bridge can also be used as a non-destructive inspection tool. All units should behave in a normal manner up to levels specified by the test (i.e., 1 W). (The normal model is based on the electro-thermal model.) Any instabilities are to be treated with suspicion and γ/α can be treated as a meaningful classification parameter.

c. Thermal follow display. This technique is used to determine EED electrical and thermal performance. A self-balancing bridge similar to that described above is used in this technique. Figure 4 shows a circuit diagram of the system. The bridge unbalance condition is set by means of R3. Amplifier A1 operates as an error detector of gain 10 and the capacitor C1 provides quadrature error rejection. This error signal is available at e_0 and is actually 10 times larger than the true bridge error. Amplifier A2 is a selective amplifier tuned to 9.6 Hz ($Q = 17$) and determines the operating circuit frequency of operation. Power to drive the bridge is provided by A3 (variable gain). Since the amplifier current is limited to ± 1 A, a 28- Ω resistor diverts a fraction

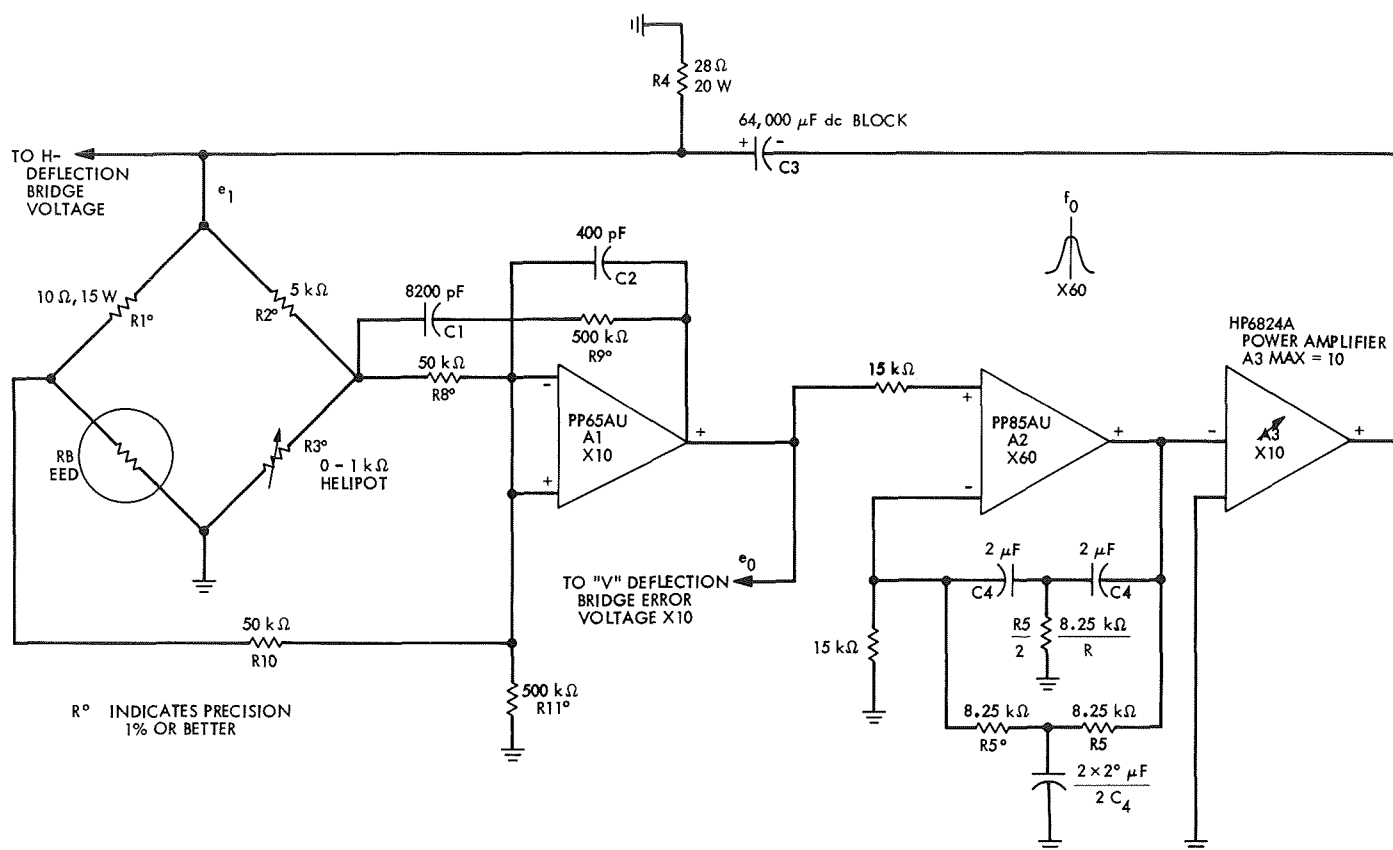


Fig. 4. Thermal follow display circuit

of this current leaving only $28/38 = 0.74$ A for the bridge. This corresponds to a swing of ± 7.4 V peak applied to the bridge e_1 .

To use the circuit with RB in location, R3 is advanced from zero to the break-in of oscillation. Both the bridge voltage e_1 and 10 times the error voltage e_0 are placed on a scope for a lissajous phase display. With a loop gain of 6000, RB is $10/6000$ or $1/6\%$ from nominal balance (below). Calling for a higher operating resistance automatically sets the applied bridge voltage e_1 up to the clipping level established by R4 and the amplifier. The resulting display can be interpreted qualitatively.

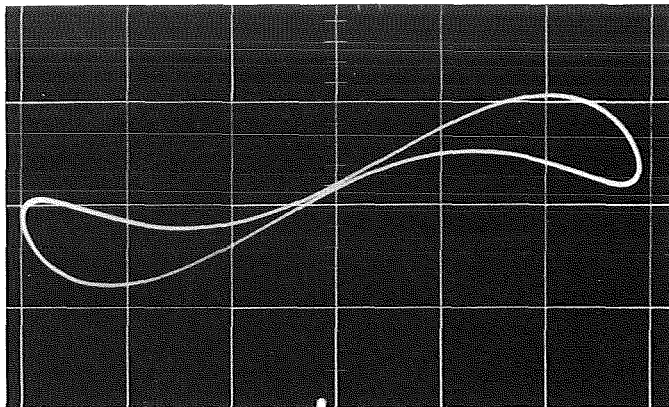
Figure 5 shows observed traces for 1-W, 1-A devices. Both break-in and test-level resistances are indicated. Figure 5 (a) is a typical normal response while 5 (b) is clearly unstable, probably due to a non-ohmic component of the bridgewire resistance. Complete follow, because of voltage nonlinearities, would result in a single-valued cubic display. The area within the lissajous appears to

be related to the γ/α parameter. Units with the smallest areas, observed from previous data, had the largest values of γ/α . Increased γ or thermal conductance results in lower operating temperatures and smaller resistance excursions. The applied voltage should be constant for a meaningful comparative display.

It would be possible to apply this test as an incoming inspection tool for 1-W, 1-A devices. The following data would be obtained:

- (1) Cold resistance: based on break-in point.
- (2) Wire resistance or weld instabilities: based on discontinuous jumps.
- (3) Qualitative thermal behavior: based on the area of the trace at a fixed excitation level (e.g., 6 V across the bridge corresponds to a power dissipation of $(6/11)^2$, $1.0 \Omega = 0.3$ W). A normal display template or wax crayon outline on the scope face could be used as a basis of comparison.

(a) NORMAL DISPLAY
 $R_{\text{BREAK-IN}} = 1.014 \Omega$
 $R_{\text{TEST}} = 1.021 \Omega$



(b) ABNORMAL DISPLAY
 $R_{\text{BREAK-IN}} = 0.972 \Omega$
 $R_{\text{TEST}} = 0.963 \Omega$ (UNSTABLE)

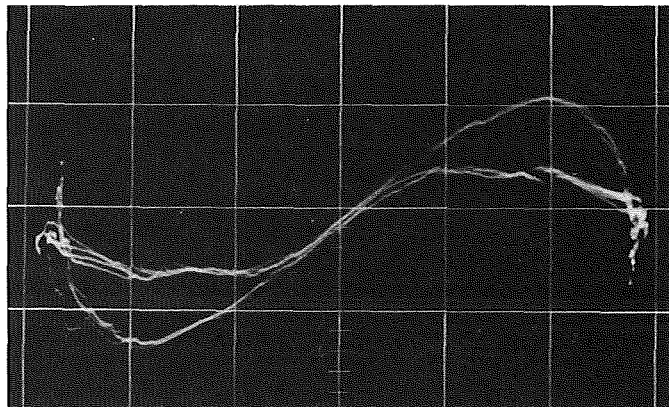


Fig. 5. Typical displays for normal and abnormal responses

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1. Rosenthal, L. A., "Thermal Response of Bridgewires Used in Electroexplosive Devices," *Rev. Sci. Instr.*, Vol. 32, pp. 1033-1036, Sept. 1961.
2. Rosenthal, L. A., "Electrothermal Measurements of Bridgewires Used in Electro-Explosive Devices," *I.E.E.E. Trans. Instr. Meas.*, Vol. IM-12, No. 1, pp. 17-21, June 1963.
3. Rosenthal, L. A., "Electrothermal Equations for Electroexplosive Devices," NAVORD Report 6684, AD 230917. Aug. 15, 1959.

B. TOPS Pyrotechnics Switching, W. S. Wuest

1. Introduction

A highly reliable electrical switching arrangement is achieved in the series-parallel redundant configuration

shown in Fig. 6. It was decided to incorporate this type of redundancy into the pyrotechnics (squib) firing circuitry of the Thermoelectric Outer Planet Spacecraft (TOPS) Project. The TOPS Project requires approximately 20 pyrotechnic events, each with its unique firing command from some external source. Normally, the switches shown in Fig. 6 would be multiplied by 20, once for each pyrotechnic event. However, if two of the switches (e.g., S2 and S4) could be replaced by two 20-position electromechanical stepping-switches, the arrangement would be functionally identical to 20 series-parallel arrangements. Also, a single external command given repetitively to the coils of the two stepping-switches would be functionally identical to 20 unique commands. A simplified sketch of the proposed arrangement is shown in Fig. 7.

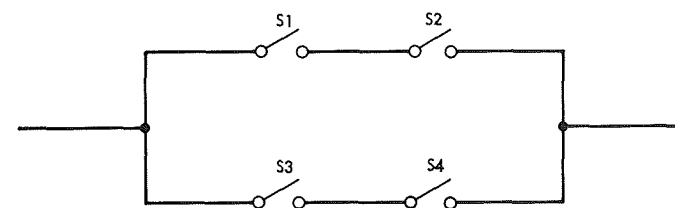


Fig. 6. Highly reliable, series-parallel electrical switching arrangement

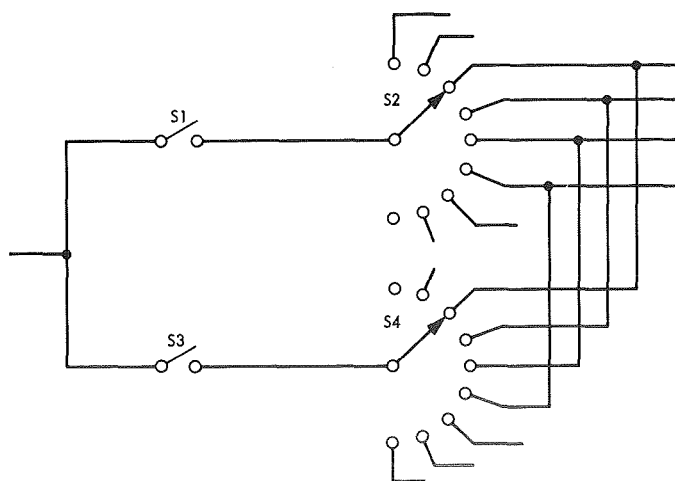


Fig. 7. Switching arrangement for 20 commands

the passive conducting state, and solid-state switches would be used for S1 and S3, the active switches. The command to actuate the solid-state switches would require a total of two commands.

2. A Preliminary Test of Feasibility

A test was performed to determine if a certain compact stepping-switch, designed for light-duty communications circuits, could passively conduct the current pulses that are required to fire the squibs. The switch, an Automatic Electric Type 44 (part PW 156315-GFHC), is an 11-position, 6-pole standard model. The contacts are of a dual-bifurcated, non-bridging, gold-plated construction. The switch was not stepped during the test to avoid the renewing action that accompanies the sliding of contact surfaces. Five of the poles at one switch position were subjected to test, the sixth pole was reserved as a control. Each of the five tested poles was subjected to 1000 cycles of current of the waveform presently used to fire two squibs in parallel.

It was assumed that if the current waveform exceeded any physical limitation of the contacts there would be a perceivable change in the resistance of the contacts. It was also assumed that the effect, if any, would be cumulative and that repeated cycling would improve the resolution of the test. The current waveform was produced by abruptly discharging a 1350- μ F capacitor bank

charged to 42.5 V through a resistance of 1.0 Ω . The result was an exponentially decaying pulse that had an initial current of approximately 40 A and a time constant of approximately 1.3 ms. The time between successive pulses was 7.1 s.

Resistance measurements were made on each of the six contacts, twice before the test, after 10 pulses, after 100 pulses, and after 1000 pulses. Resistance was measured with a Leeds & Northrup, No. 4286 Kelvin Bridge, with a four-wire circuit to avoid the resistance of the measuring leads. Comparison of repeated measurements of the same circuit indicated that the repeatability of measurement during the test was at least as good as 0.001 Ω .

3. Test Results and Conclusions

After 1000 cycles of the test waveform none of the six contacts had a resistance that differed from its original resistance by more than 0.001 Ω . If the resolution of measurement is taken as 0.001 Ω , the test indicated no perceivable change due to the current pulses. After the test the switch was stepped from its test position, no visible evidence of physical change at the points of contact could be seen with the unaided eye. It is concluded that a compact, existing design of stepping-switch is available to implement the proposed redundant switching arrangement for squib-firing circuits.

XX. Polymer Research

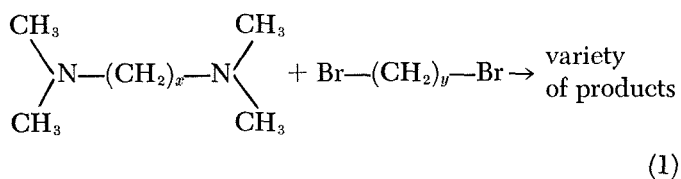
PROPULSION DIVISION

A. Kinetics of Formation of High Charge Density

Ionene Polymers, A. Rembaum, H. Rile, and
R. Somoano

1. Introduction

It has been shown (Ref. 1) that the reaction of *N,N,N',N'*-tetramethyl- α , ω -diamino alkanes with α , ω -dibromoalkanes (Reaction 1).



yields a variety of products depending on the values of the integers x and y . When x and y were less than 3, only monomeric products were formed. The lowest values of x and y leading to the formation of polymers were:

x	y
2	4
4	2
3	3
4	3
3	4

Of these combinations, only the last two yielded polyelectrolytes of weight-average molecular weight greater than 10,000 and since these high charge density polyelectrolytes (ionene-polymers) are of considerable practical importance, the formation of 3,4-ionene bromide was investigated in detail and is discussed in this article.

It was found that the rate of polymerization obeyed second-order kinetics and increased with the concentration of reagents as well as with the magnitude of the dielectric constant of the solvent. The polymers obtained at high reagent concentration and at 25°C had a higher intrinsic viscosity than those prepared in dilute solutions or at temperatures higher than 25°C. The high charge density ionene polymers are effective flocculating and electrostatic agents. In addition, they form water insoluble polyhalides characterized by bacterial growth inhibiting properties.

2. Experimental Method

A quantity of *N,N,N',N'*-tetramethyl-1,3-diaminopropane¹ was fractionally distilled at 48.6°C at 23.1 torr. A similar quantity of 1,4-dibromobutane² was fractionally distilled at 64.0°C at 2.7 torr. Solvents [methanol, dimethylformamide (DMF), and ethylene glycol] were spectro grade and were used without further purification.

¹Obtained from Ames Laboratories, Inc.

²Obtained from J. T. Baker Chemical Co.

The method used in all of the kinetic experiments follows: 3.25 g (2.5×10^{-2} moles) of *N,N,N',N'*-tetramethyl-1,3-diaminopropane and 5.40 g (2.5×10^{-2} moles) of 1,4-dibromobutane were dissolved in approximately 10 ml of spectro-grade methanol and equilibrated at 35°C for 30 min. The solutions were then quickly mixed and diluted to a total volume of 50 ml with methanol (equilibrated at 35°C). A 1-ml aliquot was then analyzed for dimethylamino end groups by the method of Akagi (Ref. 2). Additional aliquots were analyzed at appropriate intervals. After the reaction had gone to near completion, the product was isolated by evaporation of the solvent and dried at 40°C under vacuum for at least 24 h. The intrinsic viscosity $[\eta]$ determined in a 0.4-molar solution of KBr (aq), was 0.104 dl/g. Measurements of pH were made on a Sargent (model LS) pH meter with a Sargent combination electrode. All viscosity measurements were taken in the same Cannon-Ubbelohde Viscometer with permanently mounted photocells at $25 \pm 0.01^\circ\text{C}$. Viscosity data were recorded by the Hewlett-Packard Auto-Viscometer (model 5901B) and Hewlett-Packard Programmer (model 5901A).

3. Results

a. Kinetics. The kinetics of the reaction of *N,N,N',N'*-tetramethyl-1,3-diaminopropane and 1,4-dibromobutane were studied as a function of temperature (Fig. 1), solvent composition (Fig. 2), and concentration of reagents

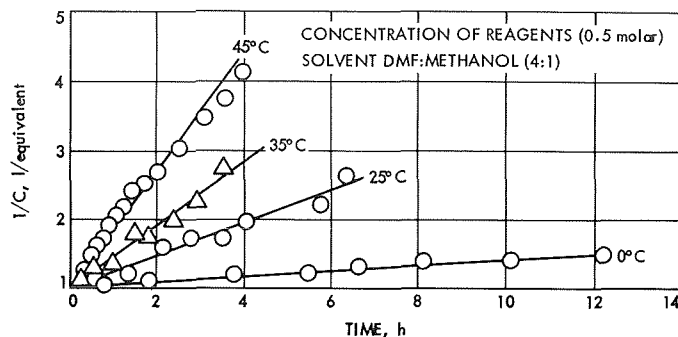


Fig. 1. Second-order plot of the reaction *N,N,N',N'*-tetramethyl-1,3-diaminopropane and 1,4-dibromobutane as a function of temperature

(Fig. 3). In all cases, except at high concentration (2 molar), second order kinetics were observed. Varying the ratio of DMF to methanol increased the rate of reaction as the volume fraction of DMF was increased (Fig. 2). The energy of activation and the frequency factor were calculated for the DMF:methanol (4:1) system. The results are shown in Fig. 4 and most of the data obtained are summarized in Table 1.

b. Viscosity. The specific viscosity in water of 3,4-ionene bromide polymer as a function of salt concentration (KBr) is compared with the viscosity of 6,6- and 3,3-ionene polymers (Fig. 5). The specific viscosity of the monomeric 6,2 reaction product is included in Fig. 5.

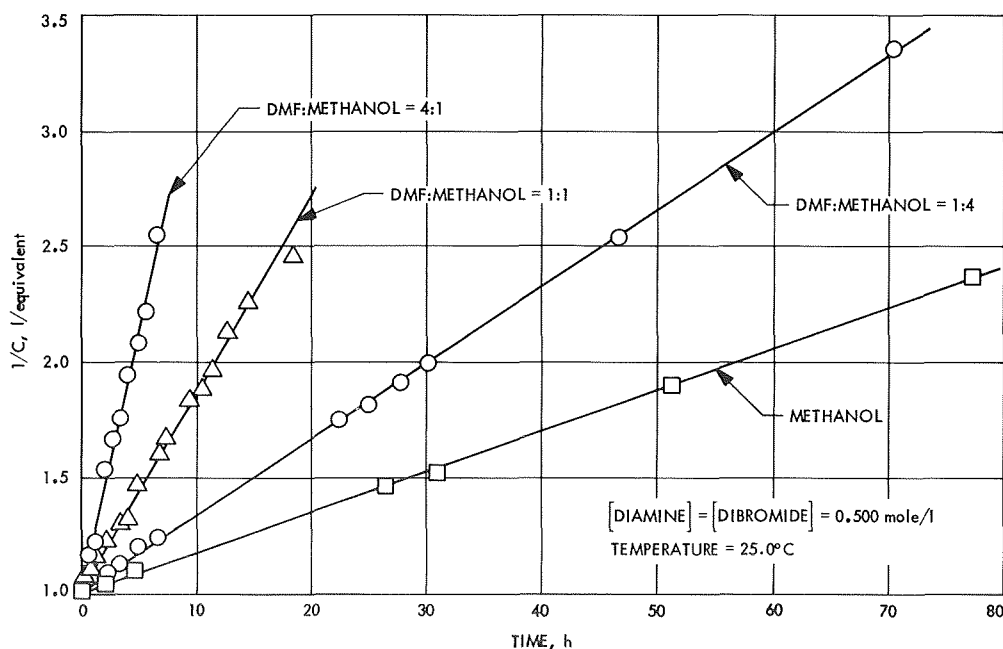


Fig. 2. Rates of polymerization of 3,4-ionene vs solvent composition

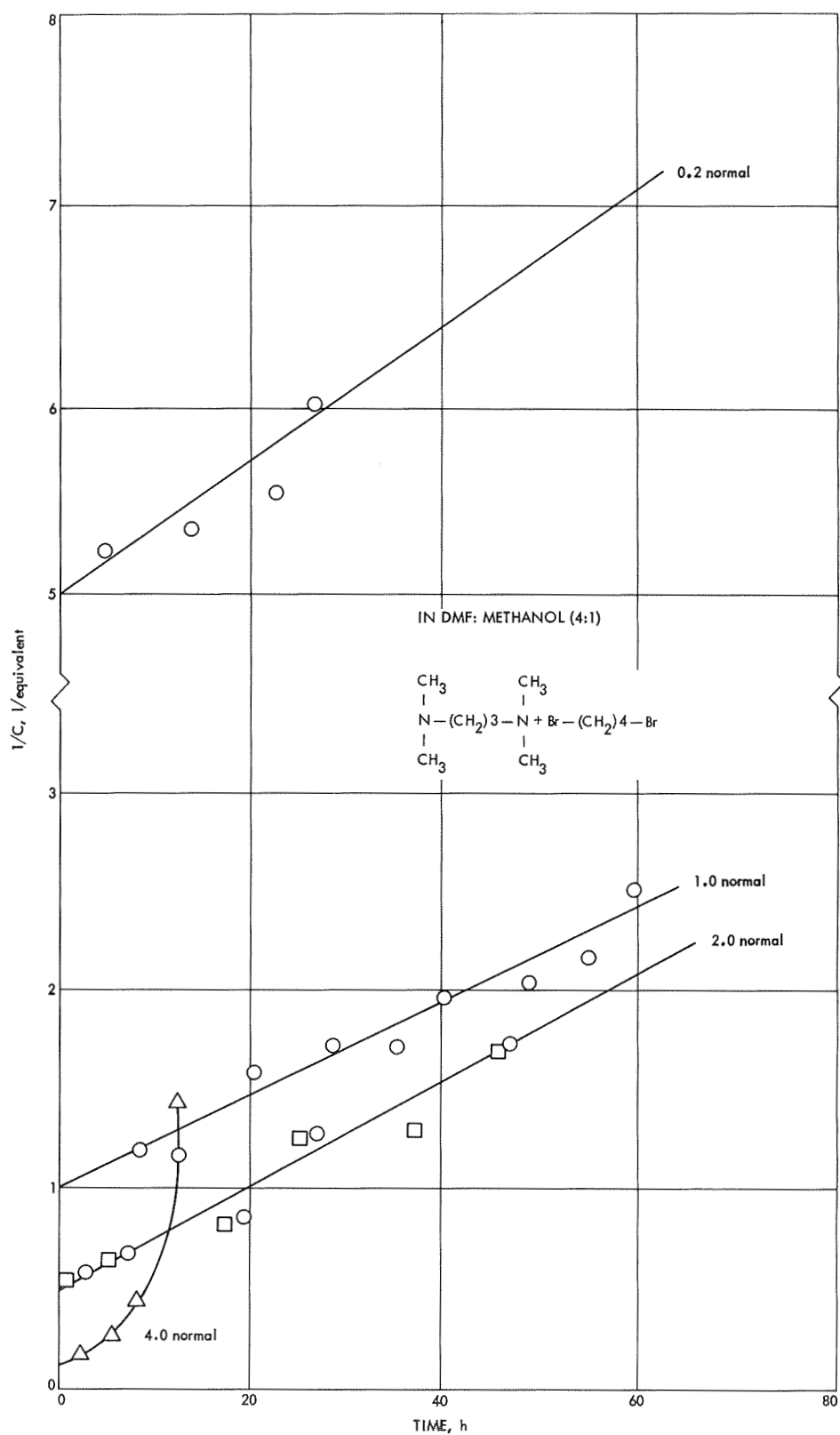


Fig. 3. Inverse of amine concentration vs time of reaction

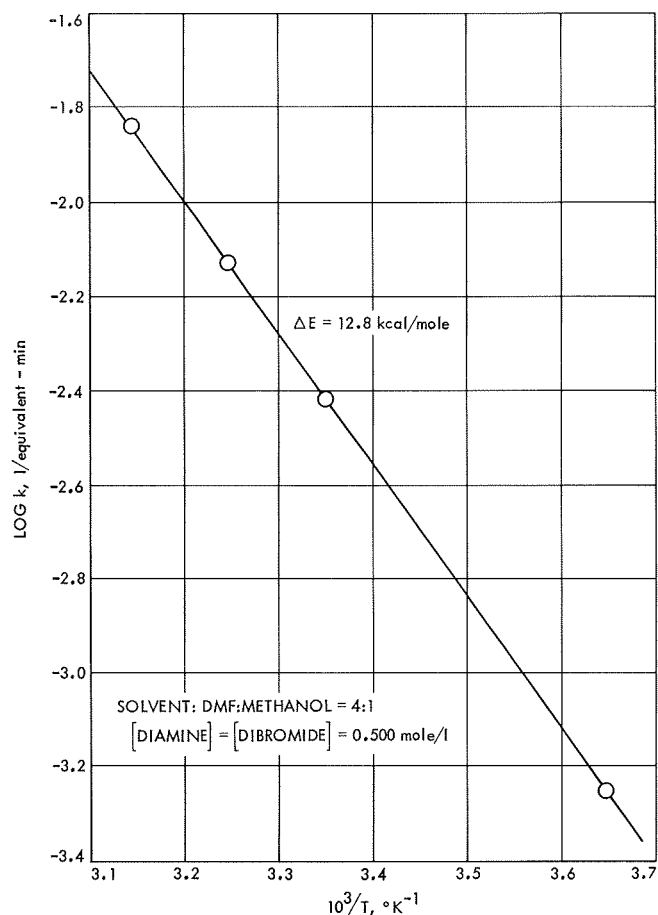


Fig. 4. Logarithm of the second-order rate constant as a function of inverse temperature for ionene bromide

c. End group analysis. The presence of bromine end groups was indicated by chain extension tests in the following way: To a sample of 3,4-ionene bromide (4 g in 10 ml of methanol) of intrinsic viscosity 0.194 (Table 1), *N,N,N',N'*-tetramethylamino-1,6-hexane was added and the mixture was allowed to react at 25°C for three days. The intrinsic viscosity of the isolated polymer increased from 0.194 to 0.233. The ionic bromide content of the unextended polymer was 42.6% (theoretical amount: 46.2%).

Some 1,4-dibromo-2-butene (0.121 g) was added to the same sample of 3,4-ionene bromide (4 g in 10 ml of methanol) and the mixture was allowed to react in an identical manner as the previous example. The intrinsic viscosity of the isolated polymer was practically unchanged.

The increase of molecular weight by reaction with a diamine and the smaller amount of ionic bromine as

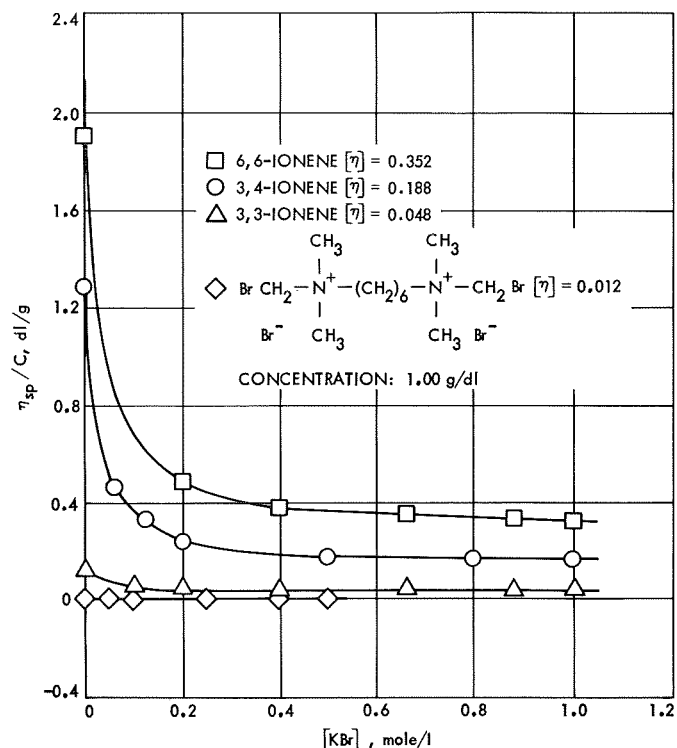


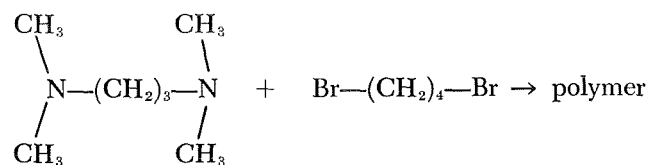
Fig. 5. Reduced specific viscosity in water (25.0°C) vs KBr concentration

compared with the total bromine content strongly suggests bromine end groups for the majority of chains. However, the presence of dimethylamino groups cannot be definitely excluded.

d. Antistatic properties. Samples of β -glass and Dacron cloth used in the heat shield of *Mariner Mars 1971* were immersed in a DMF:methanol (1:4) solution of 3,4-ionene bromide (5 g/100 ml) for 4 h and dried in vacuum overnight at 30°C. The dry specimens (weight gain after drying: 0.2% for Dacron, 0.9% for β -glass) were subjected to friction by means of a Teflon sheet (known to induce a positive potential) and wool (known to induce a negative potential). The voltages of ionene treated samples are compared with untreated and commercially aluminized samples in Table 2. (Vacuum deposition of aluminum is the presently used technique for the elimination of static charges from the heat shield.) The tests were carried out by means of a static meter in a relative humidity of approximately 25%.

e. Bacteriostatic properties. Sensitivity tests have shown strong bacteriostatic action towards *E. coli* and *S. aureus*. In addition, ionene polymers, when reacted with an aqueous solution of KI and iodine or KBr and bromine,

Table 1. Summary of data for the reaction



Solvent	Concentration, moles/l	K, $1 \text{ eq}^{-1} \text{ min}^{-1} \times 10$	Temperature, °C	Time of reaction, h	$[\eta]$, dl/g
DMF:methanol (4:1)	0.5	14.6	45	27	0.100
DMF:methanol (4:1)	0.5	7.41	35	72	0.096
DMF:methanol (4:1)	0.1	4.72	25	96	0.043
DMF:methanol (4:1)	1.0	4.20	25	96	0.188
DMF:methanol (4:1)	0.5	3.83	25	48	0.154
Ethylene glycol	0.5	1.76	25	—	—
DMF:methanol (1:1)	0.5	1.52	25	456	0.070
Methanol	0.5	0.78	35	124	0.100
DMF:methanol (1:4)	0.5	0.58	25	528	0.115
DMF:methanol (4:1)	0.5	0.56	0	—	—
Methanol	0.5	0.29	25	136	0.143
Methanol	2.0	—	25	170	0.194
Methanol: DMF (4:1)	2.0	—	25	20	0.174

Table 2. Results of static electrification tests

Fabric ^a	Electrostatic potential, V	
	Charging with Teflon	Charging with wool
β -glass		
Untreated	+20,000	-20,000
Ionene treated	+2,000	-2,000
Dacron		
Untreated	+20,000	-20,000
Ionene treated	0	0
Aluminized β -glass	0	-750
Aluminized Dacron	+8,000	-8,000 to -10,000

^aSize of sample: 12 × 12 in.

yield polymers that contain a high percentage of halogen (e.g., three atoms of iodine or bromine per positive nitrogen). In the case of 3,4-ionene bromide, the addition of KI/I₂ solution yielded a water-insoluble product; the iodine content of which was 81.0% (theoretical amount for polytriiodide: 80.4%). The 3,4-ionene triiodide when

precipitated on an organic or inorganic substrate was found to inhibit bacterial growth.

f. Flocculating properties. Ionene polymers were found to be excellent flocculating agents. Details of investigations in this area will be reported separately.

4. Discussion and Conclusions

The polymerization reaction was found to obey second order kinetics in several solvents and at relatively high concentration in agreement with Reaction (1). The increase of rate with the amount of DMF present in the solvent mixture (Fig. 2) is attributed to the increased value of dielectric constant of the mixture. The successive steps of the polymerization reaction lead to dipolar species originating from nonpolar reactants. It is expected that these dipolar species would be stabilized by an increase in the dielectric constant of the solvent, whereas the reactants would be affected relatively little. Thus, an increase in the dielectric constant should lead to an increase in rate, and it can be seen from the above data that this expectation is indeed realized. For the same

reason, a higher rate constant is obtained in ethylene glycol than in methanol (Table 1).

On the basis of data summarized in Fig. 4, the rate constant for the DMF:methanol system (4:1) may be expressed by Eq. (1).

$$K = 3.45 \times 10^6 \exp \frac{-12,300}{RT} \quad (1)$$

where R is the gas constant and T is the absolute temperature. An enthalpy of activation of 12.3 kcal found here is in agreement with the values previously reported for the Menschutkin reaction (Ref. 3).

The determination of molecular weight of 3,4-ionene bromide was based upon viscosity measurements in solution. The viscosity of polyelectrolytes is affected by the presence of salts even at low concentration. In absence of added salt, the charged groups tend to set up electrostatic force fields that cause extension of the individual chains.

An increase in anionic strength would be expected to reduce the ionization of a polycation because of the shielding effect of the anions (counterions) on the charged groups. The mutual repulsion between adjacent groups on the molecule is reduced, allowing contraction of the chain to take place. This effect is illustrated in Fig. 5 where the reduced specific viscosity of 6,6-, 3,4-, and 3,3-ionene bromide is plotted vs concentration of KBr. The sharp drop in reduced specific viscosity of 6,6-ionene as compared with 3,3-ionene is attributed to a higher molecular weight of the former. In contrast to the polymers, no change in reduced specific viscosity was observed for a diammonium salt (bottom curve Fig. 5).

With a 0.4-molar solution of KBr or higher concentration, the reduced specific viscosity undergoes a relatively small change and therefore no significant further chain contraction occurs. For this reason, the 0.4-molar concentration of KBr was selected for the determination of intrinsic viscosity $[\eta]$. The highest η in a 0.4-molar solution of KBr equal to 0.194 was obtained using a 2-molar concentration of the reagents (Table 1). Preliminary light-scattering measurements in 0.4-molar KBr yielded weight-average molecular weights in agreement with Eq. (2).

$$\eta = 4.2 \times 10^{-4} \text{ mole}^{0.65} \quad (2)$$

Similar equations were used previously to calculate weight-average molecular weights of other polyelectrolytes (Ref. 4), the viscosity of which was also measured in salt solutions.

With the intrinsic viscosity values of Table 1 and Eq. (2), it is possible to calculate the weight-average molecular weights of 3,4-ionene bromide polymers. The highest molecular weight obtained so far amounted to 12,600. This could be increased to approximately 15,000 by dialysis of an aqueous polymer solution with cellulose acetate membranes. However, the above molecular weight values serve only as an approximation and further light scattering measurements are necessary for accurate determination.

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3. Pickles, N. J. T., and Hinshelwood, C. N., *J. Chem. Soc.*, p. 1353, 1936.
4. Kuhn, W., et al., *Adv. Polym. Sci.*, Vol. 1, p. 540, 1960.

B. Determination of Molecular Weight Distribution of Poly(Isobutylene) by Gel Permeation Chromatography: Procedure and Computer Program, R. A. Rhein

1. Introduction

Gel permeation chromatography has been shown to be a very valuable tool in molecular weight determinations of poly(isobutylene), wherein the number-average, weight-average, and peak molecular weights are determined. This article describes the experimental procedure and operating parameters involved in the analysis of poly(isobutylene) by gel permeation chromatography, and describes a computer program that was used to process the recorder output data from the gel permeation chromatograph.

2. Experimental Procedure and Operating Parameters

A Waters Associates Model 200 gel permeation chromatograph, with four 4-ft $\times$ 3/8-in.-diam columns in series, each containing Styrogel 10⁵, 10⁴, 10³, and 600 Å, was used. The isobutylene polymer was dissolved in tetrahydrofuran (THF) to the extent of 0.5% (polymer weight/tetrahydrofuran volume). The flow rate was 1 cm³/min, with 1-min

injection. The relationship between retention time (in minutes) and molecular size, obtained from poly(propylene oxide) standards, is shown in Fig. 6. The relationship between molecular size and number-average molecular weight, as obtained from calibrated poly(propylene oxides), is shown in Fig. 7. Also plotted in Fig. 7 are the molecular weight vs size (obtained from retention time) for four calibrated poly(isobutylene) samples.³ It is seen that the points representing the poly(isobutylene) fall on the line for the poly(propylene oxide) size vs molecular weight. This relationship was assumed to be valid for isobutylene polymers in general. A computer program was used to process the gel permeation chromatograph data.

3. Computer Program for Processing Gel Permeation Chromatography Data

This computer program determines the number-average and weight-average molecular weights, and the weight percent-molecular weight distribution of poly(isobutylene) samples. The program, written in BASIC (and used in the CALL/360 system), instructs the computer to request as input the required gel permeation chromatograph

³Samples with a peak molecular weight of 685 and 1950 were provided courtesy of R. Roper of the Enjay Corporation Polymer Laboratories and a sample of poly(isobutylene) with a bimodal molecular weight of 580 and 2020 was provided courtesy of R. J. Lee of the Research Laboratories of Amoco Chemical Corp.

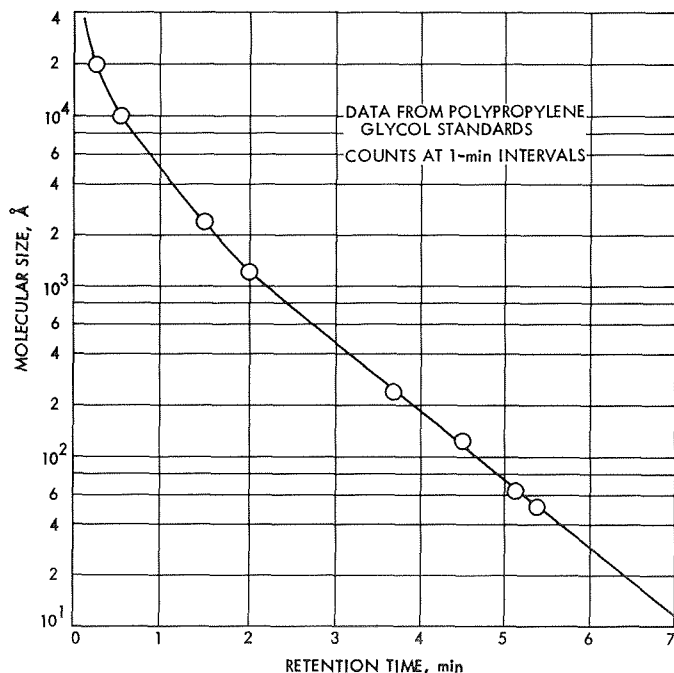


Fig. 6. Molecular size vs retention time

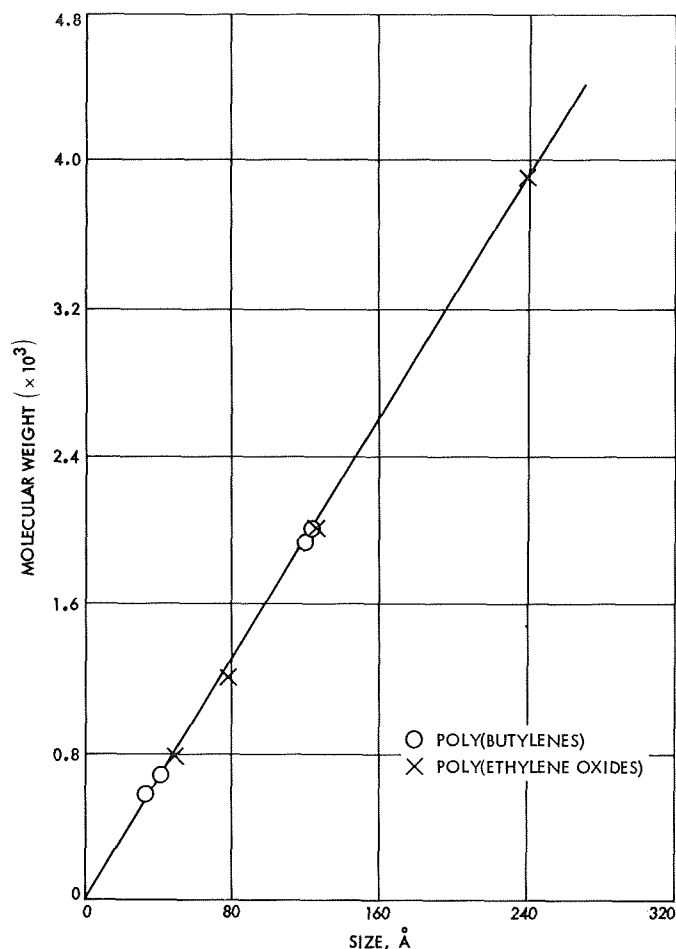


Fig. 7. Molecular weight (No. av) vs molecular size for calibrated polypropylene glycols and polyisobutylenes

(GPC) data, process it, and print the output data in a suitable format. A listing of the program, **GPC**, is provided here.

The input data required, and the corresponding computer program statement number, are listed in Table 3.

Table 3. Required input data

Statement	Variable	Description
120	A\$	Name of the person requesting the GPC computer-processed data
130	B\$	Name of the polymer sample
140	N	Number of counts to be processed
160	C (I), H (I)	Counts, along with the corresponding heights

To process the data, the computer determines the molecular size vs counts (i.e., retention time) from the programmed relationship between counts and size as shown in Fig. 6. The computer then determines the molecular weight from the molecular size by using the relationship as shown in Fig. 7.

A typical printout, for one of the experimentally obtained poly(isobutylene) samples is shown in Table 4; the corresponding gel permeation chromatogram trace is shown in Fig. 8.

Table 4. Typical computer-generated data

Counts ^a	Height	Wt %	Cumulative wt %	Size, Å	Mol wt
22.0	3.0	0.226	100.00	2380.0	38,794
23.0	20.0	1.504	99.77	1210.0	19,723
24.0	73.0	5.489	98.27	757.3	12,344
25.0	124.0	9.323	92.78	474.0	7,726
26.0	177.0	13.308	83.46	296.7	4,836
27.1	200.0	15.038	70.15	177.2	2,888
28.0	181.0	13.609	55.11	116.2	1,894
29.0	174.0	13.083	41.50	72.7	1,186
30.0	165.0	12.406	28.42	45.5	742
30.9	181.0	13.609	16.02	29.9	487
32.0	29.0	2.180	2.41	17.8	291
33.0	3.0	0.226	0.23	11.2	182

^aSubmitter: Rhein
Sample No.: 11189-2/R
Average molecular weight
Number 1229.6
Weight 3437.8

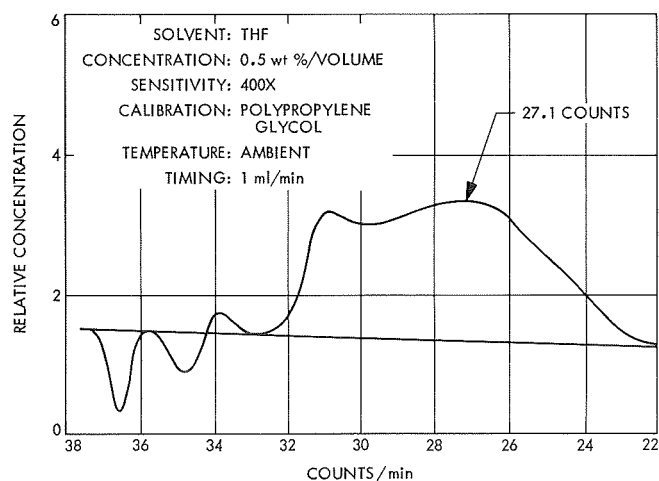


Fig. 8. Chromatogram of polyisobutylene sample

C. Long-Term Aging of Elastomers: Stress Relaxation of SBR With Thiophenol, R. Rakutis and S. H. Kalfayan

1. Introduction

It has been reported (SPS 37-59, Vol. III, pp. 194-198 and SPS 37-61, Vol. III, pp. 237-241) that dicumyl peroxide (DiCup)-cured styrene-butadiene rubber (SBR) undergoes scission and crosslinking reactions simultaneously at elevated temperatures. This kind of network change is complex and not amenable to ready analysis, the process would be simplified if crosslinking were avoided. It has been reported⁴ that crosslinking was suppressed in sulfur-cured natural rubber by the use of chain transfer agents or free radical inhibitors, such as thiophenol. This phenomenon was shown by the concurrence of the continuous and intermittent relaxation curves. On the basis of these findings, thiophenol was used in the SBR-DiCup system to suppress crosslinking.

2. Experimental Section

a. Absorption of thiophenol. The SBR films that had been crosslinked with 0.3% DiCup were immersed for approximately 1 h in thiophenol. The excess reagent was then removed under vacuum and the amount absorbed was calculated. Sample thickness was 0.022-0.024 in.

b. Stress relaxation measurements. The stress relaxometer has been described previously in SPS 37-59, Vol. III. The experimental conditions for the intermittent stress measurements were also the same as previously reported (SPS 37-61, Vol. III).

c. Infrared measurements. Uncrosslinked, raw SBR gum was extracted with acetone to remove the antioxidant. Films of the gum, from a benzene solution, were then cast on sodium chloride windows. A drop of thiophenol was added directly to one of the films. Both films were heated for 3 h at 115°C in air and the infrared spectra were taken after cooling to room temperature. A Perkin-Elmer (Model 137B) Infracord was used for the infrared measurements.

3. Results and Discussion

As seen from Figs. 9 and 10, the addition of thiophenol does not reduce the recombination or crosslinking reaction in the elastomer. It appears that with thiophenol

⁴Gates, R. D., *Sol-Degradation Studies of Polymeric Networks*, Ph. D. Dissertation, p. 155. University of Akron, Akron, O. Available from University Microfilms, Inc., Ann Arbor, Mich., 1961.

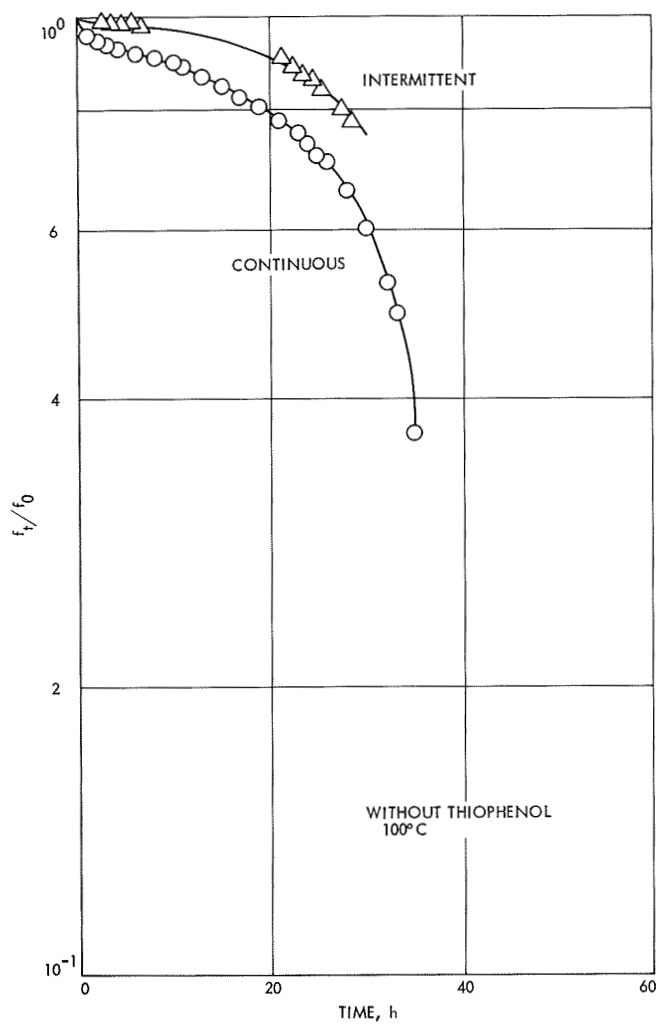


Fig. 9. Continuous and intermittent stress relaxation of peroxide-vulcanized SBR without thiophenol at 100°C

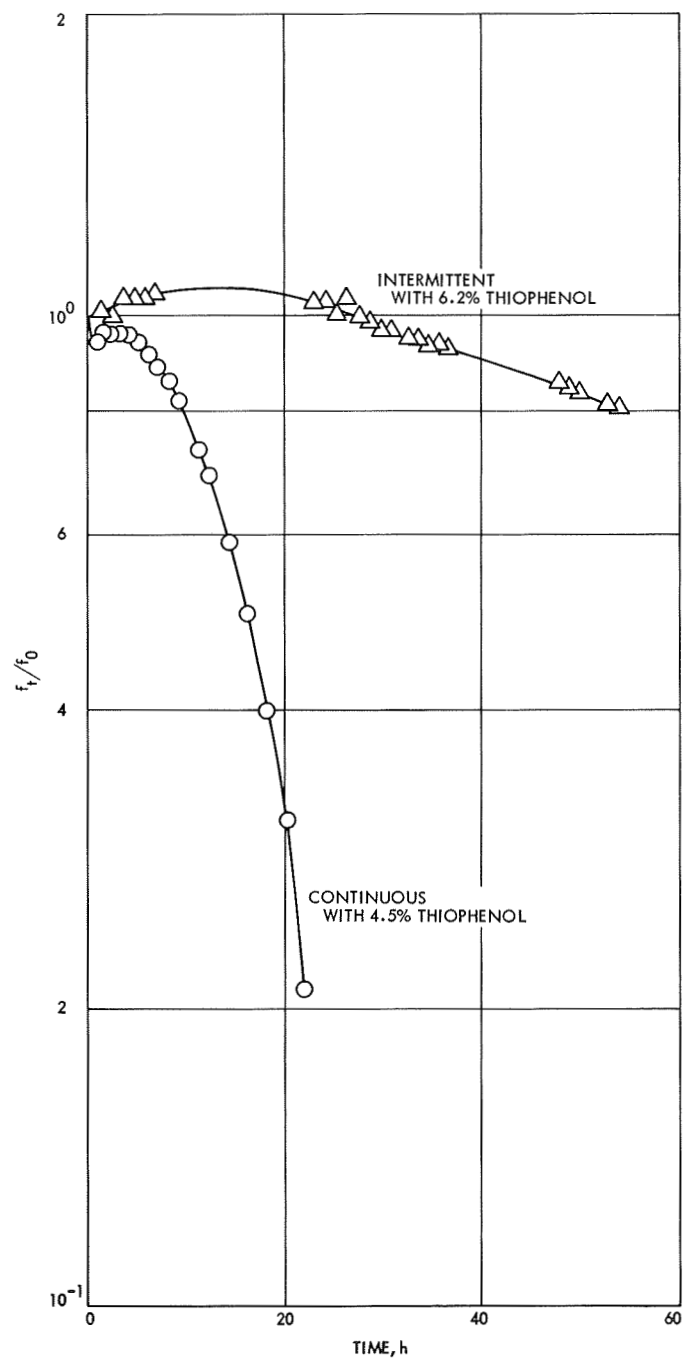


Fig. 10. Continuous and intermittent stress relaxation of peroxide-vulcanized SBR with thiophenol at 100°C

(Fig. 10), there is even more crosslinking taking place than in the control (Fig. 9). Thus, the results observed by Gates for sulfur vulcanizates of natural rubber could not be reproduced for peroxide-cured SBR. The presence of thiophenol in the raw SBR gum inhibits oxidation as

shown in Fig. 11. After 3 h at 115°C, there is only very slight change in the infrared spectrum of the sample with thiophenol compared with the unaged sample. It is possible that thiophenol prevents oxygen radicals from attacking the networks by reacting with them to form a disulfide.

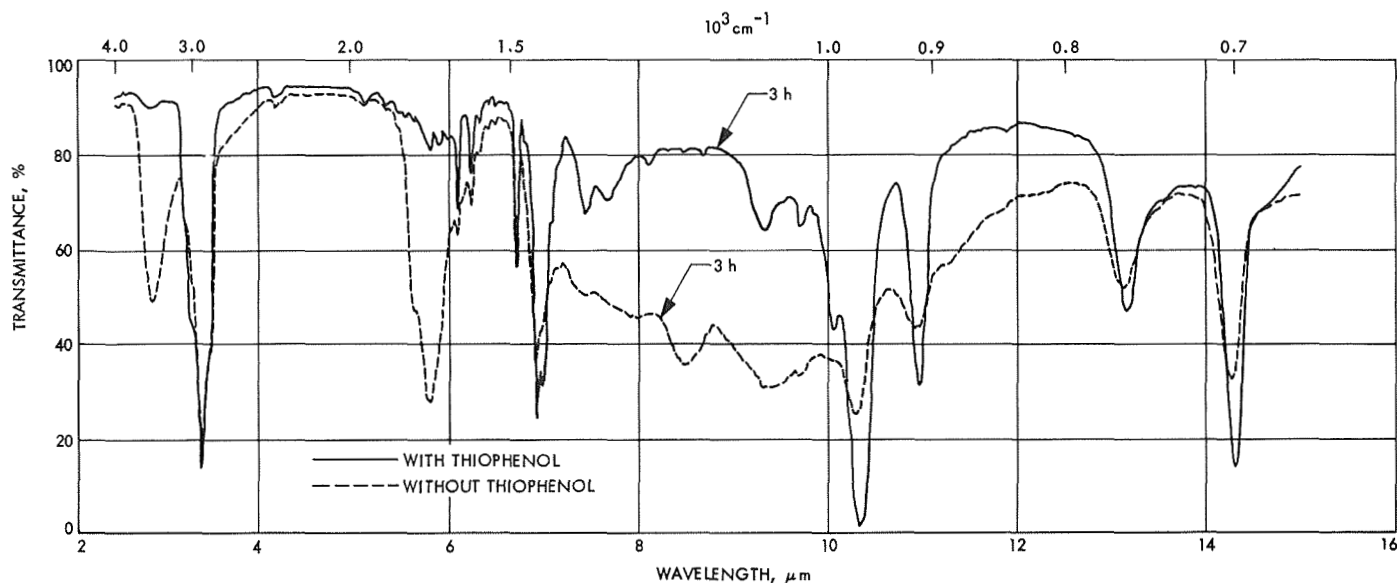


Fig. 11. Oxidation of SBR at 115°C with and without thiophenol

XXI. Research and Advanced Concepts

PROPULSION DIVISION

A. Power Conditioner Evaluation: Circuit Problems and Cures (SEPST III—Breadboard 1),

T. W. Macie and T. D. Masek

1. Introduction

The development of a lightweight, high-efficiency, power-conditioning unit (PCU) is a major part of the Solar Electric Propulsion System Technology (SEPST) Program. In addition to conditioning the power obtained from a solar panel, the PCU provides closed-loop control of the thruster. The design concept resulting in minimum gross weight for a given power dissipation, including cooling, is one that allows the components to radiate directly to space. A PC unit was built in 1966 that operated a SEPST II thruster for 500 h.¹

Subsequently, this first PCU was modified in 1968 by Hughes Aircraft to operate the JPL SE-20C oxide-cathode thruster. Testing was conducted in phase II of the SEPST Program as discussed in Refs. 1 and 2. The phase II integration and testing was successful in demonstrating stable control loop operation and PCU recycling techniques for recovery from thruster arcing. An important feature not incorporated in the breadboard unit BB-0 was the ability to operate over a range of input voltage; a must for a solar-electric deep space mission. The design

of a PCU for a 2 to 1 variation of the input voltage differs substantially from one without this feature. Because the design strongly impacts weight and efficiency, a unit that satisfies such a requirement had to be demonstrated.

Hughes Aircraft Company (under contract to JPL) has developed a PCU that satisfies the above requirements.² The breadboard unit BB-1 (delivered to JPL in June 1969) underwent an integration cycle with the 20-cm, oxide-cathode type SE-20C thruster. Integration and evaluation occurred over a period of 5 mo, including two long-term tests of 800 and 500 h, respectively.

Testing was performed in the Electric Propulsion Systems Laboratory. A diagram of the test setup is shown in Fig. 1. The power-conditioning unit's power outputs were connected through manual switches and instrumentation to the thruster. The switches allowed thruster operation with either the PCU or laboratory power supplies. Magnetic-amplifier-type current transducers were used to monitor important parameters. The PCU telemetry outputs were connected to a patch board and through buffer amplifiers to a magnetic tape recording system, used in evaluating the PCU recycling program. Commands were generated by the test console, which also provided dummy loads for initial checkout and subsequent trouble shooting.

¹"Development and Test of an Ion Engine System Employing Modular Power Conditioning," Final Report, Hughes Aircraft Co., El Segundo, Calif., Sept. 1966.

²Quarterly Technical Reports, Hughes Aircraft Co., El Segundo, Calif., July 1, 1968 to Sept. 30, 1969.

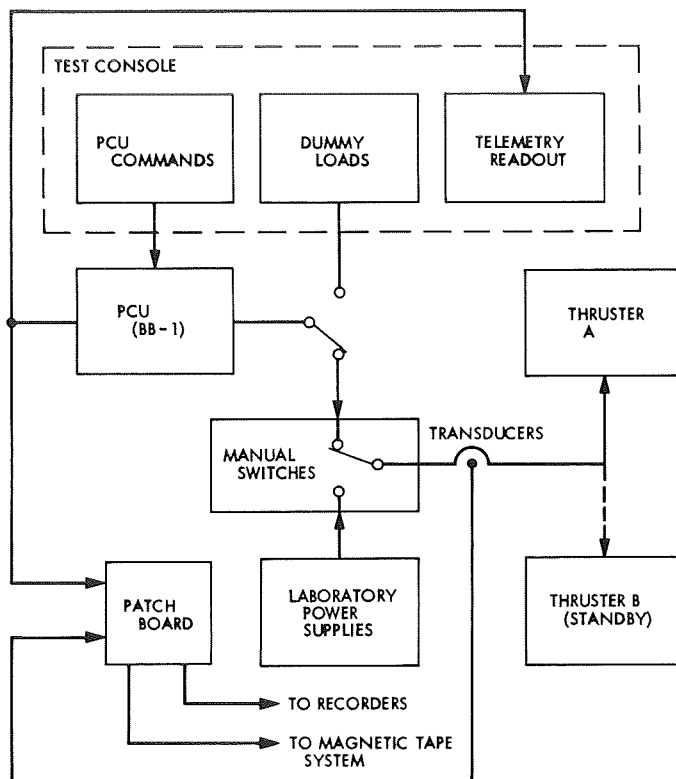


Fig. 1. Laboratory test setup

2. Power-Conditioning Unit Configuration

The power-conditioning unit is shown schematically in Fig. 2. The screen power is produced by means of eight series-connected converters; each converter consists of a 10-kHz inverter and one transformer-rectifier set. An output filter is provided for attenuation of the ripple.

The arc power is derived by means of a converter using one 10-kHz inverter feeding two transformer-rectifier units that are connected in parallel; one serves as the arc starter, the other feeds the main arc. A redundant, standby 10-kHz inverter is provided for emergency and automatically replaces the prime inverter in case of failure.

The accelerator power and the power used for magnet, vaporizer, and neutralizer are derived from a constant 35-Vdc source, obtained from the line regulator. The same line regulator also provides the housekeeping power for the electronic control circuits of the PCU. Two 5-kHz inverters supply the power for magnet, vaporizer, and neutralizer; one inverter is redundant. As in the case of arc inverters, the changeover from prime to standby in-

verter occurs automatically upon the failure of the prime unit.

The ac power for the oxide cathode is supplied by a separate 5-kHz inverter. This power is transmitted from the power conditioner to a stepdown transformer that is located near the thruster; this reduces the line losses. A standby inverter is available for redundancy (Ref. 2).

3. Malfunctions Caused by Transient Noise

a. General. Arcing of the ion thruster, principally due to contamination by material sputtered from the vacuum chamber walls, is known to cause disturbances that can interfere with the proper functioning of the electronic controls of the PCU. The arcing of the thruster causes rapid changes in current levels that, through the ground loops, were crossed into the micrologic of the PCU. The major results of this interaction were:

- (1) Erroneous changeover from prime to standby 5-kHz inverter.
- (2) "Latch-up" of RA 909 amplifier used to feed back arc current signal that controls cathode power. (It was later determined that latch-up occurred because of a defectively manufactured device.)
- (3) Noise-triggered shutdown of PCU.
- (4) Noise contamination of telemetry lines.

b. Malfunction of 5-kHz inverter. The prime 5-kHz inverter forms an ac source that feeds magnet, vaporizer, and neutralizer. In case of failure, a mechanical relay switches to a standby inverter when no ac output is sensed. The changeover is performed automatically by the electronic logic of the PCU.

The observed malfunction consisted of an unprogrammed changeover that was caused not by failure of the prime inverter but by noise. The noise-initiated changeover took place while the prime inverter delivered power. The malfunction occurred frequently and finally caused an inverter failure. One of the power transistors of the prime inverter shorted, the contacts of the transfer relay welded together, and the input fuse opened.

c. Malfunction of arc power supply. The dc arc power is derived by inverting the dc power of the solar source into a 10-kHz ac current and then converting back to a regulated dc current.

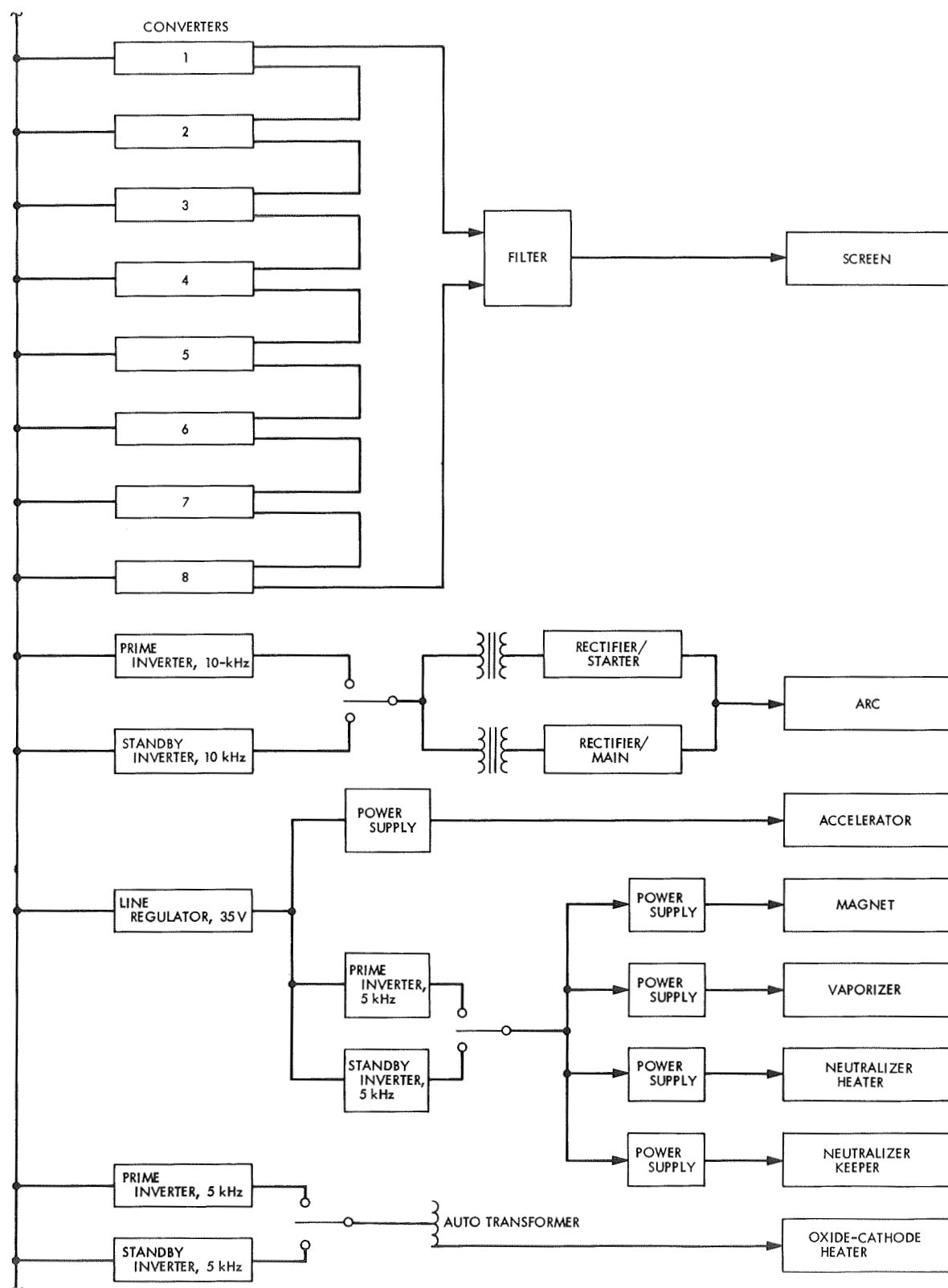


Fig. 2. Oxide-cathode power-conditioning unit

Sometime during the run, the prime 10-kHz inverter failed and the run continued on the standby. The investigation that followed revealed that one power transistor was damaged, fuses were open, the "boost" transformer had overheated and caused an internal short between the primary and the secondary winding.

The overheating and damage to the boost transformer was caused by a defective current-limiting inductor that limits the current fed to the boost transformer. The reduction of inductance occurred during the encapsulation of the unit and was never detected thereafter.

In addition, the stress upon the failed transistor was increased by another type of system malfunction: latch-up of an amplifier inside the electronic logic. The observed latch-up of the RA 909 amplifier in the feedback loop was equivalent to commanding a maximum arc current. The latch-up caused a distinct step increase of the arc current after recovery from arcing.

In some cases, this overcurrent remained undetected for periods as long as an hour, during which the arc power supply remained overloaded.

d. Shutdown of the PCU. The power conditioner was occasionally shut down by noise that penetrated into the micrologic and triggered the shutdown circuit provided for under-voltage protection. Malfunctions of this type caused no damage, but demanded manual intervention.

e. Contamination of telemetry lines. All the telemetry lines are required by specification to operate at the facility ground level. The analog signals generated inside the PCU are brought out by shielded twisted pairs and delivered to the laboratory instrumentation. All possible care is taken to prevent any external pickup. All the signal return lines inside the PCU are connected to a common signal ground ring, which in turn is externally bolted to the facility ground. Initially, the power ground ring was connected to the facility ground through the same link.

It was found that the common signal and power ground ring exhibited transients up to some 300 V above the facility ground during arcing. The facility instrumentation was not able to accept such excursions of the common mode voltages and separation of signal and power grounds became imperative.

f. Path for noise penetration. Initially, it was thought that the penetration of noise into the digital integrated circuits occurred only through the input lines and house-

keeping dc supply lines. Later, another path for noise penetration into the digital integrated circuits was found to exist; i.e., capacitance between the integrated circuit unit and the mounting board and panel of the power conditioner. When thruster arcing occurred, the high (300 V) potential between the signal ground used by the integrated circuits and the ground plane of the mounting board allowed substantial amounts of energy generated during the arcing to be stored electrostatically and, subsequently, to penetrate into the digital circuits.

g. Redesign. To make the PCU immune to the above transient interference, two steps were taken. First, additional noise suppression filters were added on all the critical inputs and housekeeping dc supply lines within the micrologic. Second, the grounding scheme of the screen power supply was modified from that of Fig. 3(a) to that of Fig. 3(b). With this change, the screen current (I_s) no longer flowed through the signal ground ring (Fig. 3 b) and the transient on the signal grounds was reduced to an acceptable level.

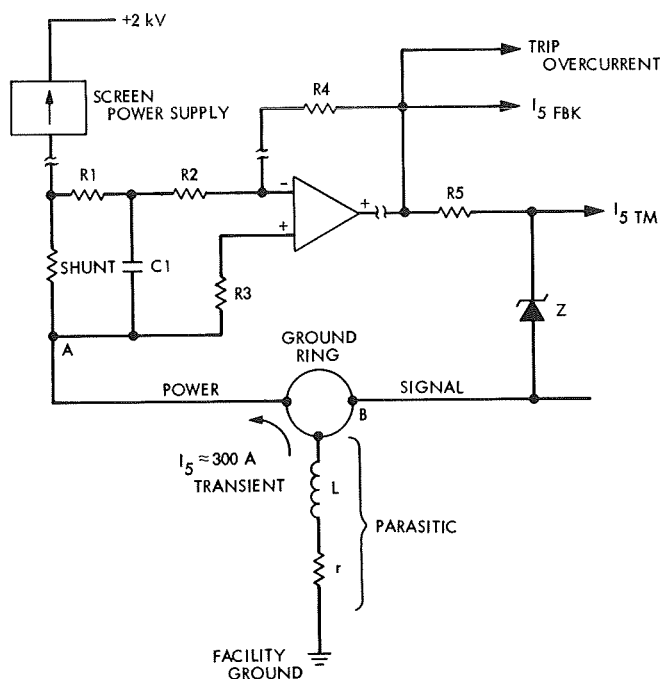
4. Other Malfunctions

a. Magnet power supply. Efficient operation of the thruster requires the use of an axial magnetic field. The arc current generates another magnetic field, both fields interact, and changes in the intensity of one field, as occurs in a thruster arc causes the total flux to vary. This, in turn, generates a back-voltage in the magnet. It was observed that large voltage spikes were induced into the output of the magnet power supply during the arcing of the thruster when the magnetic flux, generated by the arc current, changed abruptly.

Diode CR3 (Fig. 4) was able to clamp negative voltage surges (i.e., surges that caused CR3 to conduct), but surges of positive polarity were not clamped and caused voltage breakdown of that diode. Several diodes of 600-V PIV rating were punched through by excessive voltage. To cure the trouble, a zener diode capable of dissipating the feedback energy was added as shown in Fig. 4; no further failures have occurred.

b. Screen power supply ripple. The screen power supply consists of eight converters; each is rated at 250-V, 1-A dc power. The dc outputs of these converters are connected in series and the combined output of 2000-V, 1-A dc power is filtered by means of L1, C1 filter (Fig. 5). The L2 was added to suppress ripple current, that, bypassing L1, circulated through capacitive coupling between primaries and secondaries of the individual transformers.

(a) OLD CONFIGURATION



(b) NEW CONFIGURATION (C2, R6, AND R7 ADDED, OTHER COMPONENTS UNCHANGED)

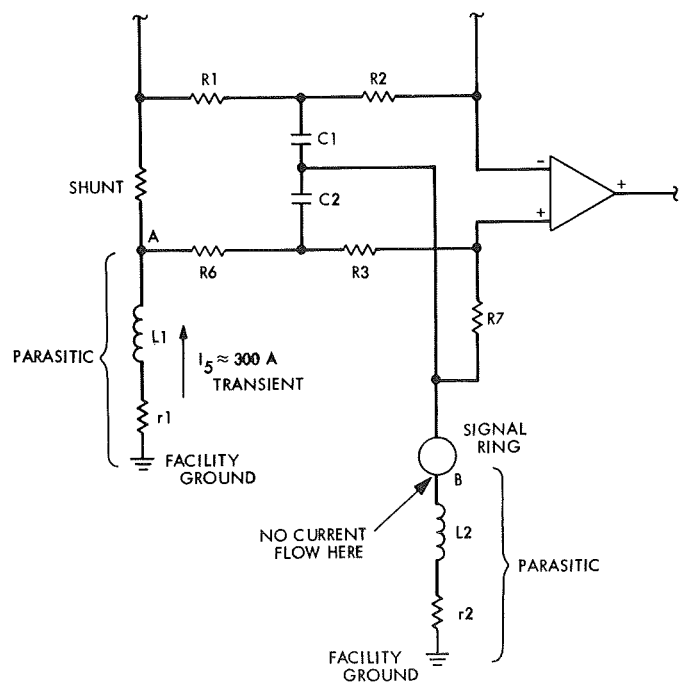


Fig. 3. Grounding scheme of screen power supply

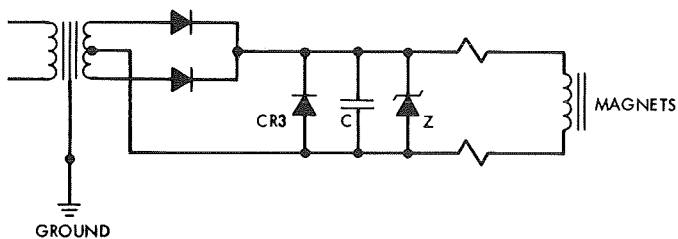


Fig. 4. Magnet power supply

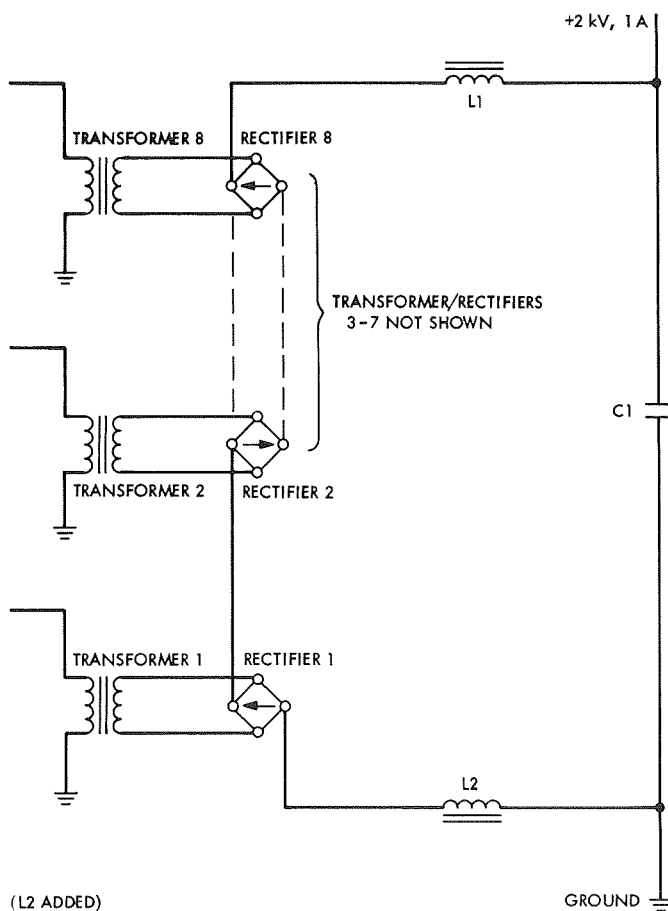


Fig. 5. Screen power supply output

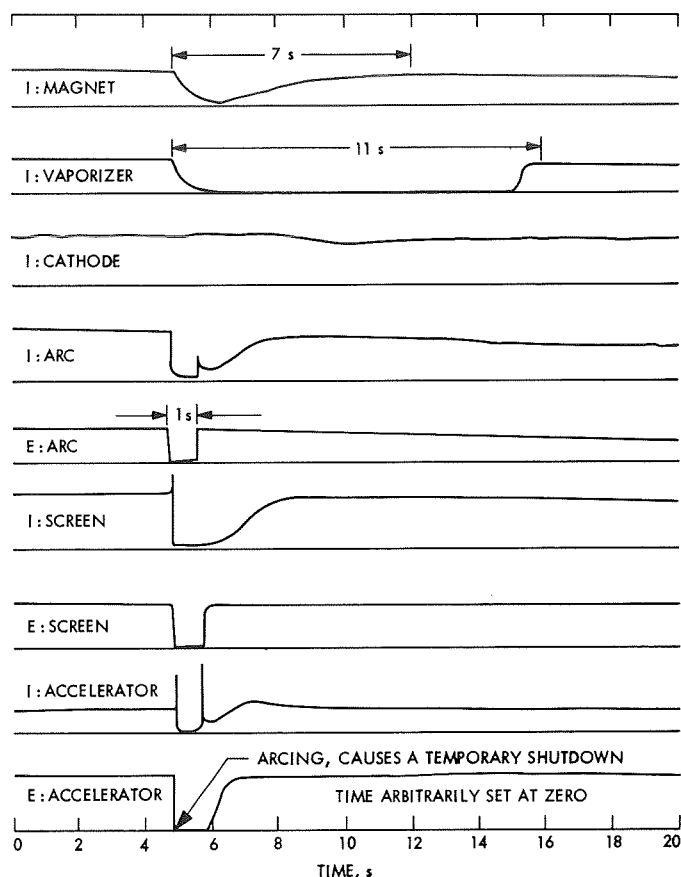


Fig. 6. Typical recovery cycle from trip after arcing

c. *Recovery from trip after arcing.* A typical recovery cycle is shown in Fig. 6. Details of work done to derive the sequence shown can be found in Refs. 1 and 2. It was found that the 11-s delay of the vaporizer (Fig. 6) caused a considerable variation in the temperature of the vaporizer and led to a slow cooling off. The delay was cut by one-half and the thermal perturbation reduced.

5. Conclusions

After all the modifications were introduced, the closed-loop system was operated for 500 h. During this period, the system performed satisfactorily with no manual intervention required. This demonstrated that the cures introduced were satisfactory. The 500-h run was terminated voluntarily after it was felt that sufficient confidence had been gained in the performance and reliability of the improved circuits to warrant their acceptance.

References

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- Pawlik, E. V., Macie, T. W., and Ferrera, J. D., "Electric Propulsion System Performance Evaluation," Paper 69-236, AIAA 7th Electric Propulsion Conference.

B. A 70-kWe Thermionic Reactor Electric Propulsion Spacecraft, J. F. Mondt

1. Introduction

A 70-kWe thermionic reactor spacecraft for unmanned electric propulsion with the primary radiator on the reactor side of the shield was discussed in SPS 37-58, Vol. III, pp. 207-213. This spacecraft configuration eliminates coolant pipe penetrations through the heat shield and also separates the high-temperature ($>1000^{\circ}\text{F}$) components from the low-temperature ($<200^{\circ}\text{F}$) components at the shield. The shield, reactor, and radiator are the high-temperature components; the power-conditioning unit, thrusters, and science package comprise the low-temperature components. This spacecraft arrangement with a large spacecraft length-to-diameter ratio ($L/D \cong 24$) has a low propulsion system specific weight. Therefore, the structural support requirements of large L/D spacecraft on the *Titan/Centaur* launch vehicle were studied.

2. Shield and Structure Weight Tradeoff

If the spacecraft is cylindrical, the shield weight is proportional to the cylinder diameter squared. The weight of the structure is directly proportional to the length. The surface area of the cylinder is fixed for a given power level. With the use of these three relationships, Fig. 7 shows the effect of spacecraft diameter and length on shield and structure weight.

The minimum shield plus structure weight is about 1800 lb for a 2.5-ft-diam spacecraft with a shroud 60-ft long. A shroud 50-ft long is being developed for the *Titan III-D/Centaur-Viking* program and a shroud 50-ft long results in a 100-lb weight penalty (Fig. 7) for this electric propulsion spacecraft. Therefore, the 3.0-ft-diam spacecraft with a shroud 50-ft long was selected.

3. Spacecraft

One of the best of several spacecraft configurations is shown in Fig. 8. The high-temperature radiator is 3 ft in diameter, the thermionic reactor is 2 ft in diameter, and the power conditioning, low-temperature radiator is 4 ft in diameter. The overall length from the bottom of the science instruments to the top of the high-gain antenna is 49 ft, 8.5 in. The nuclear radiation shield is slightly conical so that there is no nuclear particle scatter from

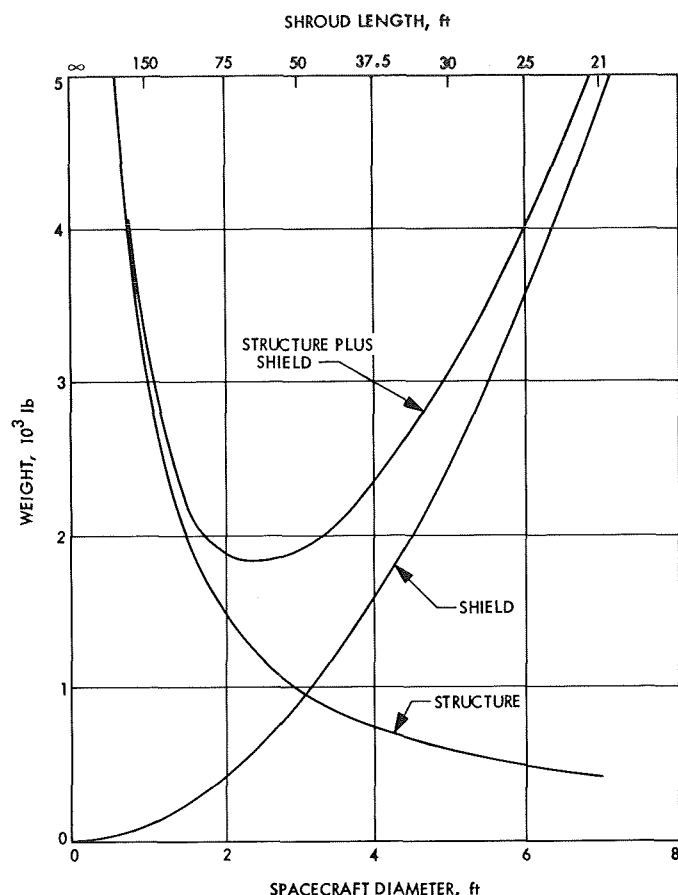


Fig. 7. Effect of spacecraft size on structure and shield weight

the thrusters to the science instruments. The mercury propellant tank is located directly behind the shield and is also used as the gamma ray shield.

The antennas and the thrusters are located so that the science instruments can have approximately a 3π -steradian field of view. Two disadvantages to this configuration are: (1) the science instruments must be covered with a titanium shroud during thrust phases of the mission to keep the mercury exhaust away from the instruments, and (2) the high-gain antenna is 50 ft from the science package and near the high-temperature radiator. More desirable arrangements are being investigated.

4. Propulsion System

The propulsion system electrical and thermal power distribution is shown in Fig. 9. The power system is designed to allow 20% electric power degradation from beginning to end of mission life. The thrust system consists of 18 4-kWe mercury bombardment thrusters,

each having its own power-conditioning panel. Fifteen thrusters (60-kWe) will be operating at one time with initially three spares. Mercury was chosen over other propellants because of the relatively well developed technology of mercury systems. The power-conditioning equipment, with silicon transistors, is mounted on aluminum panels that radiate directly to space. The transistor hot-spot temperature is maintained below 200°F. The power-conditioner design is based on prototype equipment being used in the Solar Electric Propulsion System Technology (SEPST) Program.

The power system consists of a thermionic reactor, a high-temperature radiator, and a multiduct electromagnetic pump. The outside diameter of the thermionic reactor is 2 ft and the overall reactor length is 1.5 ft. The reactor consists of 133 externally fueled, 10-in.-long thermionic converters. (Some thermionic reactor details are shown in Fig. 10.) Results of preliminary nuclear criticality calculations for this reactor are described in Section C of this chapter. Each converter is cooled by a single-loop, flow-through NaK-78 heat rejection system. The converter waste heat is radiated to space from a cylindrical radiator consisting of 266 0.42-in.-diam stainless-steel tubes. The NaK is circulated by a multiducted electromagnetic pump with 19 electrically isolated pumping sections, each section containing 7 hydraulically independent ducts. One punctured radiator tube or one weld leak (loss of coolant) causes power loss from only 1 of the 133 converters. The externally fueled thermionic reactor is the only nuclear reactor power system being investigated today that has a part power capability after any failure that causes a loss of reactor coolant. This greatly increases the power system reliability and probability of mission success.

5. System Component Weights

The spacecraft weight was first calculated without allowance for launch structure (Table 1). The spacecraft weight (9910 lb) and the spacecraft size (see Fig. 8) were used to determine the required launch structure and booster.

6. Spacecraft Structure Analysis

A preliminary analysis of the spacecraft was made to determine additional structural weight required for launch. The analysis was based on a 5-g lateral load and a 10-g axial load. Bending moment and shear diagrams were plotted and used to determine structural member size, skin thickness, and basic load paths. The resultant

weights are shown in Fig. 11, along with the center of gravity of each spacecraft component.

Table 1. Spacecraft weight breakdown

Item	Weight, lb
Propulsion system	
Power subsystem	
Thermionic reactor	1300
Heat rejection	1000
Nuclear shield	800
	<u>3100^a</u>
Thrust subsystem	
Power conditioner	650
Power distribution	150
Thrusters	250
	<u>1050^b</u>
Low-thrust propellant	3760
Net spacecraft	2000 ^c
Total	9910 ^d

^aWeight with structure is 3610 lb.
^bWeight with structure is 1380 lb.
^cWeight with structure is 2075 lb.
^dTotal weight with structure is 10,825 lb.

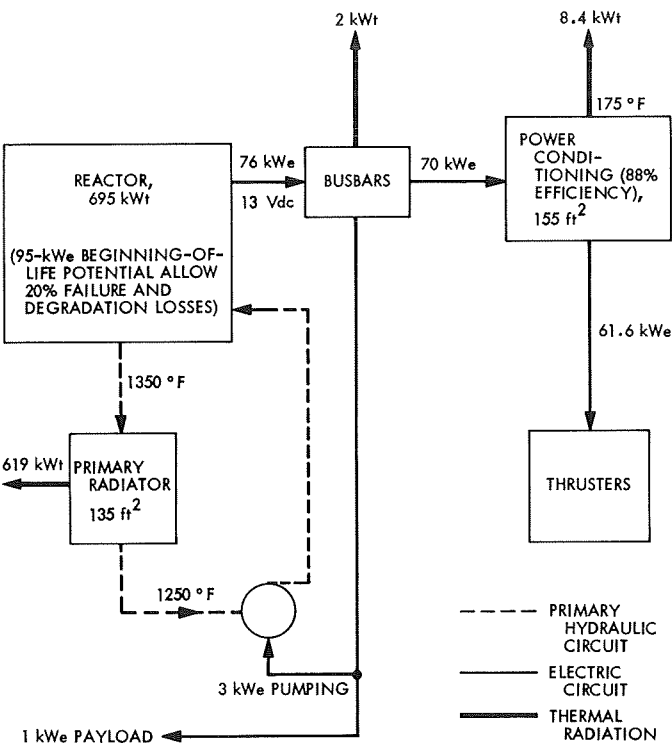


Fig. 9. Propulsion system electrical and thermal power distribution

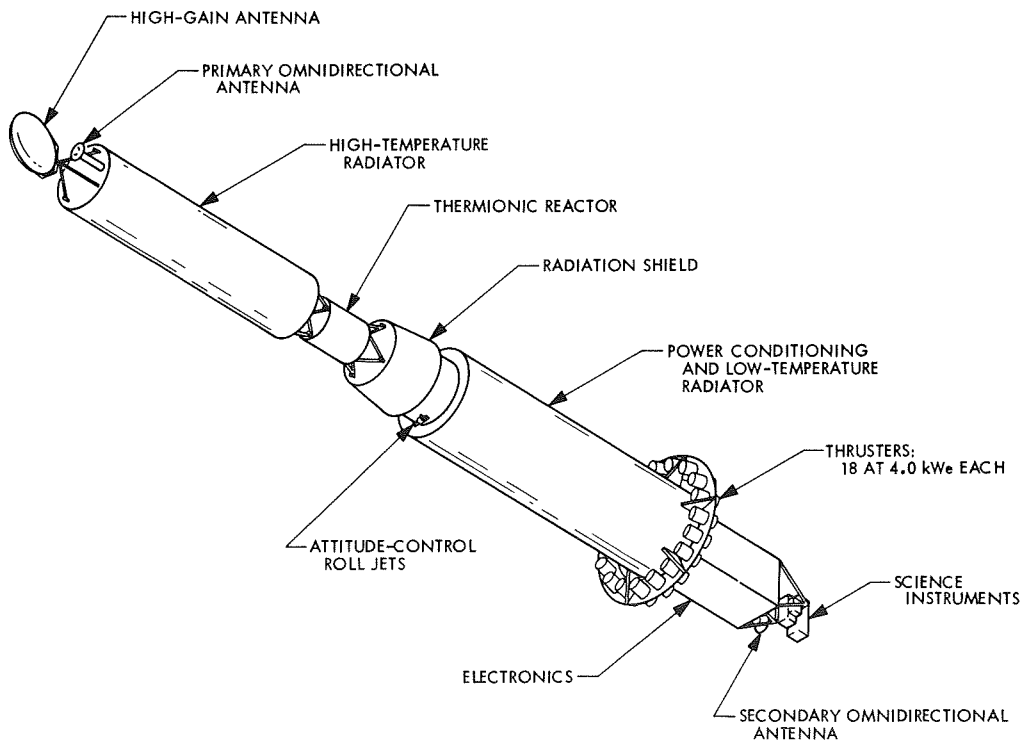
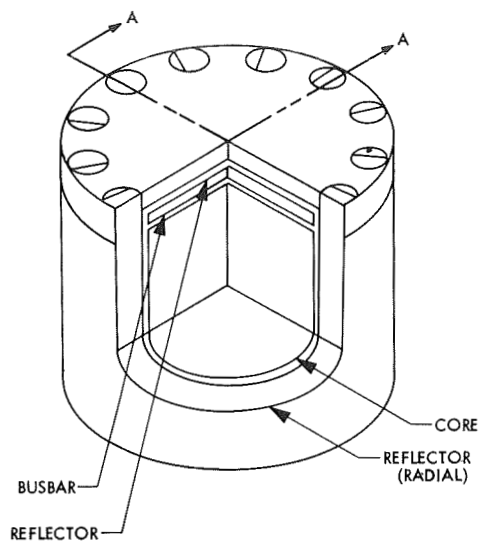
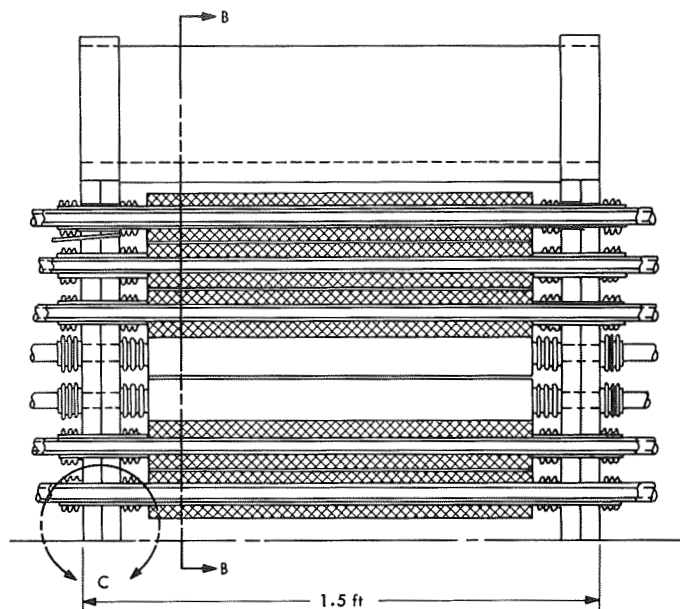


Fig. 8. Configuration of 70-kWe thermionic reactor electric propulsion spacecraft

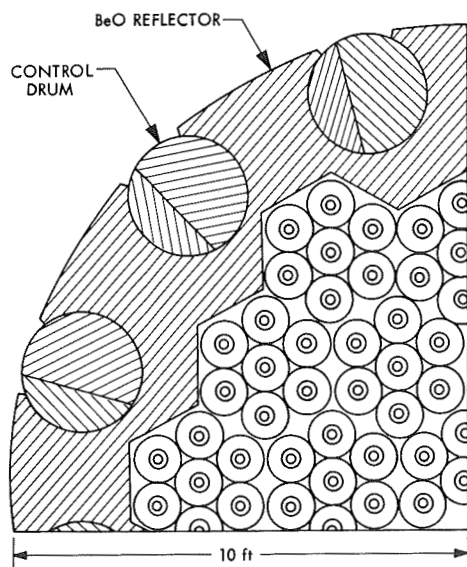
(a) SCHEMATIC
OVERALL ASSEMBLY



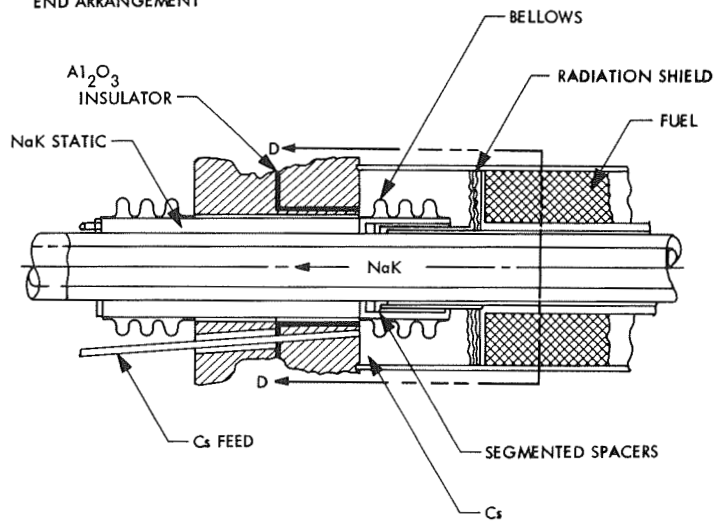
(b) 1/4 SECTION A-A



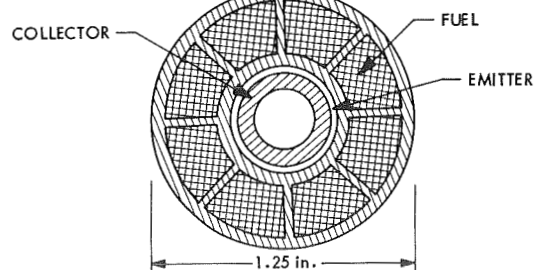
(c) SECTION B-B



(d) DETAIL C, DIODE
END ARRANGEMENT



(e) SECTION D-D



(f) CIRCUIT SCHEMATIC

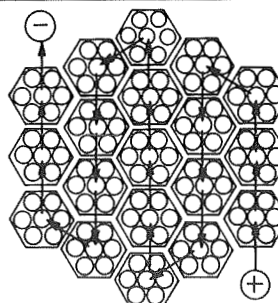


Fig. 10. Detail sections of externally fueled thermionic reactor

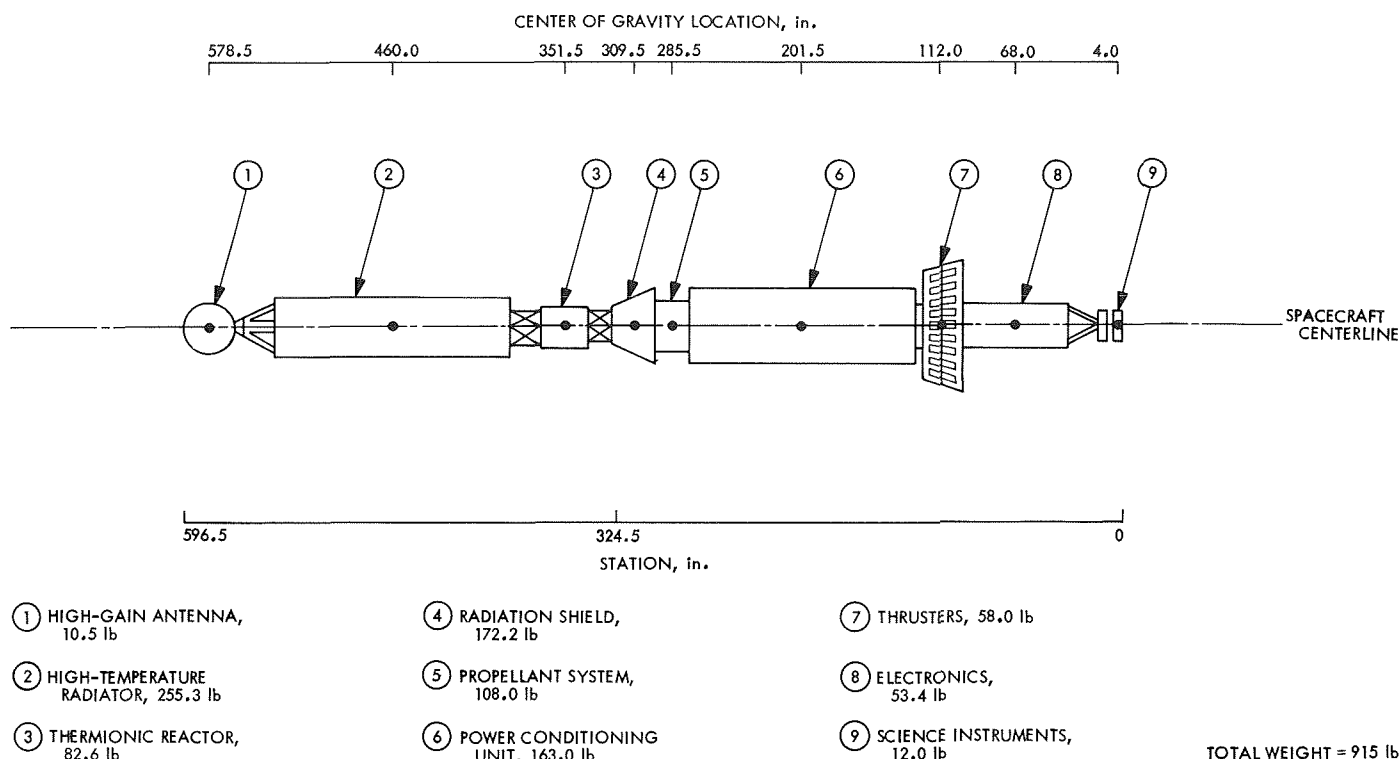


Fig. 11. Structural launch weight of each component of spacecraft

The shield, reactor, and high-temperature radiator structural material is stainless steel because their temperatures can reach 1400°F during operation. The power conditioner structural material is aluminum. Titanium is used for the electronic section because it is exposed to the thruster exhaust (mercury); the mercury propellant tanks are also of titanium.

The weight of the additional structure required for launch is 915 lb. This makes the initial spacecraft weight at earth escape, including structure, 10,825 lb. The structural analysis shows that certain parts of the structure should be strengthened. It also became obvious that the propulsion system structure must have certain strength characteristics to transmit loads from the launch structure. The basic changes to the existing structure and recommendations for new structure are listed below:

- (1) Strengthen the high-temperature radiator tube supports.
- (2) Strengthen the science support structure.
- (3) The thruster support must be stiff in torsion.
- (4) Stiffeners are required on the bottom of the reactor and shield.

- (5) Longerons should be added to the reactor and shield.
- (6) Propellant tanks should have curved ends.
- (7) The spacecraft should be supported at a bulkhead near the shield base.
- (8) The electronic section should be a welded tubular structure.
- (9) The main shear loads from truss sections to sheet metal sections must be transmitted into bulkheads by stiffeners.
- (10) The joints connecting different sections must be designed to take axial and shear loads.

7. Shroud Configuration

The spacecraft is to be boosted to earth escape with a *Titan III-D/Centaur* launch vehicle. The *Titan III-D/Centaur* is the basic *Titan* vehicle with two 5-segment, solid-fuel, strap-on boosters and a *Centaur* last stage. It can boost 12,000 lb of payload to earth escape velocity. The *Centaur* can withstand axial launch loads due to the 12,000-lb payload; however, it cannot withstand the bending moment caused by the spacecraft at launch.

Therefore, during launch both the spacecraft and the *Centaur* stage will be covered by a shroud. The shroud covering the *Centaur* stage will carry the spacecraft loads due to bending moments into the *Titan* structure. The tapered shroud section will also be used to support the spacecraft. The spacecraft is about 50-ft long; therefore, about 25 ft of shroud supporting the spacecraft will remain until escape velocity is reached. The weight of the tapered shroud section will be subtracted directly from the payload. The discarded shroud subtracts only $\frac{1}{4}$ lb of payload per pound of shroud. The proposed shroud (Fig. 12) varies in cross section to achieve minimum weight. The weight of the shroud was determined by using the same weight per square foot of shroud surface as the proposed 10-ft-diam $\times$ 50-ft-long *Viking* shroud. This may be a conservative figure because this electric propulsion spacecraft shroud may be smaller in diameter. The shroud diameter was sized by allowing at least 4 in. of clearance between the shroud and the spacecraft.

The weight of each shroud section is shown in Fig. 12. The nose section and *Centaur* section shrouds would be ejected after *Titan* shutdown and prior to *Centaur* startup. The ejected shroud weight penalty for earth escape payload is $\frac{1}{4}$ of the actual shroud weight, or 125 lb. The tapered section weighs 950 lb and must be launched to earth escape because it is required during *Centaur* operation. The total shroud payload penalty is the tapered section total weight (950 lb) plus the ejected shroud earth-escape weight penalty (125 lb). This makes the total equivalent launch weight to earth escape 11,900 lb and the *Titan III-D/Centaur* capability is 12,000 lb.

8. Conclusions

- (1) *Spacecraft weight.* The 70-kWe thermionic reactor electric propulsion spacecraft weight with launch structure is 10,825 lb (Table 1).
- (2) *Axial thrust capability.* The *Titan III-D/Centaur* has the axial thrust capability to launch the spacecraft with structure and shrouds.
- (3) *Launch dynamic load capability.* The launch dynamic load booster-spacecraft capabilities must be further investigated, but appear promising.
- (4) *Propulsion system.* The low-power (70 kWe), large L/D Thermionic Reactor Electric Propulsion Spacecraft with the *Titan/Centaur* booster appears to be capable of delivering large weight (~ 2000 lb) payloads for outer planets scientific exploration.

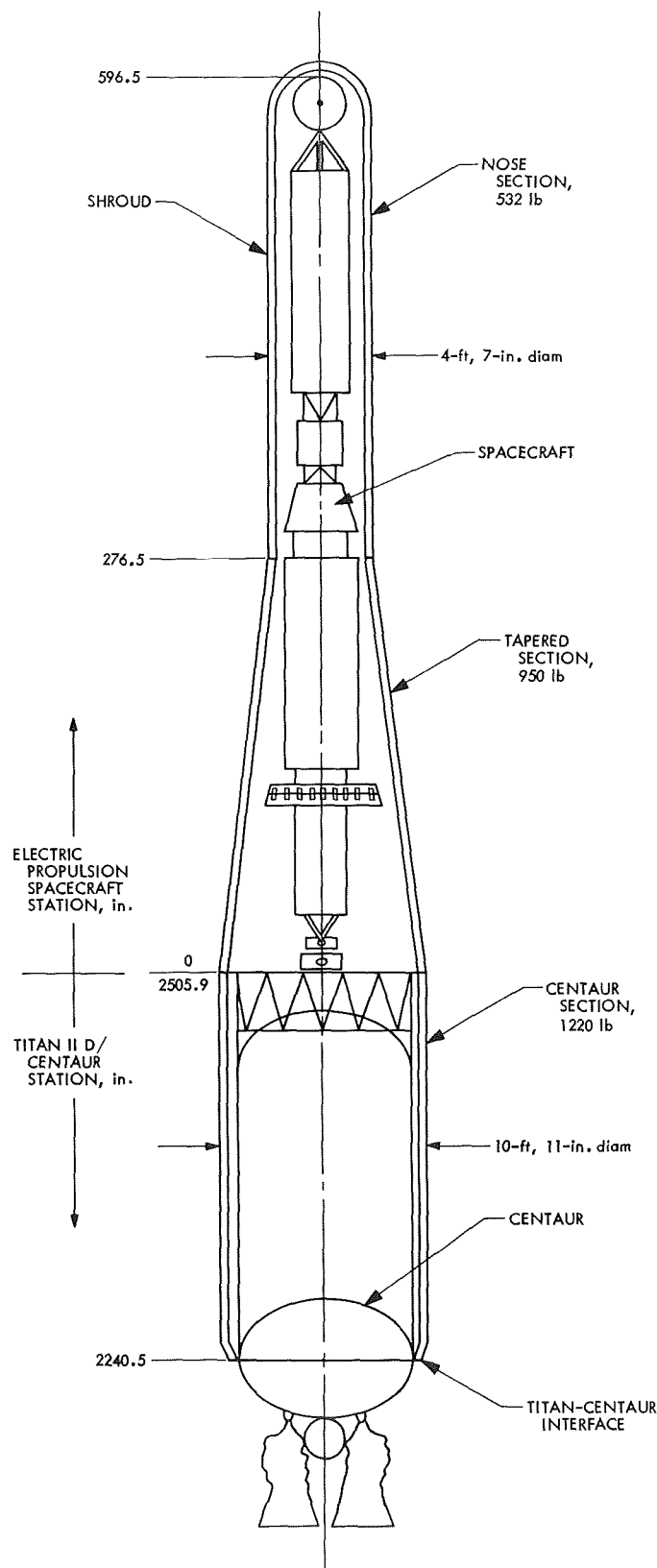


Fig. 12. Spacecraft shroud

C. Preliminary Nuclear Criticality Studies for a Thermionic Reactor With Uninsulated Externally Fueled Diodes, T. Papazoglou, H. Gronroos, and J. P. Davis

1. Introduction

This article summarizes preliminary nuclear criticality calculations performed for the JPL externally fueled, uninsulated diode, thermionic reactor design. The purpose of these analyses was to determine core dimensions, reflector thicknesses, and available reflector control drum worths with sufficient accuracy to support spacecraft design studies. Section B in this chapter describes a proposed spacecraft configuration for a 70-kWe net output thermionic reactor powerplant. Figure 10 shows details of the reactor design. The methods employed in the criticality calculations are briefly outlined, and the important results are stated.

2. Description

The basic module in the reactor is a clustered, uninsulated, externally fueled arrangement of core-length thermionic diodes (SPS 37-46, Vol. IV, pp. 142-147). Through several iterations, a cylindrical form has evolved as shown in Fig. 10. Other SPS articles have described the liquid-metal coolant pump design, spacecraft configurations, and heat transfer calculations (SPS 37-49, Vol. III, pp. 201-207, SPS 37-58, Vol. III, pp. 207-211, and SPS 37-59, Vol. III, pp. 211-214). Table 2 summarizes main dimensions and materials.

The nuclear criticality calculations employ the one-dimensional neutron transport code ANISN (Ref. 1) and the corresponding two-dimensional code DOT (Ref. 2). Both codes use the angular segmentation method (S_n method) to describe the neutron flux angular distribution. A special form of "Data FORTRAN Statement" was used to input data for both programs. A 16-group updated Hansen-Roach cross-section set (Ref. 3) formed the basic cross-section library for all computer calculations. No attempt was made at this initial stage to develop cross sections specifically for the reactor under analysis. Transport code ANISN, which is much less expensive to run, was used to obtain a mesh size and angular quadrature consistent with the desired numerical accuracy for the effective multiplication constant k_{eff} . This was done for the radial and axial dimensions with a transverse leakage connection. A relatively coarse mesh and a fourth-order S_n quadrature were found to be adequate. In a test case, a very good agreement was found between the k_{eff} -values obtained with ANISN and with DOT. However, all data

Table 2. Design data for 70-kWe net thermionic reactor with uninsulated externally fueled diodes

Item	Value
Active core height	10.0 in.
Core diameter	16.35 in. (average)
Radial reflector thickness	3.82 in. BeO (average)
Axial reflector thickness	3.0 in. BeO
Number of control drums	12
Control drum diameter	3.00 in. BeO
Poison segment	1.125 in. B ₄ C
Reactor structural material	Nb-1 Zr
Coolant	NaK 78
Number of diode clusters	19
Number of diodes per cluster	7
Power at reactor busbars	
Beginning of life	95 kWe
End of life	76 kWe
Voltage at busbars	13 Vdc
Diode structure	
Geometry	Cylindrical
Nuclear fuel	UO ₂ , externally fueled
Outside diameter	1.25 in.
Outer clad material	0.050 in. W
Emitter inside diameter	0.500 in.
Emitter thickness	0.050 in. W
Interelectrode gap	0.010 in.
Collector	0.090 in. Nb
Coolant channel diameter	0.300 in.
Axial busbar thickness	2.0 in. (total)
Calculated k_{eff} at operating temperature	
Control drums all in	1.0023
Control drums all out	1.0715

reported in this article were calculated with DOT for cylindrical (r - z) geometry. Conversion to a Univac 1108 machine and FORTRAN V from an IBM 7094 FORTRAN IV version is expected to reduce the DOT running time.

3. Analytical Reactor Model

The reactor concept under study has a small, fast, liquid-metal cooled core region surrounded radially by a beryllia reflector initially chosen to contain 12 control drums. The final number of drums will depend on the control requirements. The axial beryllia reflector is preceded by a 2.0-in. columbium layer that serves both as a supporting structure and as busbar. A 1/8-in. tungsten slab is assumed at both ends between the columbium and the core to account for structural details. The control drums are of beryllia 3.25 in. in diam. A boron-carbide segment 1.125-in. thick forms the poison material.

There are 19 clusters with 7 thermionic diodes each. The outer diameter of the cylindrical diodes was selected to be 1.25 in. after the very first preliminary criticality estimates. All results in the following apply to this dimension. The fueled core height is 10.0 in. The criticality analyses were performed for operating temperatures and dimensions. The solid masses are the same under cold shut-down and hot operating conditions, but the coolant mass changes.

Figure 13 illustrates a quarter segment of the reactor with the assigned material zones. Table 3 shows the corresponding material compositions and temperatures. The average core radius was chosen so that the area of the corresponding circle was equal to the core cross-sectional area. For some computer runs, a transition zone between the core and radial reflector was used. Three separate categories of calculations have been of interest.

a. Reflector worth calculations. The reactor is considered cylindrical with a homogeneous core. The axial and radial BeO reflectors are also taken to be homogeneous (poison segment material replaced by BeO), and of cylindrical geometry.

b. Control drum poison segment worth calculations. The minimum and maximum k_{eff} values are calculated

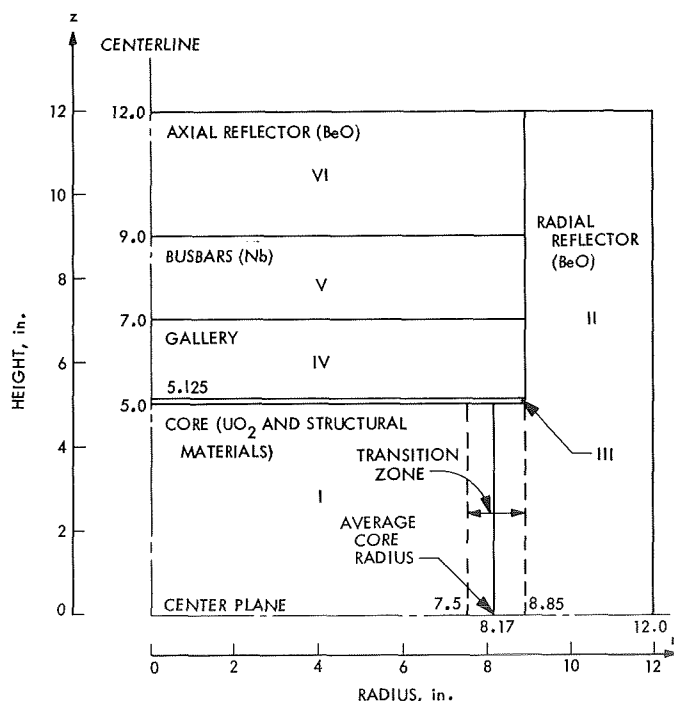


Fig. 13. Quarter segment of analytical reactor model

by considering all drum poison segments in and all segments out. The poison material is distributed in a cylindrical annulus so that its mass is preserved, and with inner and outer radii corresponding to the poison segment portion. This method has been found to be sufficiently accurate for survey type calculations (Ref. 4).

c. Power flattening studies. The objective of these investigations was to determine the distribution of nuclear fuel so that the desired thermal power distribution is obtained.

4. Results

a. Reflector worth. Figure 14 summarizes the results of the reflector worth calculations. The radial reflector is worth considerably more per inch than the axial reflector because the structural Nb and W regions shield the effect of the latter reflector. The worth of the Nb busbar is shown as the difference between the two curves for a 2-in. axial BeO reflector. All calculations have kept the dimensions constant except for varying the thickness of the reflectors. The data presented in Fig. 14 for hot operating conditions show that there is margin for underestimation of the k_{eff} values, for introduction of control drums and structural details, and for fuel redistribution. A few runs have been made in which all dimensions were varied proportionately and the mass densities were maintained. The results show that the k_{eff} values are very nearly proportional to the change in dimensions.

b. Poison segment worth. The k_{eff} value for a reactor with a 3.82-in. average thickness radial BeO reflector and a 3.0-in. axial BeO reflector, and no drum poison

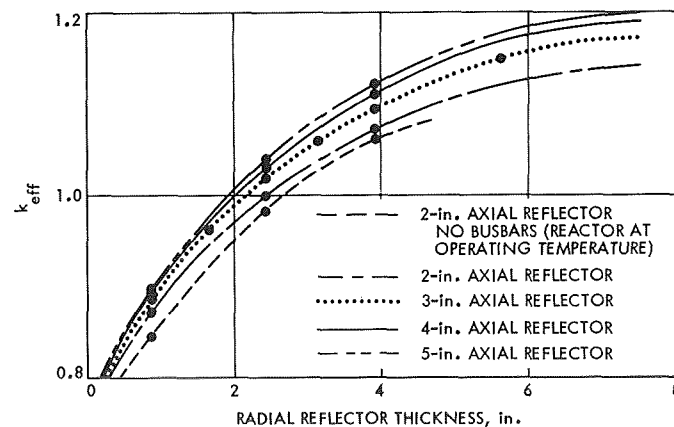


Fig. 14. Effective multiplication factor as a function of axial and radial reflector thicknesses

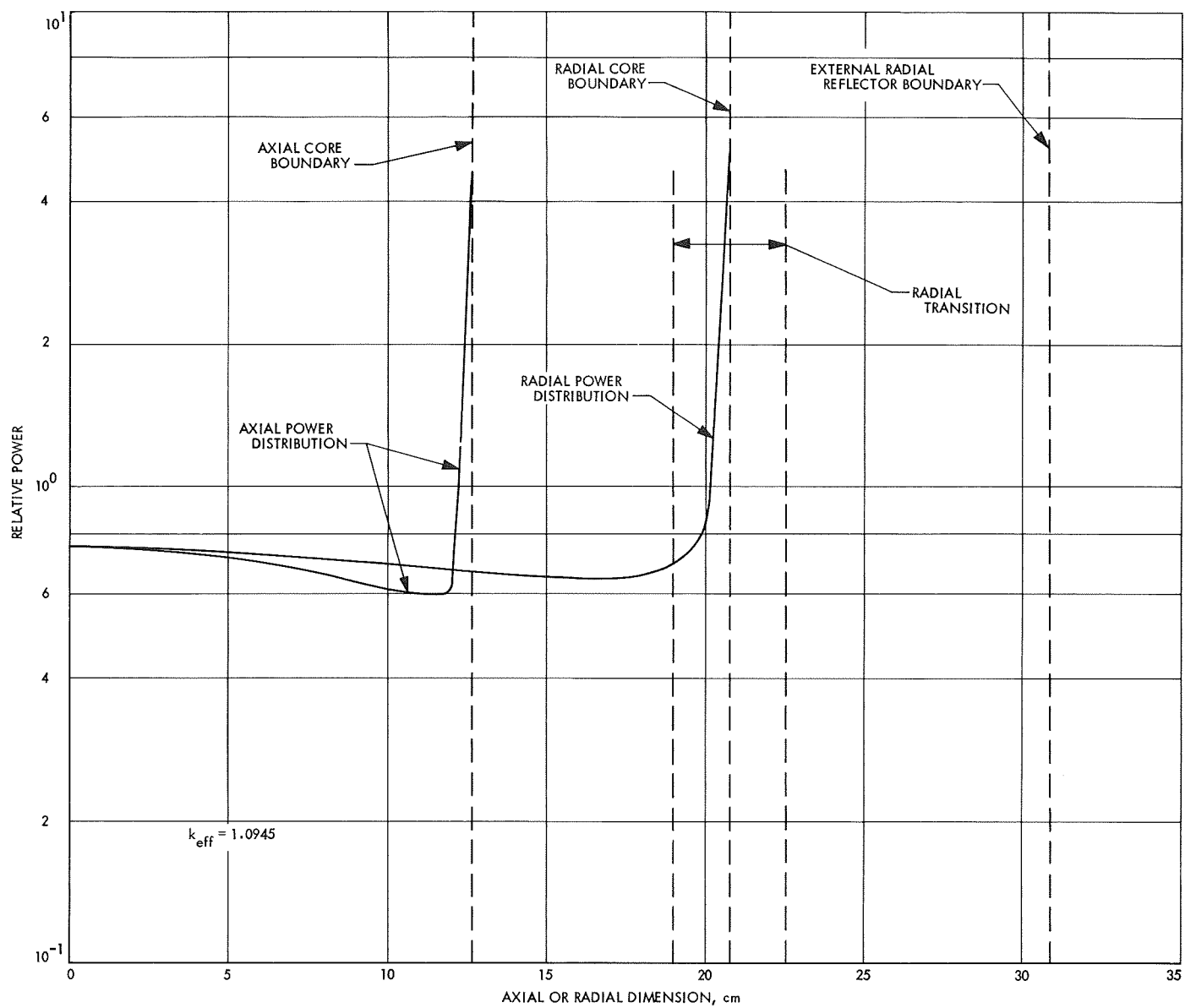


Fig. 15. Normalized axial and radial thermal power distributions

Table 3. Data for the analytical reactor model

Material zone	Materials	Temperature, °K	Density ^a , 10 ²⁴ at cm ⁻³	Radius, cm	Height, cm
I	O	2200	2.002 -02	20.79	12.70
	Nb	1000	3.867 -03		
	W	2000	1.272 -02		
	U ²³⁵	2200	9.327 -03		
	U ²³⁸	2200	7.130 -04		
	Na	1000	1.773 -04		
	K	1000	3.822 -04		
II	O	1000	7.130 -02	30.80	30.80
	Be	1000	7.130 -02		
III	Nb	1000	4.180 -03	22.54	13.02
	W	1000	6.000 -02		
	Na	1000	1.920 -04		
	K	1000	4.020 -04		
IV	Nb	1000	4.180 -03	22.54	15.56
	W	1000	1.325 -02		
	Na	1000	1.920 -04		
	K	1000	4.020 -04		
V	Nb	1000	5.450 -02	22.54	18.10
VI	O	1000	7.130 -02	22.54	30.80
	Be	1000	7.130 -02		

^aNumber following value indicates an order of 10 factor; i.e., -02 = 10⁻².

segment is $k_{\text{eff}} = 1.0945$. Introduction of the poison segment B gives a $k_{\text{eff}} = 1.0715$ with all segments out, and a $k_{\text{eff}} = 1.0023$ with all segments in; i.e., the control swing is approximately 7%, which corresponds to a worth of 10\$. It may be necessary to increase the control swing to about 10%. One way of doing this is to increase the number of drums from 12 to 18.

c. Thermal power distributions. The axial and radial thermal power profiles for a core with a uniform fuel distribution are shown in Fig. 15. Both profiles are relatively flat over most of the core, but with the characteristic peaking at the edges. Only a small degree of fuel zoning is necessary if a very nearly flat distribution is desired. However, it is most desirable to have a constant emitter temperature throughout the core. This would require a thermal power distribution continuously increasing toward the core edges. Nevertheless, the power profiles obtained do not impose a large performance penalty.

5. Summary and Conclusions

The results of initial nuclear criticality calculations for the JPL externally fueled, uninsulated diode, thermionic reactor design have been summarized. The two-dimensional neutron transport code DOT was used to obtain the data presented. An updated Hansen-Roach 16-group set formed the basic cross-section structure. The results indicate that it is possible to obtain a sufficient control margin with reflector control drums that contain a poison segment. The thermal power distributions show that a small degree of fuel zoning is necessary to obtain a nearly uniform profile.

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D. Uranium Nitride-Tungsten Compatibility,

W. M. Phillips

1. Introduction

Among the advanced nuclear power generation systems under investigation for space power is in-pile thermionics. The nuclear fuel operates at clad temperatures of up to roughly 1800°C in this system. Uranium nitride (UN) is one fuel under investigation. Fission gas must be vented from any fuel that is used because of the close dimensional tolerances that must be maintained on the clad. Because of the vapor pressure of nitrogen over the fuel at the temperatures of interest, the fission gases would be required to be vented into a closed fission gas containment system. This experiment was designed to simulate the operating conditions of a thermionic fuel element in an out-of-pile experiment. The essential features of the experiment are: (1) A tungsten-clad uranium nitride sample. (2) A thermal gradient in the fuel capsule. (3) A vent hole orifice. (4) An additional sealed chamber containing hafnium to simulate the materials in an actual fuel element that could act as a sink for the nitrogen overpressure.

2. Materials and Test Conditions

The uranium nitride used in the experiment was made from center-cut biscuit uranium arc-melted as a consumable electrode under 20 atm of nitrogen. The resulting casting was heat treated at 1450°C in a vacuum of approximately 10^{-3} torr to decompose any higher nitrides formed during arc melting. The uranium nitride was then encased in tantalum and isostatically hot-pressed to densify the material.

Tungsten used for the test capsule was made from fluoride vapor deposited tubing and high purity powder metallurgy tungsten (GE-13) for the end caps and orifice plate. The hafnium used was high purity arc cast material.

Capsule assembly was by electron beam welding at 10^{-5} torr. Testing was done in a vac-ion pumped furnace at 10^{-7} torr. The upper end cap was maintained at 1800°C. The thermal gradient was produced by asymmetrical positioning of the capsule within the hot zone and the use of a heavy capsule support stand that acted as a heat sink. This configuration (Fig. 16) resulted in a temperature of 1500°C at the lower capsule end cap. After testing, the capsule was sectioned and examined metallographically. Total time at test temperature was 1000 h.

3. Results and Discussion

Previous measurements (Refs. 1-4) on the vapor pressure and thermodynamic properties of uranium nitride present satisfactory agreement and are presented in Fig. 17. Volitization characteristics and phase equilibria

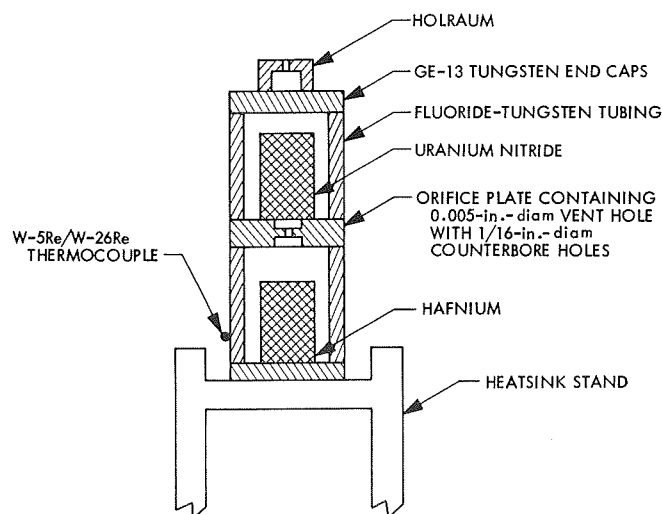


Fig. 16. Uranium nitride-tungsten test setup

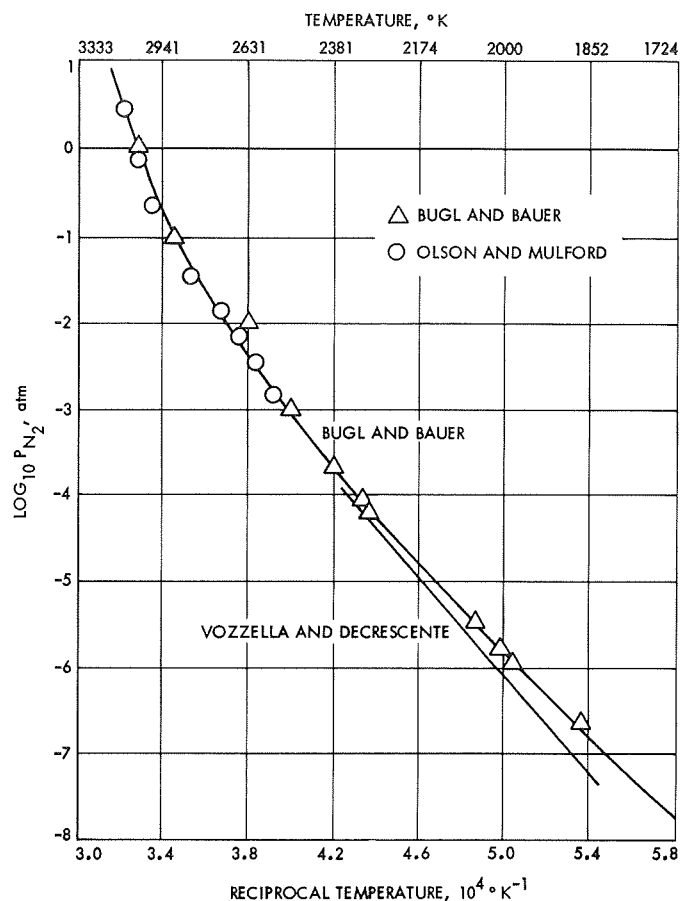


Fig. 17. Pressure of nitrogen in equilibrium with uranium nitride and uranium

have also been investigated (Refs. 5 and 6). The phase diagram of the uranium-nitrogen system in the range of interest is shown in Fig. 18.

The results of previous work indicate that the range of homogeneity of UN below $\sim 1100^\circ\text{C}$ is quite narrow but expands rapidly with temperatures in the range of interest to thermionics (1500 – 1800°C). The effect of this homogeneity range at elevated temperatures is to allow preferential vaporization of nitrogen under vacuum conditions, thereby decreasing the nitrogen to uranium ratio until the phase boundary between $\text{UN}^{(s)}$ and $\text{UN}^{(s)} + \text{U}^{(liq)}$ is reached. At this time, congruent vaporization of UN occurs with no observable amount of liquid uranium present at temperatures below 1800°C . As the temperature of vaporization is increased, observable amounts of liquid uranium appear on the vaporizing surface. The rate of condensation is dependent upon the pumping speed of the system. Increased pumping speed removes liquid uranium as fast as it is formed, increasing the temperature of congruent vaporization.

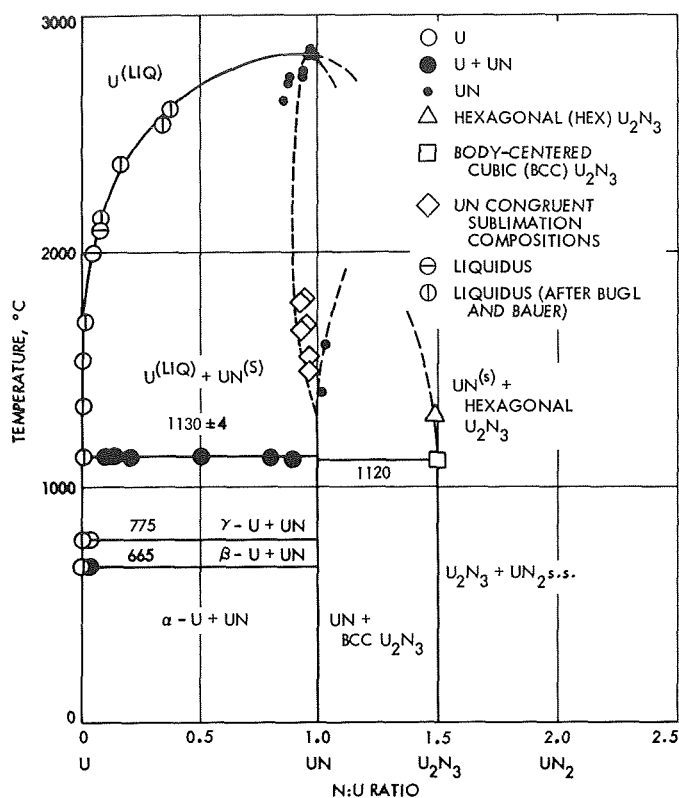


Fig. 18. Uranium-nitrogen partial phase diagram

The test system used in this experiment has a very low pumping speed for nitrogen because removal is dependent upon chemical reaction with the hafnium. Counterbalancing this is the fact that the hafnium nitride formed also has an overpressure of nitrogen. No mechanism is available for the removal of liquid uranium from the system.

Metallographic examination of the test capsule revealed a very good bond between the UN and the tungsten at the point of contact. No chemical reaction was detected in either the UN or in the tungsten. Microhardness measurements of the tungsten at the bond area indicated the Knoop hardness to be 250 ± 20 . The scatter appeared to result from various grain orientations of the tungsten rather than a function of depth from the bonding surface.

A photomicrograph of the uranium nitride is shown in Fig. 19. Metallographic examination did not detect any free uranium. Previous work (Ref. 5) detected free uranium, particularly in the grain boundaries of UN_{0.99}. Thus, the composition of the nitride after test appears very close to stoichiometric.

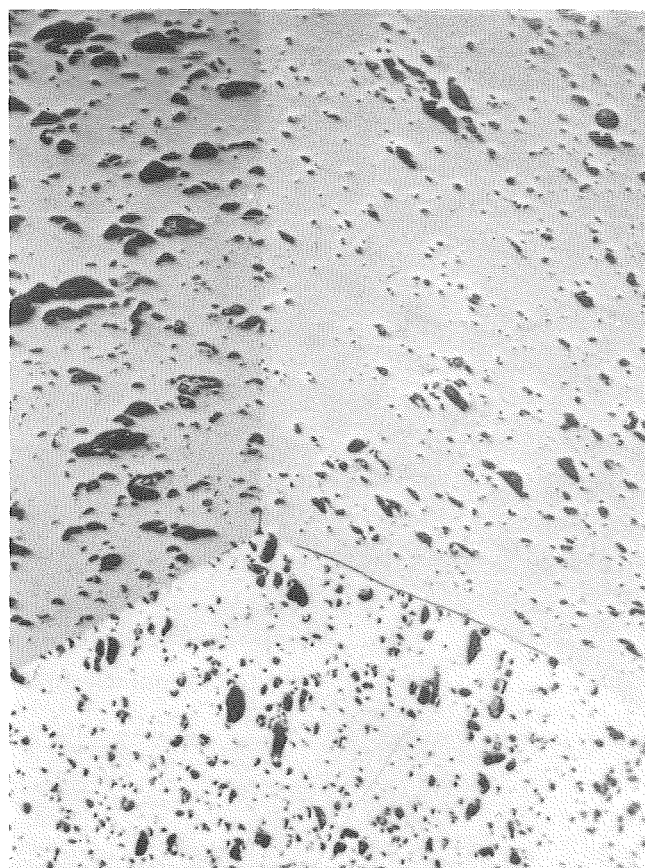


Fig. 19. Metallographic section of uranium nitride etched with lactic acid, nitric acid, and hydrofluoric acid

Redistribution of the uranium nitride took place along the counterbored hole on the orifice plate (Fig. 20). This was the only area within the capsule where redistribution occurred. The uranium nitride was well bonded to the tungsten (Fig. 21), and diffusion into the tungsten occurred. A continuous network of uranium nitride is visible in the grain boundaries of the tungsten as well as uranium nitride particles within the tungsten grains. No evidence of free uranium was observed in uranium nitride.

This behavior is not unlike that of either uranium carbide or uranium dioxide in that a measurable uranium flux through tungsten cladding occurs with both these fuels (Refs. 7 and 8). Penetration of uranium into tungsten was observable only to depth of 1-3 mils, although the flux of uranium was measured to 20-mils.

4. Conclusions

Uranium nitride is capable of producing grain boundary diffusion of uranium through a tungsten cladding

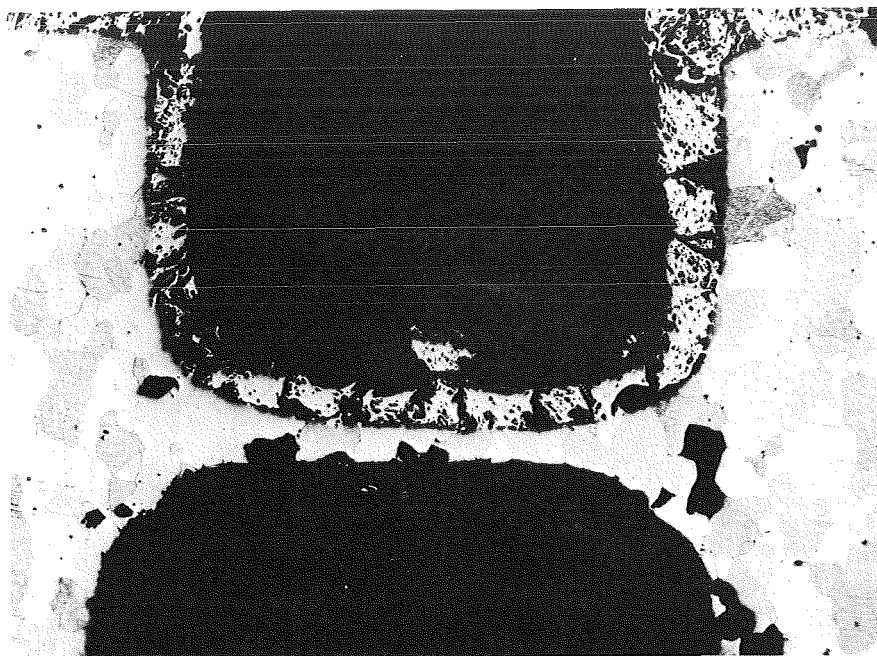


Fig. 20. Uranium nitride redistribution on the orifice plate



Fig. 21. Bond area between uranium nitride and tungsten on the orifice plate

despite apparent chemical compatibility of the uranium nitride and tungsten. Vaporization and deposition of uranium nitride has also been shown to occur.

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E. Design of Reactor Simulator for Thermionic Diode Kinetics Experiment, H. Gronroos

1. Introduction

This article describes the methods employed to simulate a thermionic reactor system by the use of the thermionic diode kinetics experiment. The experimental assembly has been described previously (SPS 37-54, Vol. III, pp. 119-124 and SPS 37-57, Vol. III, pp. 173-177), and in Refs. 1, 2, and 3. The purpose of the experiment is to study by non-nuclear simulation methods the dynamics and control of a liquid-metal cooled thermionic reactor powerplant. The assembly consists of four cylindrical thermionic diodes mounted vertically in a NaK-cooled test bank enclosed in a vacuum chamber (two more diodes will be added in the near future). Each diode is independently heated by electron beam power, and independently instrumented. The electrical output from each diode is taken to a coupling unit that permits selection of the desired inter-diode electrical coupling. From the coupling unit the electrical output may be fed either to a low-voltage electrical load or to a power conditioner. Figure 22 illustrates the main components and their interrelationships.

The electron beam power supplies have a program jack that allows an external signal to control the thermal input to the diodes. The cesium reservoir heaters and a preheater in the external NaK-loop may be similarly controlled. An analog computer is used to generate these control signals. The computer is programmed so that the experimental system with the computer in the loop matches as closely as possible the dynamic characteristics of a selected model thermionic reactor powerplant. Because of several considerable differences between the experimental assembly and a prototypical thermionic reactor, complete fidelity cannot be obtained. This is not necessary, however, because the objective is to investigate the general dynamic behavior of candidate model powerplants.

Given a reference thermionic reactor, each experimental diode is assigned a region in the reactor core and assumed to represent all diodes in that region (node). Various combinations can be chosen; some of which are illustrated in Fig. 23(a-c). The axial and radial thermal power distributions (proportional to the neutron density)

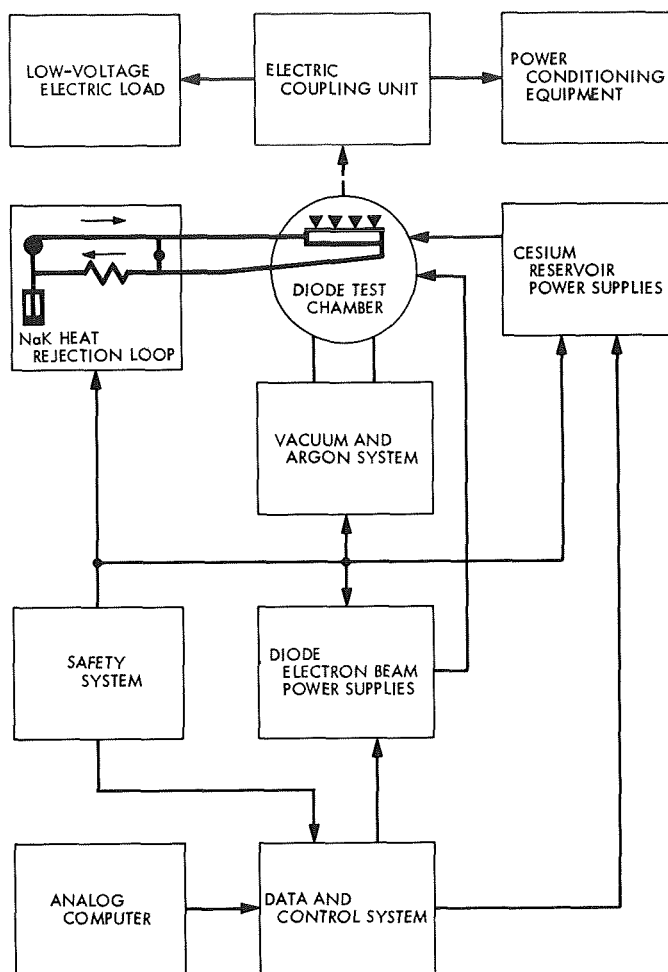


Fig. 22. Block diagram of thermionic diode kinetics experiment

are obtained from separate criticality calculations (Fig. 23 d). Temperature coefficients of reactivity for the various materials in the model reactor diode (i.e., nuclear fuel, emitter, collector, insulator, cladding, and coolant) are estimated. By the use of instrumentation thermocouples and thermal balance equations programmed on the analog computer, the temperature state for each diode material region is defined. This, together with the temperature coefficients of reactivity, defines the internal reactivity feedback. Point (one-node) neutron kinetics equations are used to determine the neutron density. The level of the signal to the electron beam power supply is proportional to the thermal power level at the assigned node in the reference reactor. The control reactivity is generated on the analog computer, and disturbances (e.g., electric load changes) are introduced as required by the experimental program. In addition, the external heat rejection loop and electric load circuit of the reference plant may be modeled.

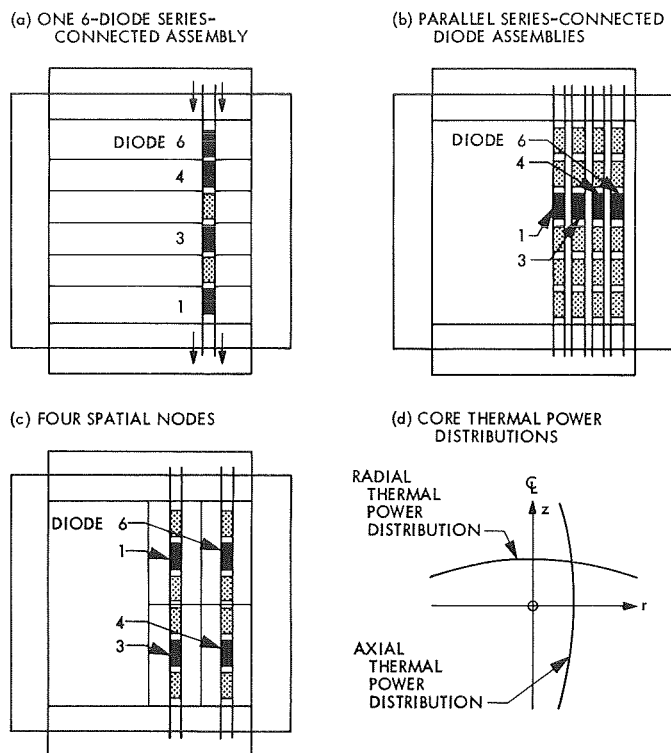


Fig. 23. Assignment of experimental diodes in model thermionic reactor

A description of the principles used in developing the equations programmed on the analog computer reactor simulator is presented. First, a relatively complete simulation scheme is described. Based on this, two simplified models are developed.

2. Simulation Method

Figure 24 shows a simplified drawing of the experimental diode and of a prototypical reactor diode heated by nuclear fission. Even if given the same emitter, spacing, and collector dimensions, there are considerable differences in the designs. To account for these to the extent feasible, the problem is broken down into three major parts:

- (1) Modeling of the fuel and emitter regions.
- (2) Modeling of the energy transport in the inter-electrode gap.
- (3) Modeling of the collector and heat rejection regions.

a. Nuclear fuel and emitter. The nuclear fuel region of the reactor diode is divided into two annular regions. The median temperature of the region closest to the

emitter is identified with that of the emitter thermocouple reading (middle location) of the experimental diode. The thickness of this fuel region is selected so that the temperature drop to the emitter surface becomes the same in the experimental diode and in the reference reactor diode. To obtain the same time constant, a unity gain lead-lag network is inserted in the signal channel from the neutron kinetics to the electron beam power supply. The purpose of such a series compensation is to cancel out the dominant pole of the electron beam power supply-filament-emitter-thermocouple assembly and replace it with a pole associated with the outer fuel region. Direct observations of response trajectories show that, despite the complicated relationships governing the experimental assembly, the thermocouple response is nearly exponential and linear with step changes in the control signal.

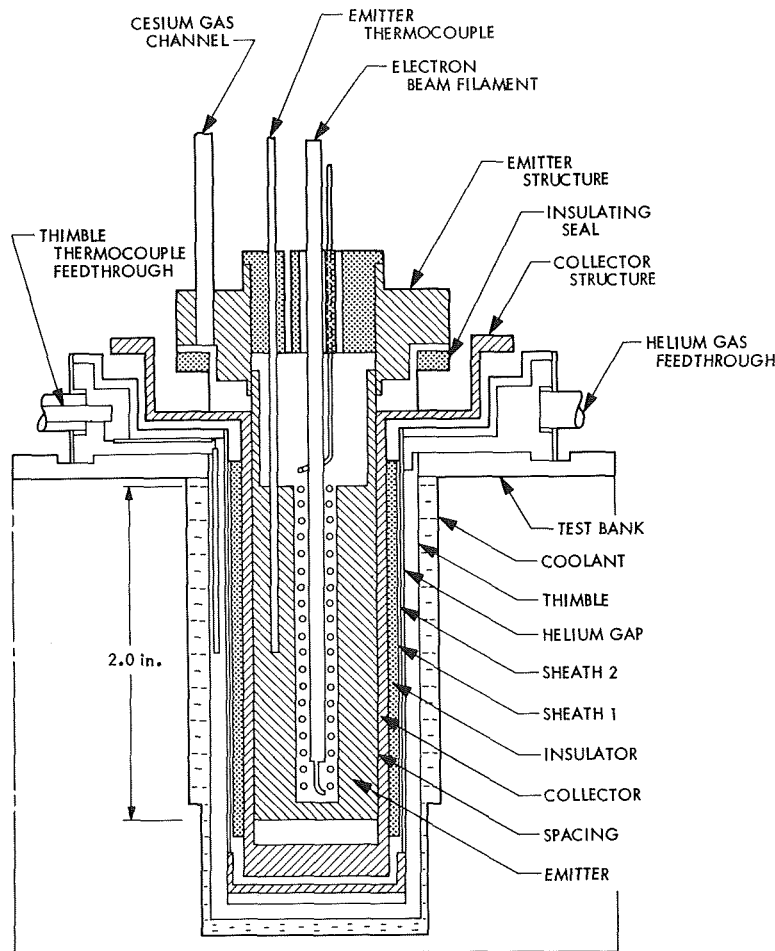
The above discussion implies that thermal balance equations must be programmed on the analog computer for the inner fuel region and the emitter region while the thermocouple can replace the equation for the outer fuel region. The balance equations are written for the average temperature of a spatial region and with lumped parameters; i.e., the spatial differentials are omitted.

b. Energy transport in inter-electrode gap. An analytical model for the energy transport across the inter-electrode gap and for the thermal lead loss is needed because there are no thermocouples defining the temperature of the collector. The selected equations are described under *Subsection 3-a*. The measured diode currents and voltages are utilized.

c. Collector and heat rejection regions. The collector and outer regions including the coolant are described by lumped parameter equations applicable for the experimental diodes. Three equations for the median temperatures are employed: (1) for a 0.030-in. part of the collector, (2) for the remainder of the collector to the thimble, and (3) for the thimble region. The equation for the latter may again be replaced by the thimble thermocouple reading. Finally, thermal balance equations can be written for the coolant region, and for any other reactor structural region of interest.

d. Limitations and assumptions. Figure 25 identifies the regions and the nomenclature used for the experimental diode and for the model reactor diode. The corresponding dimensions and materials are shown in Tables 4 and 5. Obviously, the heat transfer through the collector, insulator, and thimble for the experimental diode

(a) EXPERIMENTAL DIODE



(b) REACTOR DIODE

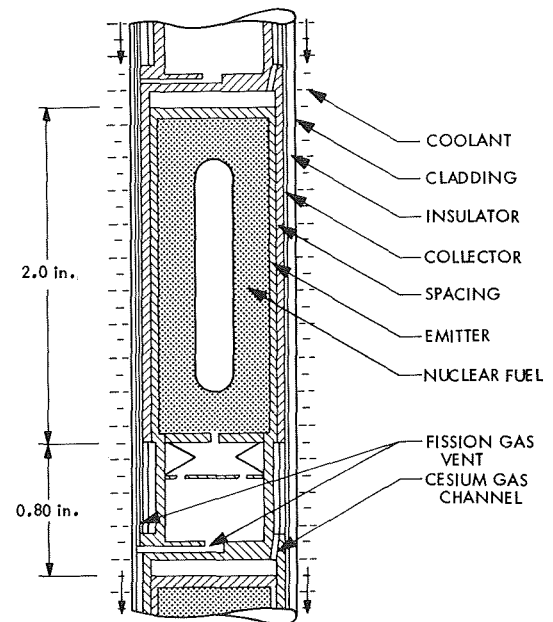


Fig. 24. Schematic of experimental thermionic diode and of reactor thermionic diode

is different than that for the reference diode. However, there is no suitable method for modifications of the kind that could be made for the emitter. Furthermore, the axial temperature distributions and the thermal lead losses are different and vary with operating point. The filament for the electron beam gun has, however, been wound so that the axial temperature distribution closely matches that of a prototypical reactor diode. If a system with tungsten emitters is simulated, one must assume that the tungsten surface is covered with a thin rhenium layer or neglect the differences in thermionic conversion performance between the tungsten and rhenium materials. Other factors affecting the fidelity of the simulation to a smaller degree are ohmic losses and the use of simplified analytical expressions. Nevertheless, the substance of the dynamic behavior is not materially affected by the stated limitations, although there are quantitative differences.

Experimental data show that for the nominal range of collector temperature variations, the effect upon energy transfer across the inter-electrode gap may be omitted. Also, the temperature drop in the simulated emitter and collector regions can be neglected. With these, and those in the foregoing listed conditions, three different simulator models reflecting decreasing degrees of complexity are described in Subsection 3.

3. Simulation Equations

a. Extended reactor model.

Fuel and emitter regions. The thermal balance equation for the inner fuel region (Fig. 25 b and Table 4) becomes (for the average temperature $\bar{T}_{11}$):

$$V_{11} \rho_{11} c_{p11} \frac{d\bar{T}_{11}}{dt} = V_{11} \gamma n - 2\pi r_{12} L h_{11} (\bar{T}_{11} - \bar{T}_{12}) \quad (1)$$

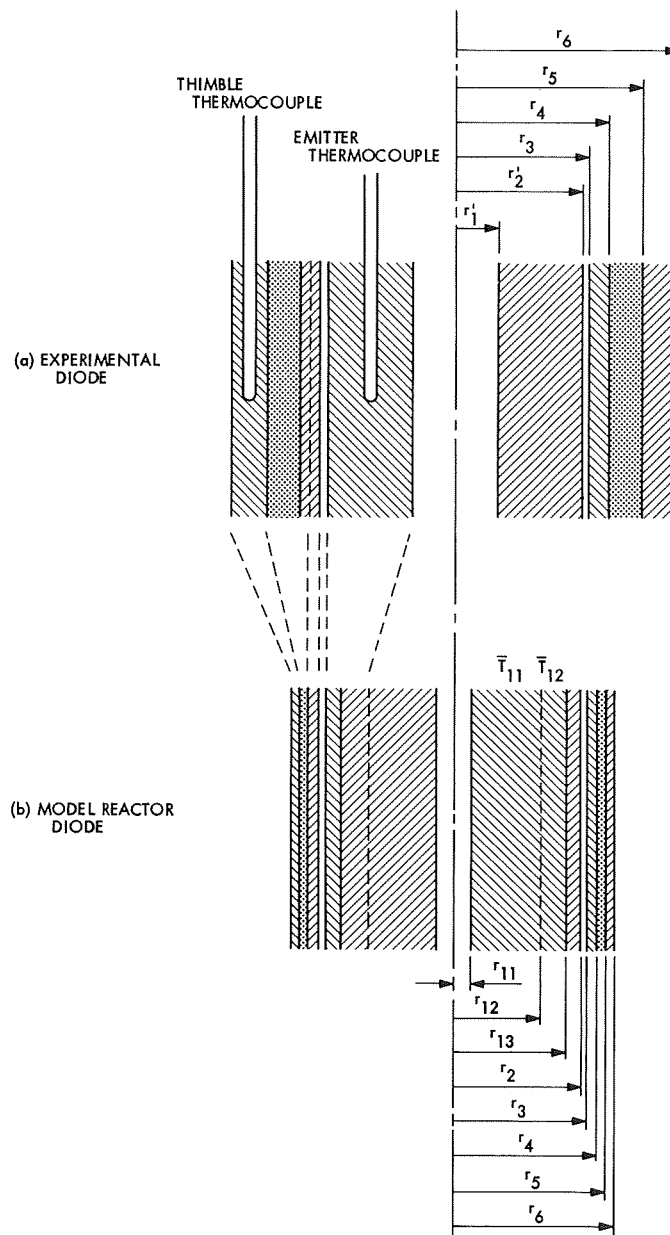


Fig. 25. Radial regions in experimental and model reactor thermionic diodes

Table 4. Dimensions and materials for the experimental diode

Region	Region number	Outer radius, cm	Region thickness, cm	Material	Reference temperature, °C	c_p , Ws/g °C	Density, g/cm ³	Conductivity, W/cm °C	Remarks
Emitter cavity	1	0.3175	0.3175	Vacuum (W)					Contains W electron beam filament
Emitter ^{a b}	2	0.9525	0.6350	Re	1600	0.1674	20.30	0.470	Contains 3 thermocouples
Gap	3	0.9779	0.0254	Cs gas	320 (Res.)	~0	~0		$h = 1.90 \times 10^{-3} \text{ W/cm}^2\text{-}^\circ\text{C}$
Collector	4	1.1303	0.1524	Nb	700	0.2723	8.42	0.56	Thin Mo layer on Nb surface
Insulator	51	1.2827	0.1524	Al ₂ O ₃	675	1.24	3.20	0.0606	
Sheath 1	52	1.3081	0.0254	Nb	650	0.2686	8.44	0.54	
Sheath 2	53	1.3589	0.0508	Kovar	650	0.51	8.0	0.11	29% Ni, 17% Co, 54% Fe
He gap	54	1.3614	0.0025	He(Al ₂ O ₃)	625			0.002975	He pressure adjustable
Thimble ^c	6	1.6154	0.2540	Kovar	610	0.51	8.0	0.11	
Coolant	7	1.9075	0.2921	NaK 78	600	0.876	0.728	0.2591	$h_{67} = 2.84 \text{ W/cm}^2\text{-}^\circ\text{C}$

^aEmitter length = 5.08 cm (2 in.).

^bEmitter W26Re/W5Re thermocouple location $r_{e,tc} = 0.6350$ cm.

^cThimble Cr/Al thermocouple location $r_{t,tc} = 1.4884$ cm.

Table 5. Dimensions and materials for the model diode

Region	Region number	Outer radius, cm	Region thickness, cm	Material	Reference temperature, °C	c_p , Ws/g °C	Density, g/cm ³	Conductivity, W/cm °C	Remarks
Fuel cavity	0	0.1903	0.1903	Vacuum (Fission gas)	2000	~0	~0	~0	Region exists only for UO ₂ fuel
Fuel	11	0.8239	0.6336	UO ₂ /UC	1800/1650	0.32/0.27	9.68/10.2	0.019/0.23	UC = 90% UC — 10% ZrC, 15% porosity
Fuel	12	0.8509	0.027				10.3	0.35	
Emitter ^a	2	0.9525	0.1016	Re/W	1600	0.19	18.7	1.03	
Gap	3	0.9779	0.0254	Cs gas	320 (Res.)	~0	~0		$h = 1.90 \times 10^{-3} \text{ W/cm}^2\text{-}^\circ\text{C}$
Collector	4'	1.0541	0.0762	Nb	700	0.272	8.42	0.56	Part of experimental diode collector
Insulator	5	1.3614	0.3073	Al ₂ O ₃	660	1.24	3.20	0.0606	Regions 4, 51–54 of experimental diode
Sheath	6	1.6154	0.2540	Kovar	610	0.50	7.76	0.21	Same as experimental diode
Coolant	7	1.9075	0.2921	NaK 78	600	0.876	0.728	0.26	Same as experimental diode

^aEmitter length = 5.08 cm (2 in.).

where n is the neutron density, γ is a conversion factor to the units W/neutron, and h_{11} is the heat transfer coefficient

$$h_{11} = \frac{k_1(r_{12}^2 - r_{11}^2)}{r_{12}} \left\{ \frac{r_{13}^2 - r_{11}^2}{4} + \left[\frac{1}{(r_{12}^2 - r_{11}^2)} + \frac{1}{(r_{13}^2 - r_{12}^2)} \right] r_{11}^2 r_{12}^2 \ln \left(\frac{r_{12}}{r_{13}} \right) + \frac{r_{11}}{(r_{12}^2 - r_{11}^2)} \ln \left(\frac{r_{13}}{r_{11}} \right) \right\}^{-1} \quad (2)$$

The neutron kinetics is simulated by the usual point kinetics equations with associated reactivity feedback terms (Ref. 4). Defining the thermocouple reading in the emitter T_{tc} to equal the average temperature of the outer fuel region $\bar{T}_{12}$, and also requiring that the temperature drops to the emitter surface be the same, gives an equation for determining r_{12} (the radius of the inner fuel region):

$$\frac{k_2'}{r_2' \ln(r_2'/r_{tc})} = \frac{k_1(r_{13}^2 - r_{11}^2)}{r_{13}} \left[\frac{r_{13}^2 - r_{12}^2}{4} - \frac{r_{11}^2}{2} + \frac{r_{11}^2 r_{12}^2}{(r_{13}^2 - r_{12}^2)} \ln \left(\frac{r_{13}}{r_{12}} \right) + \frac{k_1}{k_2} (r_{13}^2 - r_{11}^2) \ln \left(\frac{r_2}{r_{13}} \right) \right]^{-1} \quad (3)$$

where $r_2' = r_2$. For UO_2 fuel, the radius becomes $r_{12} = 0.8241$ cm. Because of the low thermal conductivity of UO_2 , a very thin outer fuel region results. If UC fuel is used, it is found that Eq. (3) has no real-valued solution for r_{12} . For that case, the thermocouple reading can simply be identified with the average UC fuel temperature.

For the average temperature $\bar{T}_{12}$ of the outer fuel region, one obtains:

$$V_{12} \rho_{12} c_{p12} \frac{d\bar{T}_{12}}{dt} = V_{12} \gamma n + 2\pi r_{12} L h_{11} (\bar{T}_{11} - \bar{T}_{12}) - 2\pi r_{13} L h_{12} (\bar{T}_{12} - \bar{T}_2) \quad (4)$$

and the heat transfer coefficient now becomes

$$h_{12} = \frac{(r_{13}^2 - r_{11}^2)}{r_{13}} \left\{ \frac{1}{k_1} \left[\frac{(r_{13}^2 - r_{12}^2)}{4} - \frac{r_{11}^2}{2} + \frac{r_{11}^2 r_{12}^2}{(r_{13}^2 - r_{12}^2)} \ln \left(\frac{r_{13}}{r_{12}} \right) \right] + \frac{1}{k_2} \frac{(r_{13}^2 - r_{11}^2)}{(r_2^2 - r_{13}^2)} \left[r_2^2 \ln \left(\frac{r_2}{r_{13}} \right) - \frac{(r_2^2 - r_{13}^2)}{2} \right] \right\}^{-1} \quad (5)$$

Equation (4) is normally replaced by the emitter thermocouple reading obtained from the experimental diodes.

The equation for the emitter temperature $\bar{T}_2$ is

$$V_2 \rho_2 c_{p2} \frac{d\bar{T}_2}{dt} = 2\pi r_{13} L h_{12} (\bar{T}_{12} - \bar{T}_2) - q_2 \quad (6)$$

where q_2 is the heat transfer between the emitter and collector sides.

Energy transport in inter-electrode gap. The term q_2 contains four main components: electron and ion cooling, radiation, lead losses, and cesium gas conduction; i.e.,

$$q_2 = q_e + q_r + q_l + q_{cs} \quad (7)$$

The electron and ion cooling term q_e was calculated for the experimental diodes (taking into account the necessary corrections) with simplified expressions derived by Wilkins and Hill (Ref. 5). A plot of q_2 versus the diode current I reveals a nearly linear relationship between these variables for a fixed emitter temperature. There are only minor influences from voltage, cesium reservoir and collector temperature variations. The term q_e is therefore taken to be

$$q_e = I \left(\phi' + \frac{2k_B}{e} \bar{T}_2 \right) = I(2.16 + 0.172 \cdot 10^{-3} \bar{T}_2) \quad (8)$$

where I is experimentally measured.

For the radiation and thermal loss terms, one obtains, respectively:

$$q_r = 2\pi r_2 L q_r''(\bar{T}_2) \quad (9)$$

$$q_l + q_{cs} = (A_l h_l + 2\pi r_2 L h_{cs})(\bar{T}_2 - \bar{T}_4) \quad (10)$$

where $q_r''(\bar{T}_2)$ is determined from thermal radiation relationships (Refs. 6 and 7) and stored on a function generator. The total of Eqs. (9) and (10) is matched to the experimental heat flux.

Collector and heat rejection regions. For the collector and the insulator, thimble, and coolant heat rejection region temperatures, the thermal balance equations become, respectively:

$$V_4 \rho_4 c_{p4} \frac{d\bar{T}_4}{dt} = [q_2 - I(V_o + R_l I)] - 2\pi r'_4 L_4 h_{45}(\bar{T}_4 - \bar{T}_5) \quad (11)$$

$$\left(\sum_{i=1}^5 V_{5i} \rho_{5i} c_{p5i} \right) \frac{d\bar{T}_5}{dt} = 2\pi r'_4 L_4 h_{45}(\bar{T}_4 - \bar{T}_5) - 2\pi r_5 L_5 h_{56}(\bar{T}_5 - \bar{T}_6) \quad (12)$$

$$V_6 \rho_6 c_{p6} \frac{d\bar{T}_6}{dt} = 2\pi r_5 L_5 h_{56}(\bar{T}_5 - \bar{T}_6) - 2\pi r_6 L_6 h_{67}(\bar{T}_6 - \bar{T}_7) \quad (13)$$

$$V_7 \rho_7 c_{p7} \frac{d\bar{T}_7}{dt} = 2\pi r_6 L_6 h_{67}(\bar{T}_6 - \bar{T}_7) - 2\omega c_{p7}(\bar{T}_7 - \bar{T}_{7in}) \quad (14)$$

$$\bar{T}_7 = \frac{1}{2}(\bar{T}_{7in} + \bar{T}_{7out}) \quad (15)$$

and other system equations as required.

In Eq. (11), the measured voltage V_o is corrected for the lead loss $R_l I$. For the thimble temperature, Eq. (13) may be replaced by an experimental thimble thermocouple reading. The heat transfer coefficients h_{45} and h_{56} are defined assuming linear temperature distributions, i.e.,

$$h_{45} = \frac{2}{r'_4} \left[\sum_{i=(3)}^{(5)} \frac{\ln(r_{i+1}/r_i)}{k_{i+1}} \right] \quad (16)$$

for regions $i = 3, 4', 4, \dots, 5$,

$$h_{56} = \frac{2}{r_5} \left[\sum_{i=(4')}^{(6)} \frac{\ln(r_{i+1}/r_i)}{k_{i+1}} \right] \quad (17)$$

for regions $i = 4', 4, \dots, 6$. The heat transfer coefficient h_{67} calculated for the experimental geometry and flow mode¹ becomes:

$$h_{67} = 2.84 \text{ W/cm}^2 - ^\circ\text{C} \quad (18)$$

Figure 26 shows a block diagram representation of the developed thermionic reactor simulator. The heavy dashed lines symbolize the location of buffer amplifiers used to prevent ground loops between the analog computers and

the experimental equipment. A separate small desk computer is employed at the experimental site primarily for the generation of the neutron kinetics, while the Analog Computing Facility (located in another building) is employed for generation of the remainder of the system equations. The equipment requirements are rather

¹G. Kikin, private communication.

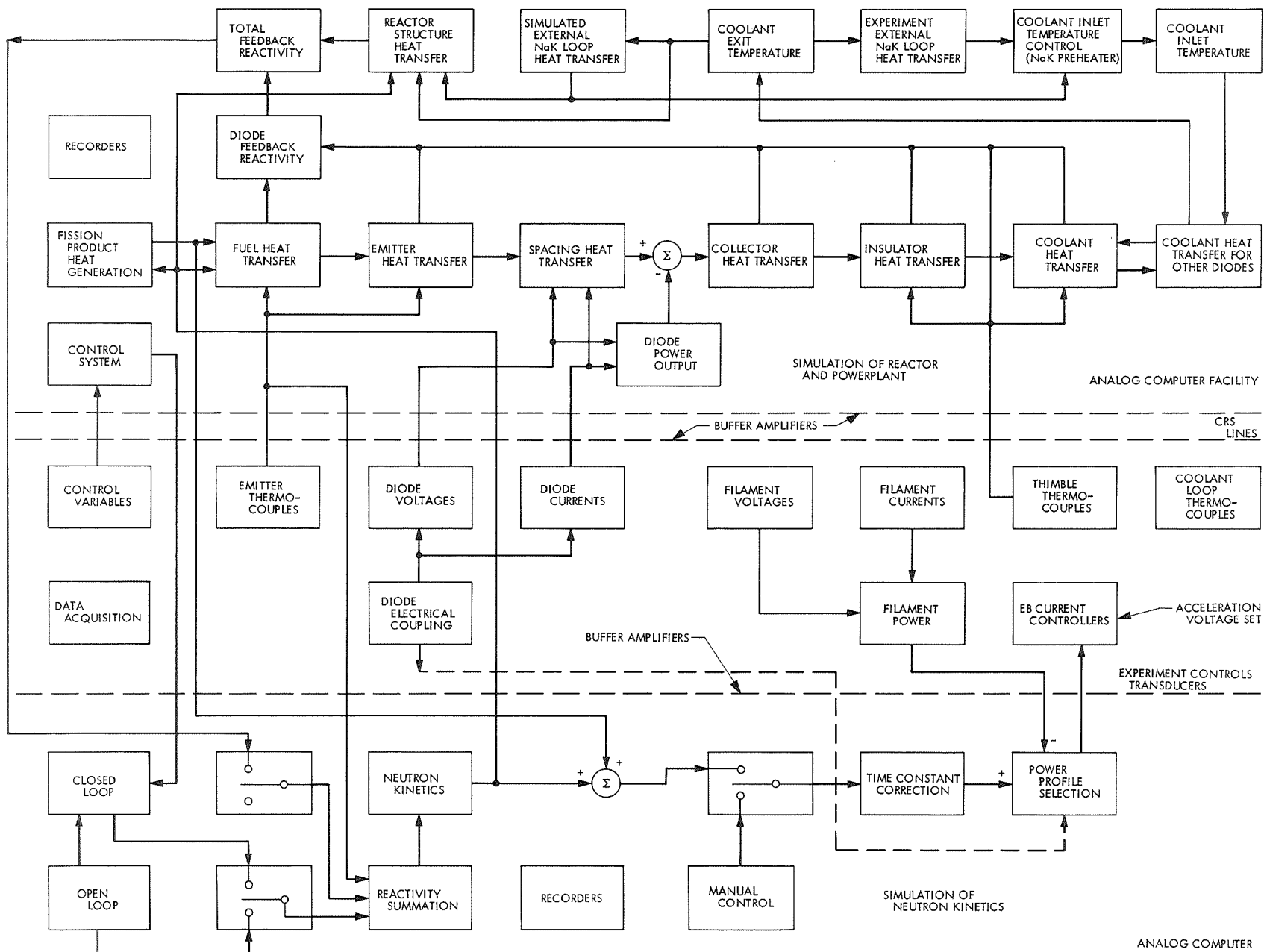


Fig. 26. Block diagram of nuclear reactor simulator for thermionic diode kinetics experiment

extensive, and the scheme illustrated on Fig. 26 was only recently implemented. A system of equations is instrumented for each diode. The coupling between them is determined by the selected electrical interconnections and the assigned location in the reactor.

b. System with reactor simulator only. A reduced simulator is obtained if only the neutron kinetics equations and Eqs. (1), (6), (11), and (12) are instrumented. The emitter and thimble thermocouples are used to define the missing temperatures and the experimental heat rejection loop is taken as representative of a prototypical plant. This is a useful approximation that requires considerably less analog computer equipment. The simulator has been implemented so that by activating a few switches either the complete system described in Subsection 3-a or the reduced version may be used.

c. Simplified reactor simulator. A much simplified reactor simulator is obtained if only the neutron kinetics equations and Eq. (1) are instrumented. The emitter thermocouple is assumed to measure the model reactor diode emitter temperature, and the thimble thermocouple is assumed to measure the temperature of the reactor structure on the heat rejection side. These three temperatures are used to define the internal reactivity feedback. If the nuclear fuel is UC, the emitter thermocouple is taken to measure the nuclear fuel temperature, and Eq. (6) is instrumented assuming a fixed collector temperature. The use of a reactor simulator simplified in this manner is supported by the analytical results obtained. They indicate that the reactivity feedback to the neutron kinetics is mainly determined by a prompt effect from the fuel-emitter structure of small magnitude, and by a delayed larger magnitude effect determined by overall core thermal expansion. Several runs have been made with the simplified reactor simulator, both with and without a control system in the loop. A representative case is described in Ref. 3.

4. Conclusions

The methods employed to design a reactor simulator for the thermionic diode kinetics experiment have been discussed. The equations programmed on on-line analog computers were described. The fully extended simulator

requires a large amount of computing equipment (in fact, all of the available analog computers); therefore, two reduced versions were developed. The simplest and most approximate version has the advantage that it can be instrumented on an analog computer at the location of the experimental assembly. The other versions require linkage to the Analog Computing Facility. The linkage and programming have recently been completed, and the next phase will be the execution of experimental runs.

The simulator design was largely dictated by the computing equipment available. Other more precise methods could have been applied, particularly if a hybrid facility had been available. However, judging from results obtained to date, it may be concluded that the experimental assembly is both an adequate and flexible tool for investigation of thermionic reactor dynamics.

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XXII. Liquid Propulsion

PROPULSION DIVISION

A. Injector Hydrodynamics Effects on Baffled-Engine Stability, *R. M. Clayton*

1. Introduction

The work upon which this article is based is an extension of previous resonant combustion studies performed at JPL (Refs. 1-5). The primary objective is one of investigating one aspect of liquid rocket injector design: the relationship of the stabilizing effectiveness of baffles with the required degree of control over the combustion distribution as exercised by control of the hydrodynamic properties of the injector.

An upper limit of hydrodynamic control had been incorporated in injectors used in the previous studies, by the use of long-tube ($L/D = 100$) orifices. These injectors had demonstrated stability to spontaneous disturbances when fitted with adequate baffling. However, the stability of these injectors to induced disturbances (as are imposed for dynamic stability rating) had never been fully established. Because it was planned to use their dynamic stability characteristics as a base for comparison with lesser-controlled injectors, it was necessary to obtain these characteristics as an initial effort. This

has been accomplished for one of three long-tube injectors; the JPL research injector designated RC-1¹.

2. Description of Experiments

The RC-1 injector/18-in.-diam cylindrical combustor assembly is described in SPS 37-56, Vol. III, pp. 204-212 and was operated with the $N_2O_4 + 50/50 [N_2H_4/UDMH]$ (unsymmetrical dimethylhydrazine) propellant combination at a nominal chamber pressure of 100 psia. The baffle configuration consisted of a fixed four-blade array of various axial lengths (1.0, 1.65, 2.15, 2.4, 2.9, 3.4, and 5.4 in.). This array is shown in Fig. 1. The blades were fabricated of $\frac{1}{8}$ -in.-thick aluminum sheet stock and welded to the injector face and to a cylindrical centerpiece. The blades were fitted so that no significant end gap (a few thousandths of an inch) existed at the chamber wall.

¹The dynamic stability of an annular-chamber version of this injector (RC-1AN) is discussed in SPS 37-55, Vol. III, pp. 245-257. All cases of resonance for these engines have been the result of discrete, identifiable initial disturbances occurring either from bombs, pops (high-amplitude combustion disturbances), or start transients. Transitions to resonance from ordinary low levels of combustion noise have not been observed even without baffles; i.e., the engines exhibit so-called linear stability.

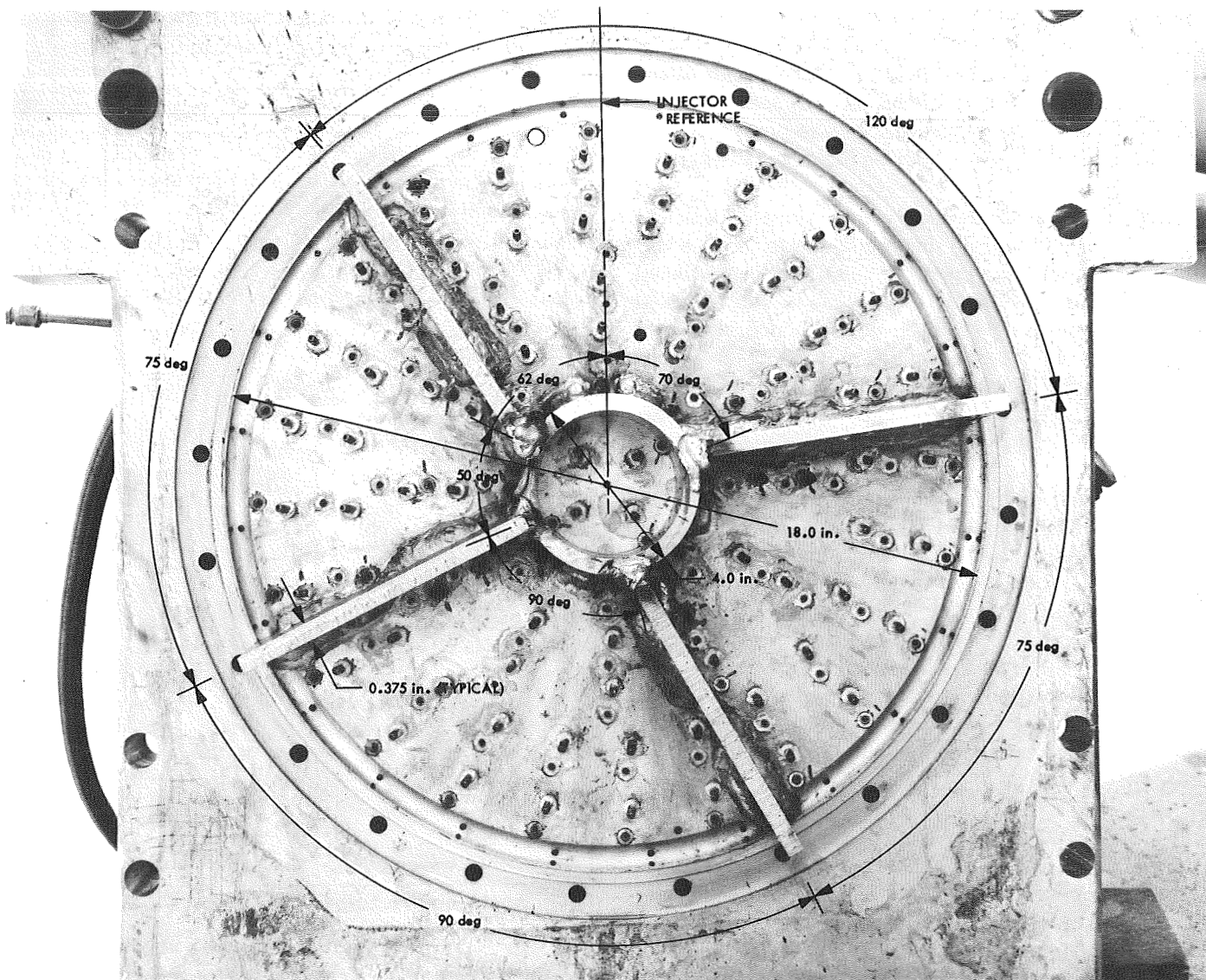


Fig. 1. Baffle array for injector RC-1

The dynamic stability of the engine was evaluated with an explosive-bomb technique. The effectiveness of each length baffle was tested by imposing bomb discharges of various sizes (2, 6.9, and 13.5 gr of pentaerythritol tetranitrate (PETN) in the form of electrically initiated blasting caps²) from various chamber locations. In practice, 5 or 6 bombs of a selected size were discharged sequentially, nominally 100 ms apart, during each engine firing. Total firing duration was ~ 2 s.

Figure 2 schematically shows the various bomb locations relative to the chamber and baffle surfaces. When the

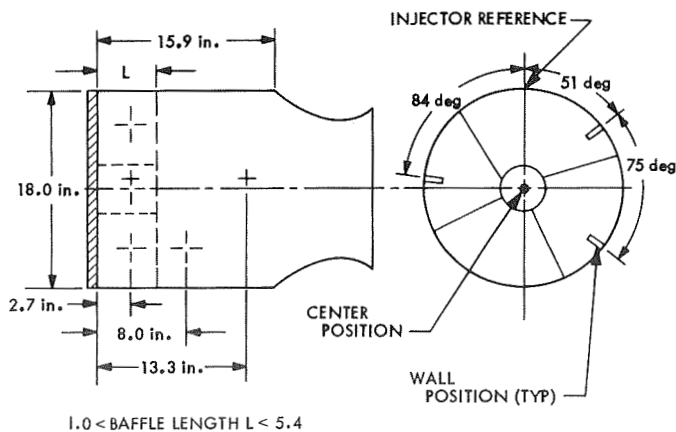


Fig. 2. Positions used in dynamic stability experiments

²2 gr, E-94 cap; 6.9 gr, E-1A cap; 13.5 gr, E-83 cap; manufactured by E. I. DuPont de Nemours and Co., Wilmington, Del.

bombs were installed on the chamber wall, the centroid of the blasting cap was located on a chamber radius ~ 1 in. from the inner wall. When installed at the center of the injector face, the centroid of the cap was located 2.7 in. from the face. A Micarta sleeve served to protect the cap from the combustion gases until the bomb was discharged.

Except for a few early firings, the sequential order of bomb discharge was from the nozzle-end to the injector-end locations, with the bomb at the injector center being discharged last. When the baffle length for marginal stability had been determined, and sustained resonance was expected for shorter baffles, only two 2-gr bombs located near the injector were utilized. (Below marginal length, any size bomb located in the upstream half of the chamber would initiate resonance.)

Provisions for multiple Photocon and Kistler measurements were available for all firings. Pertinent location information is specified as the results are presented. The physical and performance details of these systems are described in Ref. 2. Only Photocon measurements were used for the stable firings because those run durations were too long for the uncooled Kistlers. However, for the shorter duration resonant firings, arrays of both the Photocons and Kistlers were utilized to determine the pressure amplitude distribution in the baffle and chamber cavities.

To ascertain if injector operating parameters influenced baffle effectiveness, three different main mixture ratios r_m were utilized while nominally constant flow conditions from the boundary elements were maintained. Table 1 lists values for several pertinent hydraulic variables associated with these operating conditions for the nominal 100-psia chamber pressure, with performance variations also shown for reference. The choice of the particular r_m values was based on achieving a wide range of the spray mixing uniformity parameter

$$\psi = \left[1 + \frac{(\rho V^2 d)_f}{(\rho V^2 d)_{ox}} \right]^{-1}$$

described in Ref. 6. It was thought that grossly off-optimum injector operation might decrease the axial combustion gradient sufficiently to alter the baffle length required for marginal stability.

Finally, this engine characteristically produces random, high-amplitude combustion disturbances (pops) whose occurrence is a function of only the boundary flow operating conditions (SPS 37-56, Vol. III). This behavior is

Table 1. Summary of nominal injection and engine operating conditions for RC-1 dynamic stability tests

Parameter	Mixture 1	Mixture 2	Mixture 3
Main flow mixture ratio r_m	1.39	2.11 ^a	3.24
Main flow mixing uniformity parameter ^b ψ	0.3	0.5	0.7
Overall mixture ratio (includes boundary flow)	1.37	2.00	2.90
Main injection velocity ^c V , ft/s			
Oxidizer ($\rho = 91$ lbm/ft ³)	46.0	58.0	64.0
Fuel ($\rho = 56$ lbm/ft ³)	107.0	86.0	64.0
Main injection pressure drop (manifold-to-chamber), psi			
Oxidizer	76.0	120.0	146.0
Fuel	200.0	129.0	72.0
Experimental relative combustion performance ^d η_c	0.965	0.97	0.99
Other engine constants			
Number of unlike-doublet elements			
Main ($L/D = 100$)		84	
Boundary ($L/D = 30$)		24	
Chamber characteristic length, in.		41.3	
Throat area, in. ²		127.7	
Chamber contraction ratio		2.0	
Chamber pressure, psia		100.0	
^a Design value for optimum mixing uniformity. ^b $\psi \equiv \left[1 + \frac{(\rho V^2 d)_f}{(\rho V^2 d)_{ox}} \right]^{-1} = 0.5$ for optimum mixing. ^c Main orifice diameter d , in. (Oxidizer = 0.142; Fuel = 0.101). ^d $\eta_c \equiv \frac{\text{Measured characteristic velocity}}{\text{Theoretical equilibrium characteristic velocity}}$			

maximized at the design boundary flow mixture ratio $r_b = 1.27$. Because this kind of spontaneous disturbance is frequently exhibited by contemporary liquid propellant combustors, and because they are known to be effective for initiating resonant combustion, the present experiments (with the exceptions noted below) were intentionally conducted to impose spontaneous, as well as artificial, disturbances on the combustion system.

3. Results and Discussion

The overall results that show the dynamic stability characteristics of the engine are summarized in Table 2.

Table 2. Summary of the dynamic stability characteristics of the RC-1/18-in. engine with various baffle lengths

Parameter	Results											
Main mix, ratio and (ψ)	1.39 (0.3)		2.11 (0.5)								3.24 (0.7)	
Disturbance ^a	S	13.5	S		2		6.9		13.5		S	13.5
Boundary flow	Yes	Yes	Yes	No	Yes	No	Yes	No	Yes	No	Yes	Yes
Baffle length, in. ^b												
1.00			●	●								
1.65					●	●						
2.15			●		●	○			●	●		
2.40		●	●		○				○		●	○
2.90	○	○	○		○		○		○		○	○
3.40	○	○	○						○		○	○
5.40	○	○							○		○	○

^aS denotes spontaneous disturbance, numerals denote bomb size.

^bUnstable, ●; marginal, ○; stable, ○.

The engine was classified stable if, under the particular operating condition, all disturbances damped with no discernible tendency for resonance at a sustained amplitude. All firings with baffles greater than 2.9 in. long were found to be stable.

Marginal stability was assigned to the engine if any discernible sustained resonance occurred intermittently during the firing. This classification characterized the effectiveness of the 2.4-in.-long baffle in the case where pops triggered bursts of resonance ~100 ms in duration and where the 13.5-gr bomb did likewise for the low ψ condition (Table 2).

The unstable classification was assigned to any firing that exhibited continuous resonance from the time of its inception through the remainder of the firing. Except for one operating condition, this classification characterized the engine for baffle lengths less than 2.4-in. long. The exception was noted when 2-gr bombs failed to induce resonance with the boundary flow deleted, an operating condition previously observed to produce significantly lower amplitude resonance when no baffles were present (SPS 37-49, Vol. III, pp. 223-236).

Damping time and bomb size are frequently used as measures of stability margin. Plots of these variables are

shown for a particular set of firing conditions in Fig. 3. Damping time as used herein is defined as the total time (to the nearest millisecond) for chamber pressure (Photicon measurements at the 3.95-in. station) to return to its pre-disturbance appearance on a high time-resolution oscillograph record. This time was found to vary somewhat with bomb position, with the injector-center and nozzle-entrance positions generally exhibiting the shortest damping time. However, the average of the individual damping times is assumed to be a measure of the gross damping behavior of the engine. This mean value is shown as a function of the three bomb sizes by the curve in Fig. 3(a), where it is seen that the larger charge sizes produced longer damping times. The dispersion of the individual damping times around the average is also shown in Fig. 3(a) in terms of standard deviation. A plot of the pulse amplitude variation is shown in Fig. 3(b) where the average pulse amplitude also increased with bomb size. Finally, these data are crossplotted in Fig. 3(c), which shows that the average damping time varied approximately linearly with the average pulse amplitude for a fixed (2.9 in.) baffle length. This suggests that, for this engine, damping time is highly correlated with the time constant of the chamber-nozzle combination.

Interestingly, the damping behavior for the pops exhibited during this same group of firings showed a close

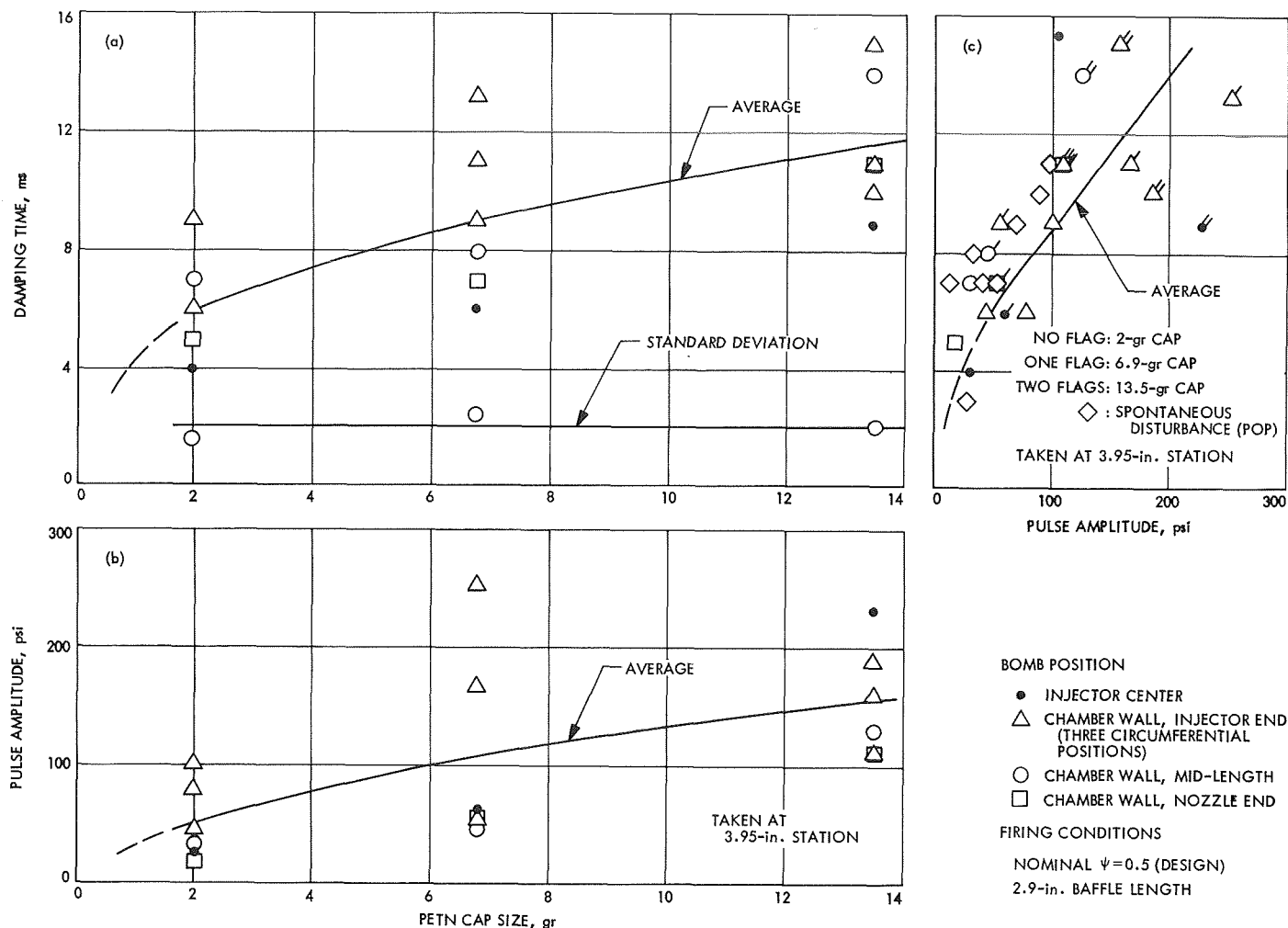


Fig. 3. Pulse amplitude and combustor damping characteristics for various size bombs, measured at the 3.95-in. chamber station

similarity to that for the bombs as indicated by the diamond-shaped data points in Fig. 3(c). This behavior also indicates that the relatively small size bombs utilized in these experiments closely simulate the natural disturbance. Moreover, the results shown in Table 2 verify the similarity between pops and bombs for initiating the transition to sustained resonance.

The average damping time and dispersion, and the average initial-pulse amplitude vs baffle length for the three values of nominal ψ are summarized in Fig. 4. It can be seen that amplitude clearly increased (for a fixed 13.5-gr size) as baffle length decreased, but that the average damping time remained nearly constant until the length for marginal stability was reached. At this length, a slight increase in damping time was observed but the more pronounced change appeared to be an

increase in dispersion of the times for the various bomb locations. In any event, the variations in damping time were small for all the stable firings. This parameter, therefore, does not appear to be a good measure of margin of stability for injectors with large hydraulic impedance.

Baffle effectiveness was not clearly influenced by the injector operating condition ψ . This is interpreted to be evidence that, for this particular combustion system, changes in that operating condition do not appreciably alter the axial combustion distribution. The fact that the steady-state relative combustion performance η_c (Table 1) varied only about $\pm 1.5\%$ over the broad range of ψ tested tends to verify this.

With a baffle length of less than 2.4 in., the engine was unstable to pops and bombs (with the exception noted in

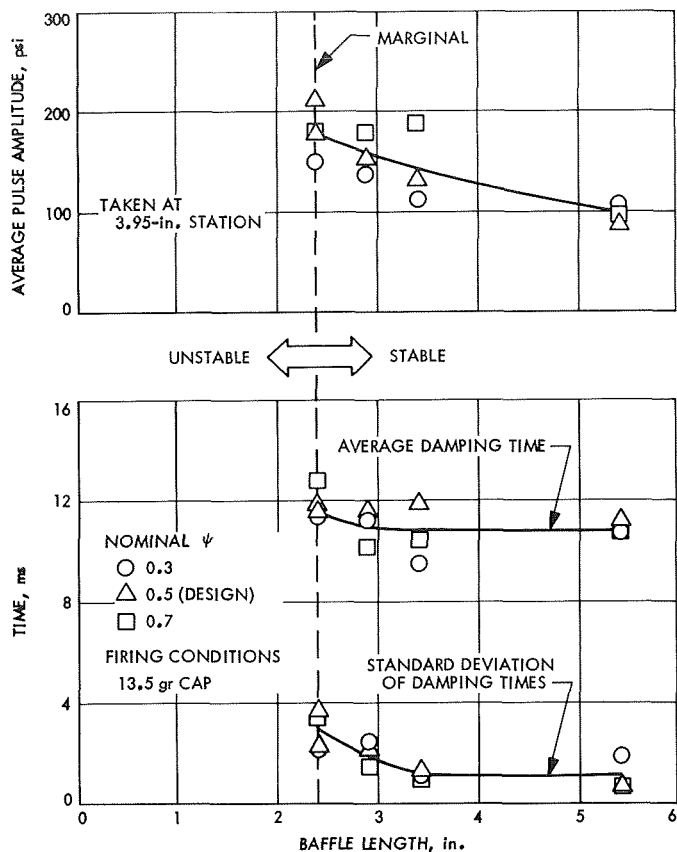


Fig. 4. Pulse-amplitude and combustor damping characteristics for various baffle lengths and constant 13.5-gr bomb size; measurements made at 3.95-in. chamber station

Table 2, where the 2-gr charge size failed to induce resonance with the boundary flow deleted). For all baffle lengths between 1 and 2.4 in., the mode of resonance was always a transverse-wave motion (single wave) with the so-called antinodes occurring on the chamber diameter approximately bisecting the two 75-deg width baffle cavities (Fig. 1). However, for the 1-in.-long baffle, this transverse wave system rotated about the chamber axis at the rate of ~ 35 times per second; i.e., the location of the nodal diameter slowly rotated in a constant clockwise direction.

Both the period (round-trip travel time across the chamber) and the amplitude of the transverse wave varied with the baffle length. Figure 5 shows the variation in period in terms of frequency vs baffle length, with and without the boundary flow. This trend of decreased frequency with increased baffle length is consistent with other experimental results (Ref. 7 and SPS 37-55, Vol. III) and at least one theoretical treatment

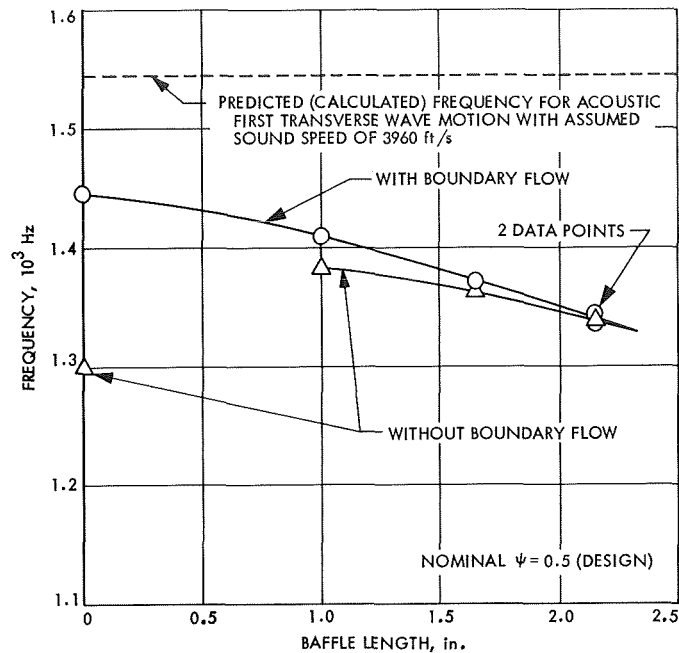


Fig. 5. Variation of predominant (transverse) frequency with baffle length

(Ref. 8). It evidently occurs even with high-amplitude, nonlinear waves when the wave systems in the baffle cavities and in the downstream chamber cavity are gas dynamically coupled. In the case of the present engine, it is believed that the chamber-cavity wave system is the controlling system (sustained by energy release downstream of the baffles) and that this motion is driving the baffle cavity wave system. For other combinations of combustion distributions and relative cavity geometries, wave systems supported by energy release within the baffle cavity may control uncoupled oscillations of the main chamber cavity. In this case, the predominant frequency of the engine resonance may be due to large-amplitude, transverse waves in the baffle cavity(ies) and wave periods may be unaffected by the length of the baffle³, although some limiting ratio of length-to-width cavity dimensions may be required to support such a wave.

Resonance amplitude variations as a function of chamber wall position for several lengths of baffles are shown in Fig. 6 for near design flow conditions. The pressures were measured with Kistler transducers located near the antinode of the wave system; therefore, they are maximum amplitudes for each baffle condition. For comparison, the amplitude distribution for the unbaffled chamber

³This situation is believed to have been encountered in recent experiments with a different engine and baffle configuration.

(SPS 37-49, Vol. III) is also shown. The latter distribution was obtained for the fully developed rotating detonation-like wave, characteristic of the unbaﬄed engine. In addition to preventing this severely destructive mode from developing, the baﬄles progressively reduced the amplitude of the standing mode as their length was increased toward the length to stabilize the engine. Analogous results with the boundary flow deleted showed that resonance amplitudes are very similar. This is in contrast to the unbaﬄed engine where the spinning wave amplitude is decreased by a factor of two in the absence of the boundary flow.

The hump in the pressure distribution at about the mid-chamber station (Fig. 6) is not understood at this time. It suggests a possible longitudinal wave effect, but no evidence of such a wave is present in the phase or period measurements from the pressure records.

4. Summary and Conclusions

a. Baffle length. The baffle length (four-blade array) to provide marginal dynamic stability for the RC-1, 18-in. combustor is 2.4 in., independent of injector operating conditions. A high margin of stability is provided with 2.9- to 5.4-in.-long baﬄes (the maximum length tested).

b. Damping time. For the RC-1 injection scheme, which is well decoupled (hydrodynamically) from chamber pressure variations, damping time does not vary significantly with baffle length or injector operating conditions for a fixed-size initial disturbance. A slight increase in damping time and its dispersion for individual disturbances was noted at the baffle length for marginal stability, but the most significant indication of marginality was the presence of intermittent bursts of resonance initiated by bombs or pops.

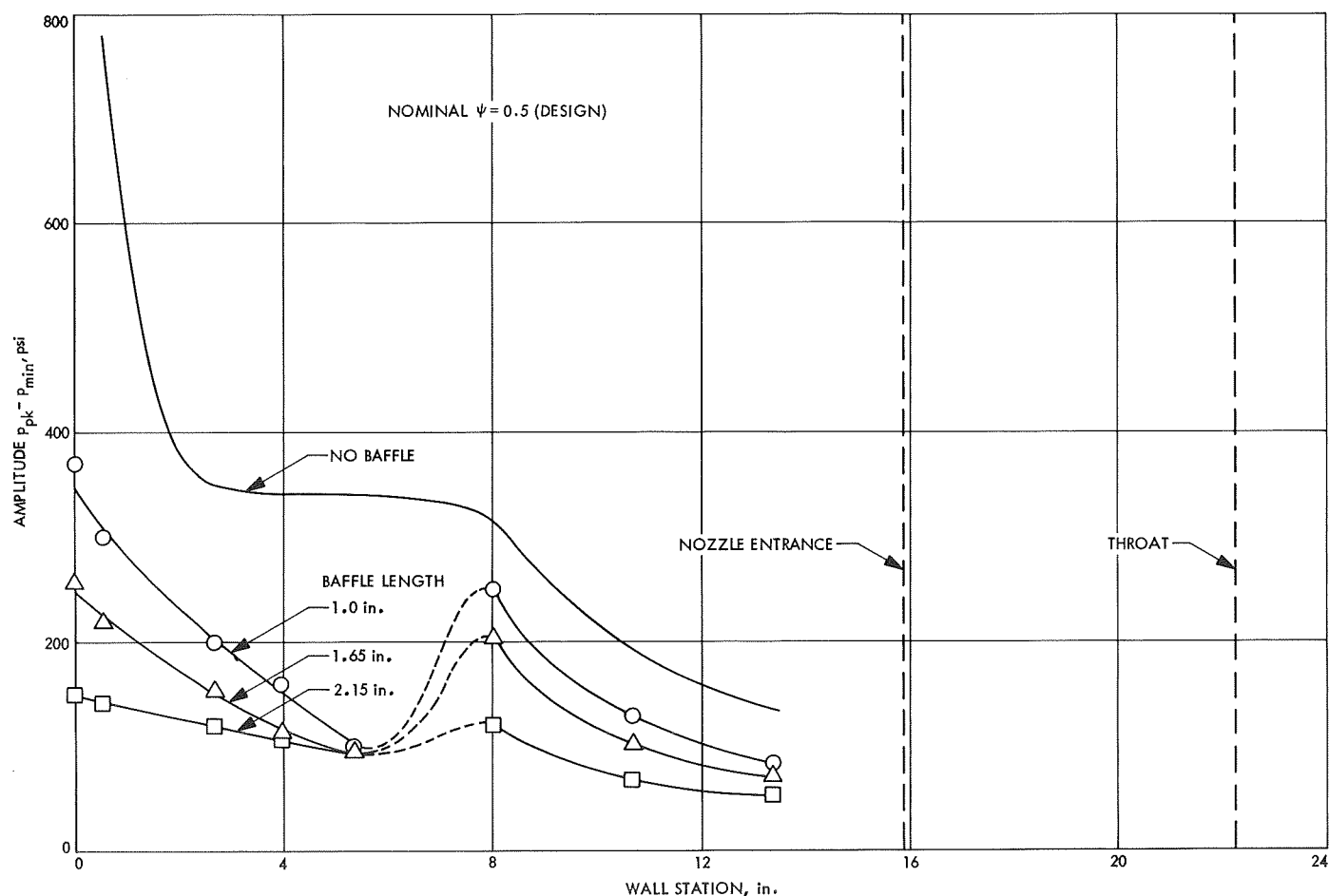


Fig. 6. Pressure distribution at wall of chamber for various baffle lengths; measurement positions coincide with pressure antinode of transverse wave

c. *Baffle effectiveness dependency.* The above characteristics and the progressive decrease in resonance amplitude for longer baffles are believed to show the dependency of baffle effectiveness on the axial combustion distribution rather than on gas dynamic damping. They are also believed to support a qualitative design criteria for maximizing baffle effectiveness: Sufficient hydrodynamic control of the injected flow must be provided so that the spatial distribution of mass and energy sources in the early reaction region is reproducibly compact and steadily maintained (relative to the baffle geometry) under perturbed combustion conditions.

d. *Natural disturbance simulation.* Relatively small bombs (2–10 gr PETN) simulate the typical natural disturbance exhibited by the RC-1 combustor and similar combustion systems. Either kind of disturbance is equally effective for initiating resonance.

e. *RC-1 results.* These RC-1 results will provide a good basis for comparison with results from planned experiments with lesser-controlled injection schemes where the combustion distribution is expected to be less reproducible and more spatially responsive (hence, less steady) to chamber pressure perturbations.

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B. Nozzle Exhaust Plumes in a Space Environment, W. Simon

In the space environment, the interaction of nozzle exhaust gases with spacecraft components causes problems in such diverse areas as flight mechanics, materials, and structures, and communications. Plumes impose or alter forces and loads on spacecraft surfaces. Materials and structures can be degraded by thermal, chemical, and mechanical interaction with the plume. In some instances, plumes can also influence command, control, guidance, and telemetry systems.

Presently, techniques such as those based on the method of characteristics are available to predict nozzle exhaust plume flow fields. They are generally based on the assumption that the nozzle exhaust flow field is inviscid and that the boundary layer that may exist is so small as to be insignificant and can be neglected without impairing the accuracy of the analysis. This assumption, however, is not valid in the case of many small nozzles typical of those used aboard spacecraft today. For these nozzles, the boundary layer flow can occupy 15% or more, 50% not being uncommon, of the nozzle internal flow area. The gases in the boundary layer are at lower Mach number than those in the nozzle core due to their viscous interaction with the nozzle wall. Therefore, the exhaust gases in the boundary layer will cause the plume to spread further than an analysis based on inviscid flow will indicate. There are no available prediction methods for complete plumes from nozzles with substantial boundary layer flow. Experimental data are very limited and for many conditions of interest are nonexistent. The Liquid Propulsion Section has, therefore, undertaken an experimental and analytical program to predict and characterize the nature of exhaust plumes from nozzles with relatively large internal boundary layer flows. This

program is a necessary step to increase the accuracy of the methods used to predict plume-spacecraft interaction effects for present and future flight projects.

The main element of the program is the experimental effort utilizing the unique Molsink facility at JPL. This is being supplemented by university research grants and advanced technology contracts with industry. The University of Southern California is studying, from an analytical standpoint, the expansion of boundary layer gases around the edge of a nozzle and also condensation effects in nozzle exhaust plumes. Their efforts will be correlated with the experimental data obtained by JPL. The Lockheed Missiles and Space Company is currently completing a contract to improve their plume prediction computer program. The results from this program will be used in conjunction with the experimental test data.

The scope of the complete JPL program is shown schematically in Fig. 7. Because of limited space and problems of increasing available "feed-throughs" in the Molsink, the basic nozzle flow characteristics are measured in a separate vacuum facility, in this case the Space Simulator. To measure these characteristics, it is only necessary to have a facility that will allow the nozzle to flow full since plume measurements are not required. It was necessary to calibrate the pressure vs nozzle flow rate characteristics of the Molsink as well as of the other vacuum facilities before it was practical to design and manufacture the test nozzles. The facilities were calibrated using nitrogen gas. Calibration of the space simulators at JPL⁴ indicated that the 25-ft Space Simulator was satisfactory for taking nozzle thrust and exit flow field measurements, but not for plume measurements. The Molsink calibration indicated that plume measurements could be taken and background pressures maintained at a minimum equivalent altitude of approximately 80 mi (10^{-5} torr), indefinitely, with nitrogen flow rates as high as 3.3 g/s.

A total of 24 nozzles has been designed and fabricated. The complete set of nozzles will allow the investigation of the influence of all major parameters on the characteristics of nozzle exhaust plumes. The nozzles range in area ratio from 15:1 to 240:1, with the majority being 60:1. Most of the nozzles are conical and, therefore, have a positive pressure gradient, but contoured nozzles with an adverse pressure gradient are also included. The conical nozzles have wall half angles as low as 15 and as high as 40 deg. Some of the nozzles have no exit base area while

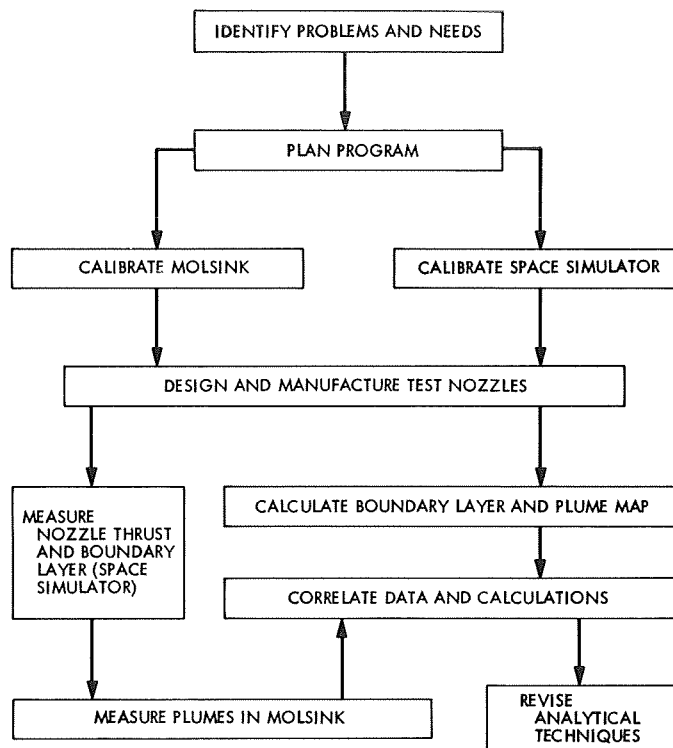


Fig. 7. Nozzle flow and vacuum exhaust technology program

others, internally identical, have base areas as large as 50% of the exit area. Throat diameters of the nozzles are 0.100, 0.200, and 0.500 in. Through suitable choice of nozzle chamber pressures, boundary layer thicknesses of well over 50% of the nozzle flow area are established. The five nozzles comprising the first set have been installed and tested in the JPL 25-ft Space Simulator. Thrust, wall pressure profiles, and exit plane pressure surveys were made; these data are now being analyzed.

Subsequent to these tests, one of the nozzles was installed in the Molsink where the plume characteristics were measured using an electron beam system and a quartz crystal micro-balance system. The instrumentation systems were designed and built for this program but can be adapted for nozzle plume tests of small nozzles of actual spacecraft propulsion systems. The electron beam system uses a collimated beam of electrons that is projected into the plume, resulting in an illuminated path whose intensity and spectral characteristics are related to gas density, composition, and temperature. The fluorescence of the beam is measured with a photomultiplier whose output is normalized by the electron beam current and calibrated against known gas density. The movements

⁴For detailed results of this calibration and of other tests, see Chapter XVII-A.

of both the beam and photomultiplier allow the determination of the density of any desired point in the plume. The quartz crystals of the quartz crystal micro-balance system are mounted at various locations on the walls of the Molsink. They detect the mass flow per unit area by

a change in their frequency. The combination of the measurements taken in the Molsink and the Space Simulator allow a complete determination of the nozzle and plume characteristics. Molsink tests to develop operational methods and check out the system have been initiated.

XXIII. Systems Analysis Research

MISSION ANALYSIS DIVISION

A. Brute-Force Least-Squares Estimation,

W. G. Melbourne and C. B. Solloway

1. Introduction

In estimation problems, a *brute-force* method is occasionally employed to determine the minimum sum-of-the-squares estimate of a parameter. Suppose that $p + r$ parameters are to be estimated. The values of the r parameters z (r usually equal to 1), are fixed, while the p parameters x are adjusted by the usual weighted least-squares procedure to obtain the weighted least-squares estimate $\hat{x}$. Such an approach may be useful when, for example, the partial derivatives of the observable with respect to z are not available. Also, z is weakly determined and gives rise to a near-singular normal matrix. In this case, it is desirable to examine possible structure on the residuals as a function of the choice of values for z , etc. The weighted sum of the squares of the residuals after the least-squares solution Q is evaluated. The values of the r parameters are adjusted to neighboring values and the least-squares process is repeated, giving a new value of $\hat{x}$ and Q . This procedure is continued in a systematic fashion until the minimum of Q is found with respect to z . The values of $\hat{x}$ and $\hat{z}$ that correspond to the minimum value of Q are equivalent to the values that would have been obtained from a least-squares process involving the simultaneous estimate of x and z .

The inverse of the matrix describing the dependence of the quadratic form Q on the parameters z is the covariance matrix of z . This matrix is obtained from the

$r \times r$ submatrix of the covariance matrix of x and z that would have been obtained from a simultaneous solution. The exact equivalence of the covariance matrix of z with the matrix describing the quadratic form Q does not appear to be widely recognized. The purpose of this discussion is to present a straightforward proof of this equivalence.

2. Proof

Let y be the $N \times 1$ vector describing the difference between the N observations O and the N predicted observations C based on the *a priori* values chosen for the p and r parameters, $N > p + r$; thus, $y = O - C$. It is assumed that the *a priori* values of the parameters to be estimated are sufficiently close to the true values that linear theory may be assumed. This is not a crucial assumption, because, in the nonlinear case, the concern would be with limiting values (after successive iterations) that reduce to the linear results presented here. Let x and z be the small corrections to the *a priori* values to be estimated. That is, letting X and Z be the true values of the parameters and X_a and Z_a be the *a priori* values, then

$$\left. \begin{aligned} x &= X - X_a \\ z &= Z - Z_a \end{aligned} \right\} \quad (1)$$

The vector y is then given by

$$y = Ax + Bz + n \quad (2)$$

where A and B are $N \times p$ and $N \times r$ partial derivative matrices and n is the noise on the observations assumed here to have a gaussian distribution with mean zero and an $N \times N$ covariance matrix Λ_o . It is convenient to suppress Λ_o in the subsequent discussion by multiplying Eq. (2) from the left by $\Lambda_o^{-1/2}$ and redefining the variables so that:

$$\left. \begin{aligned} \Lambda_o^{-1/2} y &\rightarrow y \\ \Lambda_o^{-1/2} A &\rightarrow A \\ \Lambda_o^{-1/2} n &\rightarrow n \\ \Lambda_o^{-1/2} B &\rightarrow B \end{aligned} \right\} \quad (3)$$

By this normalization it is seen that the expected value of nn^T , $E[nn^T] = \Lambda_o^{-1/2} \Lambda_o \Lambda_o^{-1/2} = I$, and the weighted least-squares process reduces to a simple least-squares process. All subsequent expressions may be rewritten for the weighted least-squares problem by use of the transformations in Eq. (3).

It is assumed that $(A^T A)^{-1}$ and $(B^T B)^{-1}$ exist (i.e., A is of rank p and B of rank r). The case where these normal matrices are singular and also the case where one uses the pseudo-inverse of A with rank less than p is not discussed here. The least-squares estimate of x for a fixed z is given by

$$\hat{x} = (A^T A)^{-1} A^T y \quad (4)$$

The term $\hat{x}$ may be written in terms of its true value x and the corrupting effects due to the true (but unknown) values of the r parameters Z and the noise n by replacing y with Eq. (2) to obtain

$$x = \hat{x} + (A^T A)^{-1} A^T (Bz + n) \quad (5)$$

Note that $\hat{x} = \hat{x}(z)$; that is, the estimate $\hat{x}$ depends on the *a priori* value assumed for those parameters not estimated. The error in $\hat{x}$ due to the error z resulting from the chosen *a priori* value Z_a is given by $(A^T A)^{-1} A^T Bz$. The matrix $(A^T A)^{-1} A^T B$ provides the sensitivity of errors in $\hat{x}$ to errors in Z_a ; the value $\hat{x}$ minimizes the sum of the squares of the residuals whose numerical value is determined from the equation

$$Q = (y - A\hat{x})^T (y - A\hat{x}) \quad (6)$$

The term Q is also a function of z and this may be expressed by the use of Eqs. (2) and (5) in Eq. (6) to obtain:

$$Q(z) = (Bz + n)^T [I - H] (Bz + n) \quad (7)$$

where H is given by

$$H = A(A^T A)^{-1} A^T \quad (8)$$

The projection matrix H is of rank p ; the p nonzero eigenvalues of H are $+1$. For an ensemble of experiments holding z fixed, the expectation of Q is taken to obtain

$$\begin{aligned} E[Q] = \bar{Q} &= z^T B^T [I - H] Bz + E\{n^T [I - H] n\} \\ &+ 2E\{n^T [I - H] Bz\} \end{aligned} \quad (9)$$

Equation (9) (with the use of the statistical properties of n) becomes

$$\bar{Q} = z^T B^T [I - H] Bz + \text{trace} [I - H] \quad (10)$$

or, with the use of the properties of H ,

$$\bar{Q} = z^T B^T [I - H] Bz + N - p \quad (11)$$

The $E\{n^T [I - H] n\}$ term in Eq. (9) is simply the expected contribution to Q of the components of n that are orthogonal to the linear space spanned by the vector x . Hence, for the case of $z = 0$, it is the expected value of the post solution sum of the weighted square residuals. The numerical value of this term is $N - p$ as indicated in Eq. (11). This explains the rationale for dividing the sampled value of Q by $N - p$ to obtain a measure of goodness of fit. The term Q is also a function of z ; it has a minimum when $z = 0$, that is, when the *a priori* choice for Z_a is the true value.

Because N is usually very large, the actual sample value Q should be close to $\bar{Q}$. In practice, the sample value of Q shown in Eq. (6) is plotted for the various values chosen for Z_a and its quadratic behavior defines both its minimum and its curvature at the minimum. For the one-dimensional case, a parabola is often fit to the numerical values of Q as an aid in locating the minimum and its curvature. An alternate but equivalent approach uses the derivative of Q with respect to z and

searches for the zero crossing of this quantity. Differentiating Eq. (6) with respect to z and remembering that both y and $\hat{x}$ are functions of z , the following is obtained:

$$\frac{\partial Q}{\partial z} = 2(y - A\hat{x})^T \left(\frac{\partial y}{\partial z} - A \frac{\partial \hat{x}}{\partial z} \right) \quad (12)$$

which, from Eqs. (2) and (5), becomes

$$\frac{\partial Q}{\partial z} = 2[y - A\hat{x}]^T [I - H]B = 2[Bz + n]^T [I - H]B \quad (13)$$

Equation (13) also follows directly from Eq. (7): Q will have a minimum with respect to z when the *a priori* choice for Z_a is such that

$$z^* = -\{B^T[I - H]B\}^{-1} B^T[I - H]n \quad (14)$$

This may be verified by setting Eq. (13) to zero. Equivalently (Eq. 1), the value to be chosen for Z_a that minimizes Q is the true value Z , plus the corrupting effects of n as expressed by

$$\hat{Z}_a = Z - z^* = Z + \{B^T[I - H]B\}^{-1} B^T[I - H]n \quad (15)$$

From the definition of least-squares estimators, it follows that $\hat{x}$ and $\hat{Z}_a$ associated with the minimum value of Q correspond exactly to the estimates $\hat{x}$, $\hat{z}$ obtained from the simultaneous least-squares process,

$$\begin{pmatrix} \hat{x} \\ \hat{z} \end{pmatrix} = \begin{pmatrix} A^T A & A^T B \\ B^T A & B^T B \end{pmatrix}^{-1} \begin{pmatrix} A^T \\ B^T \end{pmatrix} y \quad (16)$$

Further, the matrix $B^T[I - H]B$, which can be seen from Eq. (13) to be $\frac{1}{2}$ the second vector derivative of Q with respect to z , corresponds to the inverse of the $r \times r$ covariance matrix of $\hat{z}$ in the simultaneous least-squares case. That is, let Λ be the covariance matrix of the simultaneous estimate of the $p + r$ parameters. Then Λ is given by:

$$\Lambda = \begin{pmatrix} A^T A & A^T B \\ B^T A & B^T B \end{pmatrix}^{-1} = \begin{pmatrix} \Lambda_{\hat{x}\hat{x}} & \Lambda_{\hat{x}\hat{z}} \\ \Lambda_{\hat{z}\hat{x}} & \Lambda_{\hat{z}\hat{z}} \end{pmatrix} \quad (17)$$

and $\Lambda_{\hat{z}\hat{z}}$ identified as the $r \times r$ submatrix in Λ shown in Eq. (17) is given by:

$$\Lambda_{\hat{z}\hat{z}}^{-1} = B^T[I - H]B = \frac{1}{2} \frac{\partial^2 Q}{\partial z^2} \quad (18)$$

To prove this, let $c = A^T A$, $b = A^T B$, $a = B^T B$; let the inverse of

$$\begin{pmatrix} c & b \\ b^T & a \end{pmatrix}$$

be given by

$$\begin{pmatrix} \gamma & \beta \\ \beta^T & \alpha \end{pmatrix}$$

Then

$$\begin{pmatrix} c & b \\ b^T & a \end{pmatrix} \begin{pmatrix} \gamma & \beta \\ \beta^T & \alpha \end{pmatrix} = \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix} \quad (19)$$

from which

$$\begin{cases} b^T \beta + a \alpha = I \\ c \beta + b \alpha = 0 \end{cases} \quad (20)$$

Hence,

$$(a - b^T c^{-1} b) \alpha = I \quad (21)$$

or

$$\alpha = \Lambda_{\hat{z}\hat{z}} = (a - b^T c^{-1} b)^{-1} = [B^T B - B^T A (A^T A)^{-1} A^T B]^{-1} \quad (22)$$

or

$$\Lambda_{\hat{z}\hat{z}}^{-1} = B^T [I - A (A^T A)^{-1} A^T] B = B^T [I - H] B \quad (23)$$

which was to be demonstrated.

3. Conclusions

Although the equivalence of the estimates $\hat{X}$ and $\hat{Z}$ derived from the brute force and the simultaneous process is obvious by virtue of the definition of least squares, it is an easy exercise to demonstrate this equivalence algebraically. In the same way that Eq. (22) was derived, it also follows that:

$$\gamma = [A^T A - A^T B (B^T B)^{-1} B^T A]^{-1} \quad (24)$$

$$\beta = -(A^T A)^{-1} A^T B \alpha \quad (25)$$

$$\beta^T = -(B^T B)^{-1} B^T A \gamma \quad (26)$$

In the brute-force method, it is recalled (Eqs. 1, 5, 14, and 15) that the estimates for X and Z are given by

$$\left. \begin{aligned} \hat{X} &= X_a + \hat{x}(z^*) \\ \hat{Z} &= \hat{Z}_a \end{aligned} \right\} \quad (27)$$

when a minimum is obtained for Q . Hence, $\hat{Z}_a$ is the *a priori* choice for Z that gives the value z^* shown in Eq. (14) for z . In the simultaneous case, the estimates are given by:

$$\left. \begin{aligned} \hat{X} &= X_a + \hat{x} \\ \hat{Z} &= Z_a + \hat{z} \end{aligned} \right\} \quad (28)$$

where $\hat{x}$ and $\hat{z}$ are obtained from Eq. (16). Let the equivalence in Z be shown first (Eqs. 16, 19, and 2):

$$\begin{aligned} \hat{z} &= (\beta^T A^T + \alpha B^T)y = (\beta^T A^T A + \alpha B^T A)x \\ &\quad + (\beta^T A^T B + \alpha B^T B)z + (\beta^T A^T + \alpha B^T)n \end{aligned} \quad (29)$$

which, from Eqs. (25) and (22), reduces to

$$\hat{z} = z + (\beta^T A^T + \alpha B^T)n \quad (30)$$

It follows from Eq. (28) that

$$\hat{Z} = Z_a + z + (\beta^T A^T + \alpha B^T)n = Z + (\beta^T A^T + \alpha B^T)n \quad (31)$$

From Eq. (15), the brute-force method yields:

$$\hat{Z} = \hat{Z}_a = Z + \{B^T[I - H]B\}^{-1} B[I - H]n \quad (32)$$

This (Eqs. 8, 22, 25) is identical to Eq. (31), proving that the same estimate for Z will result from either method. For x (Eqs. 16, 24, 2, and 27), for the estimate using the simultaneous method, the following is obtained:

$$\hat{x} = x + (\gamma A^T + \beta B^T)n \quad (33)$$

For the brute-force method (from Eq. 5 with the use of the minimizing value $\hat{z}^*$ for z given in Eq. 14):

$$\hat{x}(z^*) = x + (A^T A)^{-1} A^T \{-B(\beta^T A^T + \alpha B^T)n + n\} \quad (34)$$

which (upon using Eqs. 24, 25, and 26), reduces to:

$$\hat{x}(z^*) = x + (\gamma A^T + \beta B^T)n \quad (35)$$

thus proving the equivalence of the estimates from the two methods.

The brute-force method does not give information regarding the cross-correlation properties between $\hat{x}$ and $\hat{z}$ and it gives an optimistic covariance matrix for $\hat{x}$. It is seen from Eqs. (17), (19), and (24) that the true covariance matrix for $\hat{x}$ is given by

$$\Lambda_{\hat{x}}^{-1} = A^T[I - K]A \quad (36)$$

where K is a projection matrix analogous to H and is given by

$$K = B(B^T B)^{-1} B^T \quad (37)$$

However, the covariance matrix for $\hat{x}$ obtained in the brute-force method is simply $(A^T A)^{-1}$ (see Eq. 4). It can be easily shown that $(A^T A)^{-1} \leq \{A^T[I - K]A\}^{-1}$ with the equality holding only when x and z are uncorrelated. The difference between these two covariances depends on the correlation properties between x and z , which cannot be readily assessed without performing the simultaneous solution.

B. Average Access Times for a Magnetic Disk Storage Device, P. Gottlieb and H. Lass

1. Introduction

An important element of computer program running time is the average access time for information to be read from, or written on, a disk storage device. The main component of this access time is the time required for the read-write head to move to the track containing the information requested by the program. This time is directly proportional to the number of tracks that must be traversed. However, this proportionality is not quite linear, and varies from one type of disk to another.

The average number of tracks traversed, and the average travel times for a few typical types of disks have been calculated (Ref. 1) under the assumption that both the locations of the read-write heads and the location of the information requested by the program are selected at random.

This article shows that closed-form analytic expressions can be obtained for the average number of tracks traversed (under the same assumptions of random head

and requested locations). The mean distance traveled along a track has also been computed. These analytic expressions should be useful both for (1) estimating program running time and (2) designing system programs and storage systems.

2. Mean Number of Tracks Traversed for a Single Head

Consider a circular disk with N concentric tracks and one read-write head. If the desired location is not on the track containing the head, then the head will have to move to the appropriate track. It is assumed that the probability of the desired location being on any given track is $1/N$ for all tracks, with the same assumption for the position of the desired location. The probability that the head and desired location are on the same track is $1/N$, and the probability that they are on different tracks is $(N-1)/N$; only the latter case is of interest. Assuming that the head moves in an optimal fashion, it is desired to compute the expected value and the variance for the number of tracks traversed by the head until the desired location is reached.

Thus, the problem is equivalent to the following: Two of N keys placed at random in a row are the select keys. What is the mean number of keys between the select keys? The expected number of tracks traversed is one more than this mean number. Let x be the number of keys up to the first select key, y the number of keys between the select keys, and z the remaining number of keys. Obviously x , y , and z are identically distributed. Furthermore, $x + y + z = N - 2$, so that $E(x) + E(y) + E(z) = N - 2$, yielding $E(y) = (N - 2)/3$. Hence, the mean number of tracks traversed is:

$$\mu = \frac{N-1}{N} \left(\frac{N-2}{3} + 1 \right) = \frac{N^2 - 1}{3N} \quad (1)$$

Equation (1) will be derived by a different method that will enable the variance for the number of tracks traversed to be obtained. The probability that the head and the desired location are exactly k spaces apart is $2(N-k)/N^2$, since the head can be on tracks $1, 2, \dots, N-k$, with the desired location on tracks $k+1, k+2, \dots, N$, respectively, and conversely. Thus,

$$\mu = E(k) = \frac{2}{N^2} \sum_{k=0}^{N-1} k(N-k) = \frac{2}{N^2} \left[N \sum_1^N k - \sum_1^N k^2 \right] = \frac{2}{N^2} \left[\frac{N^2(N+1)}{2} - \frac{N(N+1)(2N+1)}{6} \right] = \frac{N^2 - 1}{3N} \quad (2)$$

$$E(k^2) = \frac{2}{N^2} \sum_{k=0}^{N-1} k^2(N-k) = \frac{2}{N^2} \sum_{k=1}^N k^2(N-k) = \frac{2}{N^2} \left[\frac{1}{6} N^2(N+1)(2N+1) - \frac{1}{4} N^2(N+1)^2 \right] = \frac{1}{6} (N^2 - 1) \quad (3)$$

Hence, the variance is given by:

$$\sigma^2 = E(k^2) - [E(k)]^2 = \frac{(N^2 - 1)(N^2 + 2)}{18N^2} \quad (4)$$

3. Mean Number of Tracks Traversed for K Heads Located on Randomly Selected Tracks

Suppose K heads are located at tracks numbered $\alpha_1, \alpha_2, \dots, \alpha_K$ with $\alpha_1 < \alpha_2 < \dots < \alpha_{K-1} < \alpha_K$, where α_i is random (order statistics). The desired location is on track number β , with β a number chosen at random from $1, 2, \dots, N$. It is assumed that the head nearest the desired location moves directly to this location. If two heads are equidistant from the desired location then one or the other moves to the desired location. If

$$z = \min(|\beta - \alpha_1|, |\beta - \alpha_2|, \dots, |\beta - \alpha_K|) \quad (5)$$

then $\mu = E(z)$ is the mean number of tracks traversed to reach the desired location.

In Fig. 1, let N_1 be the number of tracks up to (but not including) track α_1 , N_2 the number of tracks between track α_1 and track α_2 , etc. Then, N_i ($i = 1, 2, \dots, K+1$) are identically distributed random variables, with

$$\sum_{i=1}^{K+1} N_i + K = N \quad (6)$$

so that

$$E(N_1) = \frac{N-K}{K+1} \quad (7)$$

with a similar result for $E(N_2), \dots, E(N_{K+1})$.

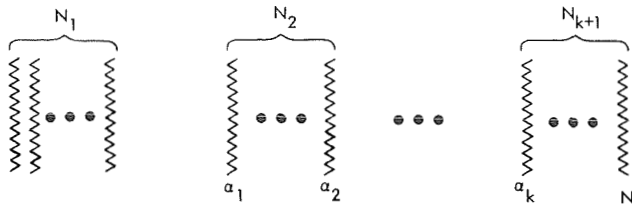


Fig. 1. Schematic representation of track formats

If the desired location is on track 1, with probability $1/N$, the number of tracks traversed by the head located at track α_1 is N_1 . If the desired location is on track 2, the number of tracks traversed by the head located at track α_1 is $N_1 - 1$, etc. For a given N_1 the mean number of tracks traversed is

$$\frac{1}{N} [N_1 + (N_1 - 1) + (N_1 - 2) + \cdots + 1 + 0] = \frac{N_1(N_1 + 1)}{2N} \quad (8)$$

A similar result applies if the desired location is on track β with $\beta \geq \alpha_K$, with N_1 replaced by N_{K+1} . A different result occurs if $\alpha_1 \leq \beta \leq \alpha_2$, or $\alpha_2 \leq \beta \leq \alpha_3$, ..., or $\alpha_{K-1} \leq \beta \leq \alpha_K$. For an examination of $\alpha_1 \leq \beta \leq \alpha_2$ for various values of N_2 , see Fig. 2. For $N_2 = 1$, the probability is $(1/N)$ that one move is required to reach the desired location; for $N_2 = 2$, the mean number of moves is $(1/N) \cdot 1 + (1/N) \cdot 1 = 2/N$; for $N_2 = 3$, the mean number of moves is $(1/N) \cdot 1 + (1/N) \cdot 2 + (1/N) \cdot 1 = 4/N$; for $N_2 = 4$ the mean number of moves is $(1/N) (1 + 2 + 2 + 1) = 6/N$; and for $N_2 = 5$ the mean number of moves is $(1/N) (1 + 2 + 3 + 2 + 1) = 9/N$.

If the sequence 1, 2, 4, 6, 9, 12, 16, 20, ..., is examined, it is found that the first differences are 1, 2, 2, 3, 3, 4, 4, ..., and the second differences are 1, 0, 1, 0, 1, 0, ..., so that

$$\nabla^2 u_r = \frac{1}{2} [1 + (-1)^{r+1}], \quad r = 1, 2, 3, \cdots \quad (9)$$

the solution of which is

$$u_r = \frac{1}{8} + \frac{r}{2} + \frac{r^2}{4} - \frac{1}{8} (-1)^r \quad (10)$$

Hence, for a fixed N_2 , the mean number of moves (tracks traversed) is

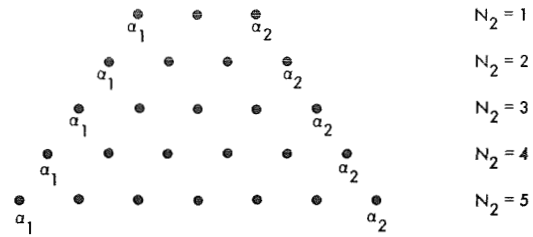


Fig. 2. Examination of $\alpha_1 \leq \beta \leq \alpha_2$

$$\frac{1}{N} \left[\frac{1}{8} + \frac{1}{2} N_2 + \frac{1}{4} N_2^2 - \frac{1}{8} (-1)^{N_2} \right] \quad (11)$$

For fixed $N_1, N_2, \dots, N_{K+1}$ the mean number of tracks traversed until the desired location is reached is

$$\frac{1}{2N} N_1 (N_1 + 1) + \frac{1}{2N} N_{K+1} (N_{K+1} + 1) + \frac{1}{N} \sum_{j=2}^K \left[\frac{1}{8} + \frac{1}{2} N_j + \frac{1}{4} N_j^2 - \frac{1}{8} (-1)^{N_j} \right] \quad (12)$$

Since N_i ($i = 1, 2, \dots, K + 1$) are identically distributed, an average over all possible values of N_i yields the result that the mean number of tracks traversed until the desired location is reached is

$$\begin{aligned} \mu &= \frac{1}{N} E \left[N_1 + N_1^2 + \frac{1}{8} (K - 1) + \frac{1}{2} (K - 1) N_1 \right. \\ &\quad \left. + \frac{1}{4} (K - 1) N_1^2 - \frac{1}{8} (K - 1) (-1)^{N_1} \right] \\ &= \frac{K - 1}{8N} + \frac{K + 1}{2N} E(N_1) + \frac{K + 3}{4N} E(N_1^2) \\ &\quad - \frac{K - 1}{8N} E[(-1)^{N_1}] \end{aligned} \quad (13)$$

To compute the expected values arising in Eq. (13), the probability that $N_1 = j$ must be determined, [e.g., $p_j = \text{Prob}(N_1 = j)$]. Now there are

$$\binom{N}{K}$$

ways of placing the heads on the N tracks, with no two heads on the same track. In order that $N_1 = j$ one head must of necessity be placed on track $\alpha_1 = j + 1$, and the remaining $(K - 1)$ heads must occupy positions on the remaining $N - (j + 1)$ tracks to the right of track $(j + 1)$.

This can be performed in

As a check,

$$\binom{N-j-1}{K-1}$$

$$\sum_{j=0}^{N-K} p_j$$

ways, so that

$$p_j = \frac{\binom{N-j-1}{K-1}}{\binom{N}{K}} \quad (14)$$

is computed. From the combinatorial formula

$$\binom{N-j}{K} = \binom{N-j-1}{K-1} + \binom{N-j-1}{K} \quad (15)$$

$$\sum_{j=0}^{N-K} p_j = \frac{1}{\binom{N}{K}} \sum_{j=0}^{N-K} \left[\binom{N-j}{K} - \binom{N-j-1}{K} \right] \equiv 1 \quad (16)$$

is obtained. Verification of Eq. (7) yields:

$$\begin{aligned} E(N_1) &= \sum_{j=0}^{N-K} j p_j = \frac{1}{\binom{N}{K}} \sum_{j=0}^{N-K} j \left[\binom{N-j}{K} - \binom{N-j-1}{K} \right] = \frac{1}{\binom{N}{K}} \left[\sum_{j=1}^{N-K} j \binom{N-j}{K} - \sum_{j=0}^{N-K-1} j \binom{N-j-1}{K} \right] \\ &= \frac{1}{\binom{N}{K}} \left[\sum_{j=0}^{N-K} j \binom{N-j}{K} - \sum_{j=1}^{N-K} (j-1) \binom{N-j}{K} \right] = \frac{1}{\binom{N}{K}} \sum_{j=1}^{N-K} \binom{N-j}{K} = \frac{\binom{N}{K+1}}{\binom{N}{K}} = \frac{N-K}{K+1} \end{aligned} \quad (17)$$

The term $E(N_1^2)$ is then computed from

$$\begin{aligned} E(N_1^2) &= \sum_{j=0}^{N-K} j^2 p_j = \frac{1}{\binom{N}{K}} \left[\sum_{j=0}^{N-K} j^2 \binom{N-j}{K} - \sum_{j=0}^{N-K-1} j^2 \binom{N-j-1}{K} \right] = \frac{1}{\binom{N}{K}} \left[\sum_{j=1}^{N-K} j^2 \binom{N-j}{K} - \sum_{j=1}^{N-K} (j-1)^2 \binom{N-j}{K} \right] \\ &= \frac{1}{\binom{N}{K}} \left[2 \sum_{j=1}^{N-K} j \binom{N-j}{K} - \sum_{j=1}^{N-K} \binom{N-j}{K} \right] = \frac{1}{\binom{N}{K}} \left[2 \binom{N+1}{K+2} - \binom{N+1}{K+1} + \binom{N}{K} \right] \\ &= 1 + \frac{(N+1)(2N-3K-2)}{(K+2)(K+1)} \end{aligned} \quad (18)$$

Finally,

$$E[(-1)^{N_1}] = \frac{1}{\binom{N}{K}} \sum_{j=0}^{N-K} (-1)^j \binom{N-j-1}{K-1} \quad (19)$$

Making use of the above results yields:

$$\mu = \frac{K-1}{8N} + \frac{N-K}{2N} + \frac{K+3}{4N} \left[1 + \frac{(N+1)(2N-3K-2)}{(K+2)(K+1)} \right] - \frac{K-1}{8N} \frac{1}{\binom{N}{K}} \sum_{j=0}^{N-K} (-1)^j \binom{N-j-1}{K-1} \quad (20)$$

For $K=1$, $\mu = (N^2-1)/3N$ is obtained, which agrees with Eq. (2). For $K=2$,

$$\mu = \frac{5N^2 - 3N - 11}{24N} - \frac{1}{16N^2(N-1)} \{2N + [(-1)^N - 1]\} \quad (21)$$

is obtained. Substituting $N = 6$ into Eq. (21) yields $\mu = 47/45$. This result has been verified by a direct enumeration.

For reasonable values of N , K , it is a simple matter to obtain μ from Eq. (20). No attempt has been made to obtain the variance for the case $K \geq 2$.

Reference

1. Frank, H., *Computing Machinery*, Vol. 16, No. 4, pp. 602-620, Oct. 1969.

XXIV. Systems Analysis

MISSION ANALYSIS DIVISION

A. The 1982 Jupiter Orbiter Mission, *R. K. Wilson*

1. Introduction

As space flight technology advances, the desire to explore and to investigate the outer planets and their natural satellites will increase. Jupiter, because of its unique qualities of large size, low density, rapid spin rate, captured radiation belts, and large number of natural satellites (four are the approximate size of the moon), will be the most potentially profitable target for exploration.

By selecting an arrival date approximately 30 days before conjunction, the approach conditions at encounter can be fixed. The launch period constraint and launch energy available further constrains the selection and design of the interplanetary trajectory. The captured radiation belts and the dynamic orientation of the natural satellites of Jupiter provide a constraint on the final orbit design.

To facilitate a detailed study of Jupiter, an extended coverage in time will be necessary. This capability would be provided by an orbiter mission as opposed to a fly-by mission. To maximize science return, an orbit design to include both imaging and field and particle experiments will be selected. To facilitate this study, the matching conic approach was selected with the launch planet phase consisting of the standard 100-nmi circular parking orbit.

2. Interplanetary Trajectory Selection

A mapping of the absolute minimum launch energy requirements was made for the years 1975–2000. The period of minimum launch energy arises during the 1981–1982 launch opportunity (Fig. 1). This period appears to be feasible from a required lead-time standpoint and state-of-the-art constraints.

A launch energy contour map was constructed for the 1981–1982 launch opportunity (Fig. 2). In addition to the

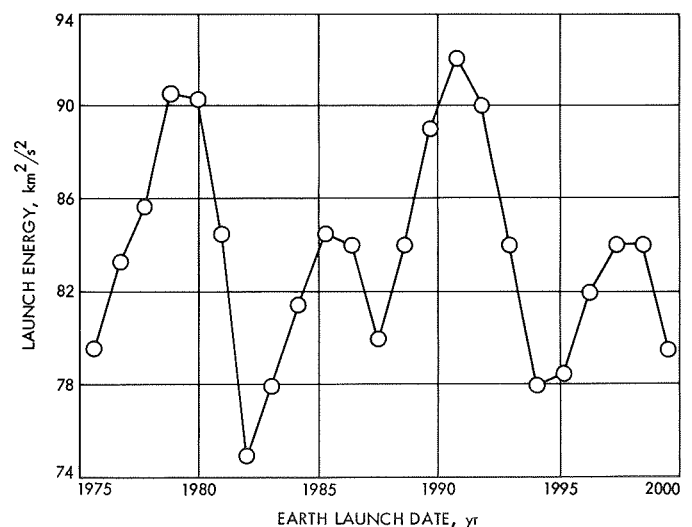


Fig. 1. Absolute minimum energy requirements for earth-Jupiter mission

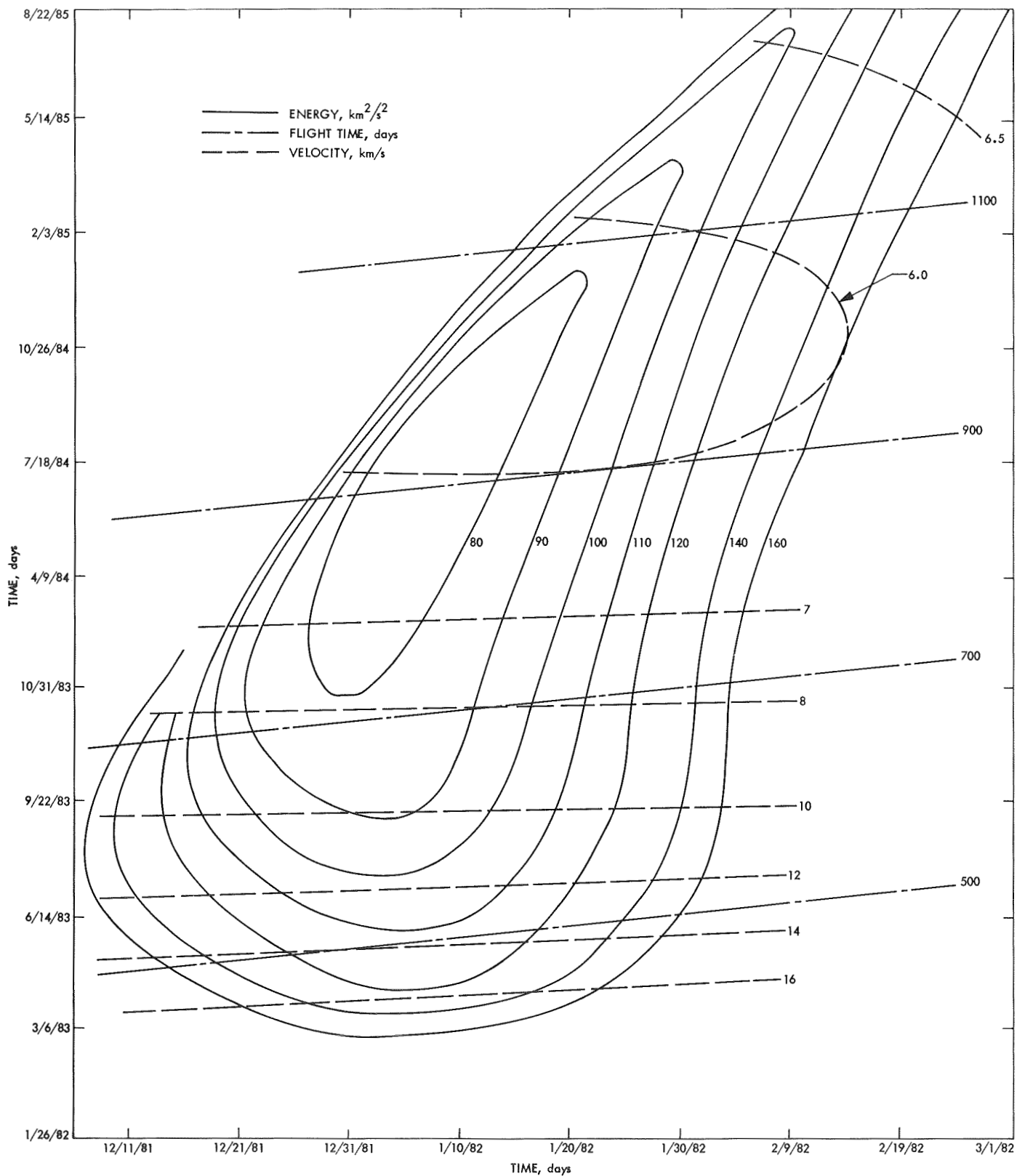


Fig. 2. Arrival vs launch date with overlaid launch energy contours

Table 1. Baseline trajectory parameters

Parameter	Value
Nominal launch	12/25/81 to 1/8/82
Flight time, days	753 ± 8
Arrival date	1/1/84
Launch energy, km^2/s^2	85
Arrival velocity, km/s	7.3414
Encounter communications distance, km	908×10^6

launch energy information, other parameters of flight time and arrival hyperbolic excess velocity are shown.

Interplanetary trajectory selection is further constrained by flight time, launch energy, orbit-insertion velocity change, and launch period parameters. By observing the above constraints, the baseline trajectory was developed as shown in Table 1.

3. Orbit Selection

From the environmental constraint imposed by the radiation belts, the periapsis altitude was set at a minimum within the constraints to provide for imaging experiments. As a result of this low periapsis altitude, a relatively high apoapsis is considered to provide for minimization of the insertion velocity change. This would provide a relatively short period orbit that would be capable of field and particle experiments. A further minimization of the velocity requirement was accomplished by assuming a periapsis-periapsis transfer at insertion with no apsidal rotation of the orbit required. From the parameter study performed on arrival velocity, an apoapsis altitude of 100 radii (Jupiter) was selected. Because of possible encounters with the natural satellites of the planet, the final apoapsis altitude was changed slightly. Table 2 gives the final orbital parameters.

Figure 3 shows the final Jupiter orbit design. From this orbit design, satellite encounter is possible within the first few orbits as shown in Fig. 4.

4. Further Considerations

a. Inclination changes. Should possible orbit plane changes be desired, Fig. 5 illustrates the velocity change ΔV requirements necessary per degree change of inclination.

b. Occultation. The spacecraft position as a function of time (after encounter) has been superimposed upon

Table 2. Final orbital parameters

Parameter	Value
Periapsis altitude, R_J^a	4
Apoapsis altitude, R_J	98.8
Velocity change, km/s	1.477
Period, days	45.39
Inclination (to planet equator), deg	3.296
^a R_J = Jupiter radii.	

the trajectory plane plot (Fig. 6) to provide insight into spacecraft positioning for science requirements.

Because of Jupiter's large mass, large size, rapid mean motion, and close proximity to the sun, occultation periods vary quite rapidly throughout the 3-yr active orbital life of the spacecraft. The occultation period is further altered by secular effects that result from the planet's oblateness. As a result of this oblateness, there is a dynamic rotation of the orbit about its primary. This rotation can be measured in terms of the rate of change of the longitude of the ascending node $\dot{\Omega}$ and the rate of change of the argument of periapsis $\dot{\omega}$.

Figure 6 shows the trajectory plane plots as they have rotated at 6-mo intervals from insertion through the active orbital life of 3 yr. (Each orbit is shown perpendicular to its plane; thus, the ellipse is rotating into the page as well as counter-clockwise.) For Jupiter, the change in longitude of ascending node $\dot{\Omega}$ and argument of periapsis $\dot{\omega}$ are -6.513 and 12.994 deg, respectively.

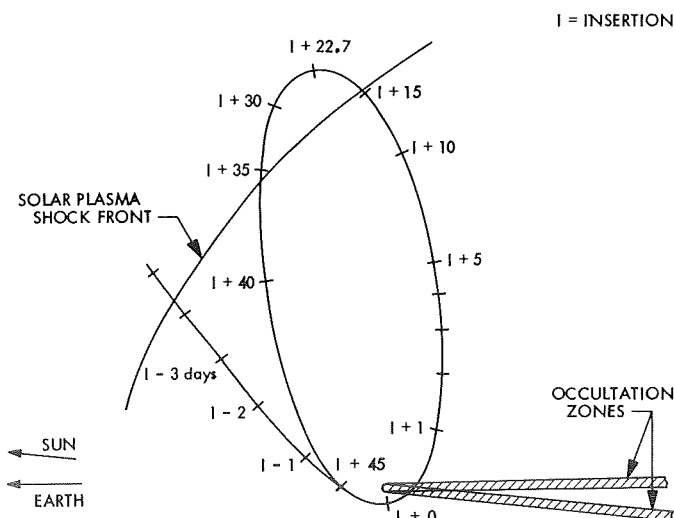


Fig. 3. Typical Jupiter orbiter geometry

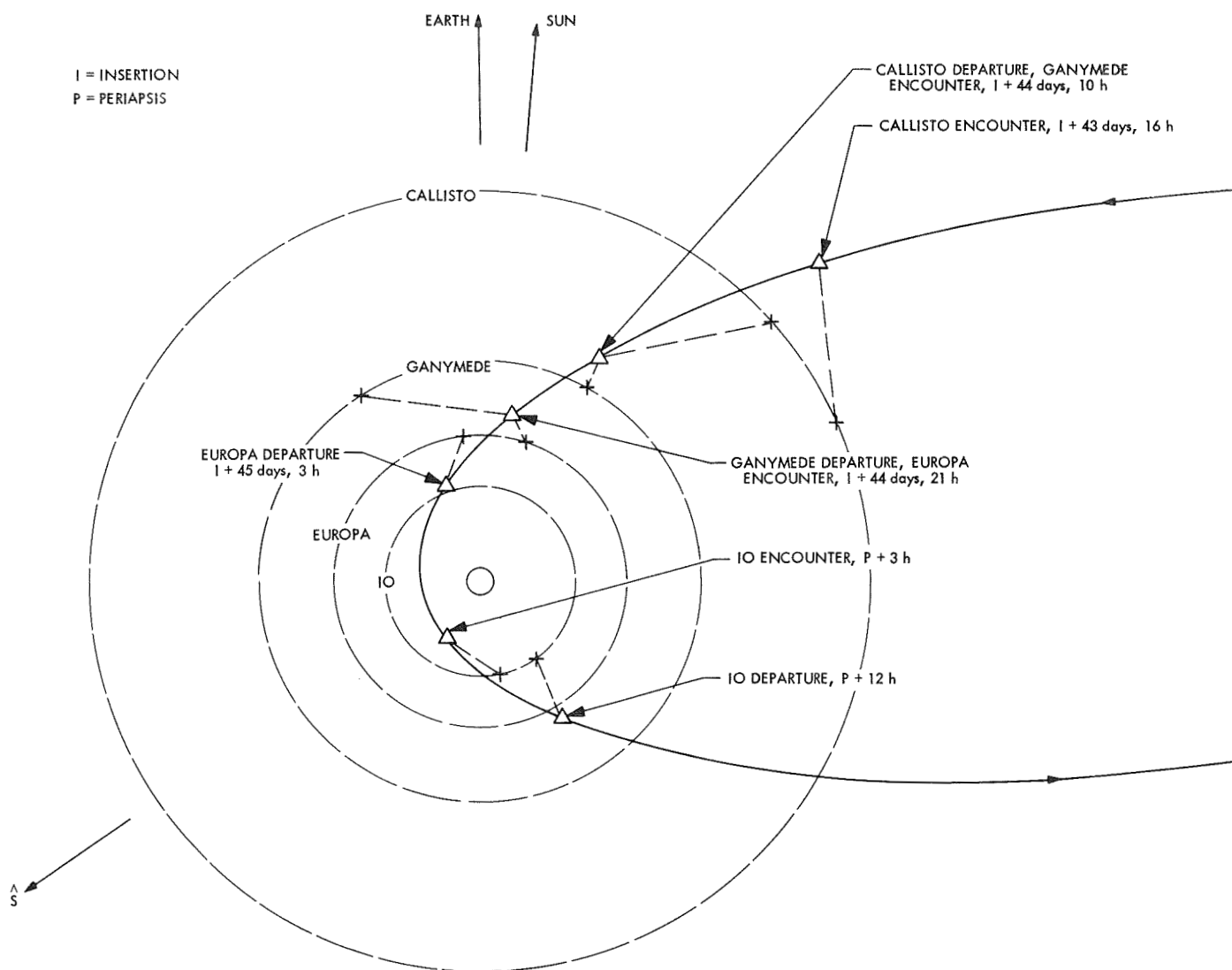


Fig. 4. Jupiter satellite encounter geometry

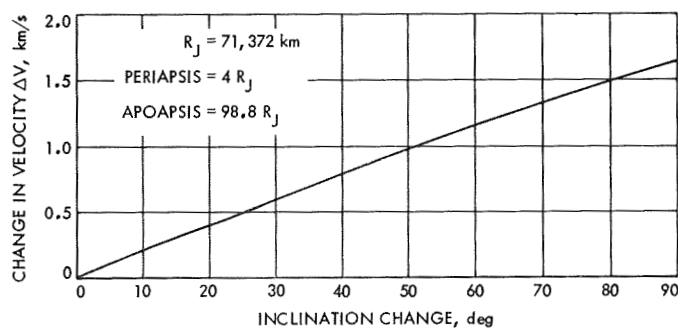


Fig. 5. Velocity change required for change in inclination

It is apparent (Fig. 6) that time in solar occultation will be increasing from its minimum at periapsis to some maximum value as the shadow's intersection of the orbit approaches apoapsis. (This effect decreases as the spacecraft's orbital plane becomes more inclined to the planet's orbital plane.)

Figure 7 is similar to Fig. 6 except that earth occultation is shown for the 3-yr active orbital life of the spacecraft. The reason for this precession is the same as that for the solar occultation just described.

Figure 8 is a composite graph showing the duration of solar, earth, and co-occultation (i.e., solar and earth occultation occurring simultaneously) as a function of time in orbit. The maximum time in occultation does not occur during the active life of the orbiter for this particular baseline orbit. However, attention should be focused on this area for any future designs to minimize the cooling problems that might result from extended periods in solar occultation. Extended periods of earth

occultation should also be avoided from a telecommunications standpoint. The occurrences of co-occultation are critical because the spacecraft must then operate in an inertial mode and rely upon the onboard gyros for guidance. The longer the duration of these periods the greater the inaccuracy in the trajectory (inertial mode accuracy being inversely proportional to time). The Jupiter orbiter provides a co-occultation period of 11.3 h (Fig. 8). This period does not seem extreme for the present study; however, for future studies, design requirements should be adjusted for this potential problem area.

5. Conclusion

A 1982 Jupiter orbiter mission is feasible within the constraints outlined (minimum launch energy, 15-day launch period, and minimum hyperbolic excess velocity V_∞ at encounter). Satellite encounter with the four major satellites is possible with the selected orbit. Imaging experiments as well as field and particle experiments are feasible.

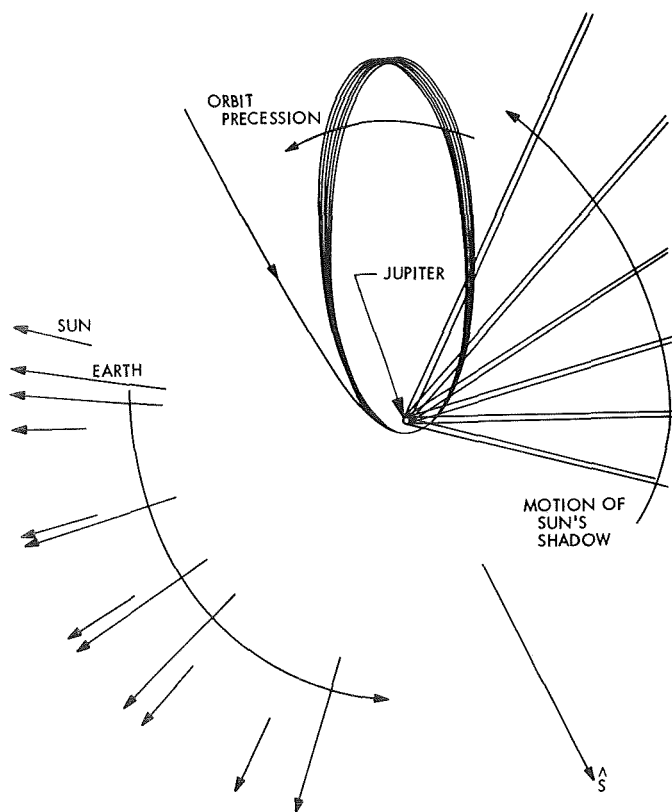


Fig. 6. Jupiter trajectory plane plot—solar occultation

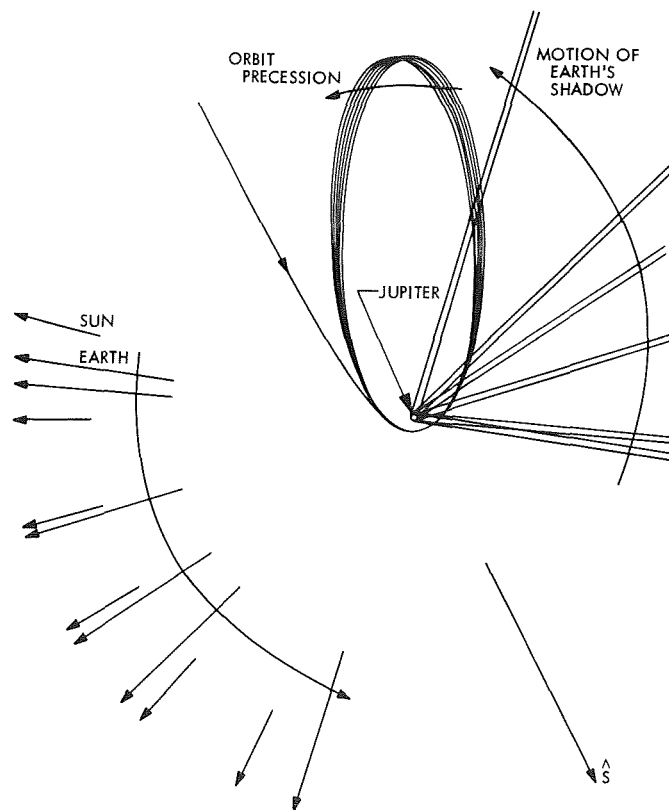


Fig. 7. Jupiter trajectory plane plot—earth occultation

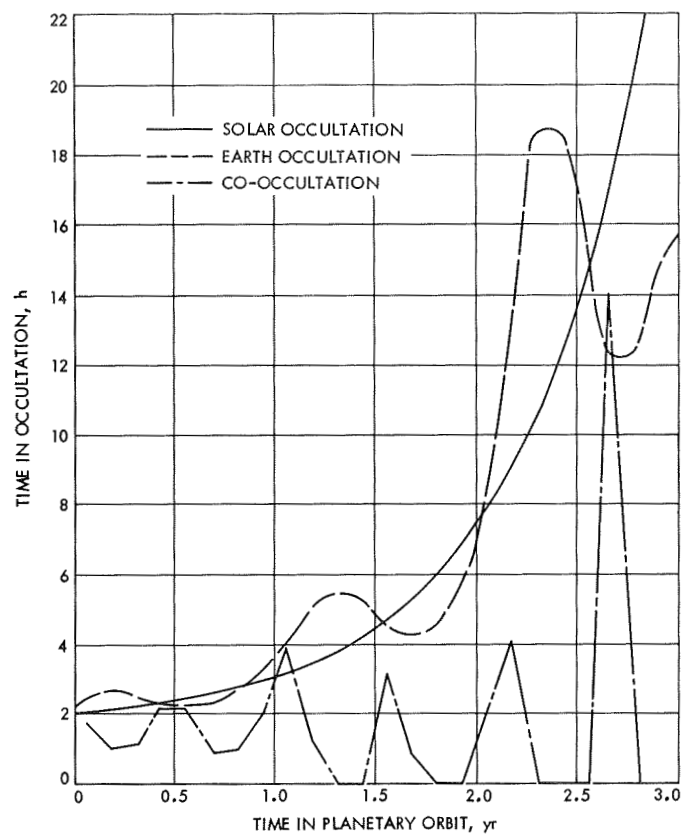


Fig. 8. Solar occultation time, earth occultation time, and co-occultation time vs time in orbit due to oblateness of planet

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